Unit 8, Lesson 4

Rational and Irrational Numbers

Let's learn about irrational numbers.

4.1 Math Talk: Positive Solutions

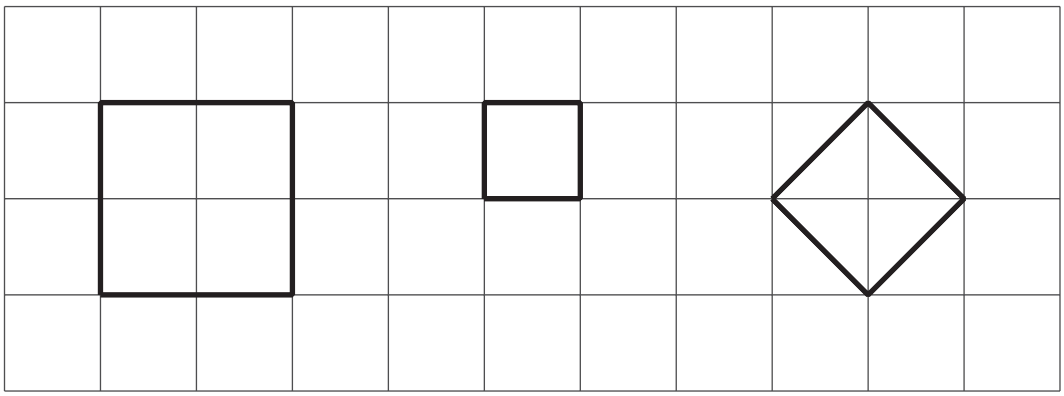
Solve each equation mentally.

- $x^2 = 36$
- $x^2 = \frac{9}{4}$
- $x^2 = \frac{1}{4}$
- $x^2 = \frac{49}{25}$

4.2 Three Squares

For each square:

1. Label the area.
2. Label the side length.
3. Write an equation that shows the relationship between the side length and the area.



4.3 Looking for a Solution

Are any of these numbers a solution to the equation $x^2 = 2$? Explain your reasoning.

- 1
- $\frac{1}{2}$
- $\frac{3}{2}$
- $\frac{7}{5}$

4.4 Looking for $\sqrt{2}$

A **rational number** is a number that can be expressed as a positive or negative fraction.

1. Find some more rational numbers that are close to $\sqrt{2}$.

2. Can you find a rational number that is exactly $\sqrt{2}$?

Are you ready for more?

If you have an older calculator and evaluate the expression $\left(\frac{577}{408}\right)^2$, it will tell you that the answer is 2, which might lead you to think that $\sqrt{2} = \frac{577}{408}$.

1. Explain why you might be suspicious of the calculator's result.

2. Find an explanation for why $408^2 \cdot 2$ could not possibly equal 577^2. How does this show that $\left(\frac{577}{408}\right)^2$ could not equal 2?

3. Repeat these questions for $\left(\frac{14142135623730}{10000000000000}\right)^2 \neq 2$, an equation that even many modern calculators and computers will get wrong.

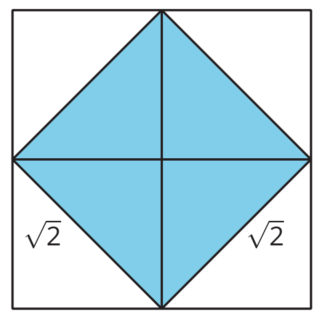
Lesson 4 Summary

A square whose area is 25 square units has a side length of $\sqrt{25}$ units, which means that $\sqrt{25} \cdot \sqrt{25} = 25$. Since $5 \cdot 5 = 25$, we know that $\sqrt{25} = 5$.

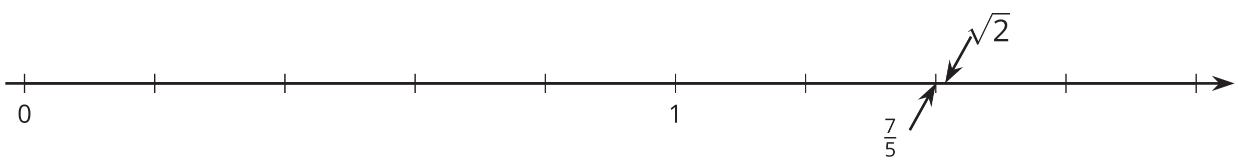
$\sqrt{25}$ is an example of a rational number. A **rational number** is a fraction or its opposite. In an earlier grade we learned that $\frac{a}{b}$ is a point on the number line found by dividing the interval from 0 to 1 into b equal parts and finding the point that is a of them to the right of 0. We can always write a fraction in the form $\frac{a}{b}$, where a and b are integers (and b is not 0), but there are other ways to write them. For example, we can write $\sqrt{25} = \frac{5}{1} = 5$ or $-\frac{1}{\sqrt{4}} = -\frac{1}{2}$. Because fractions and *ratios* are closely related ideas, fractions and their opposites are called *rational* numbers.

Here are some examples of rational numbers: $\frac{7}{4}, \ 0, \frac{6}{3}, 0.2, -\frac{1}{3}, -5, \sqrt{9}, \ -\frac{\sqrt{16}}{\sqrt{100}}$

Now consider a square whose area is 2 square units with a side length of $\sqrt{2}$ units. This means that $\sqrt{2} \cdot \sqrt{2} = 2$.



An **irrational number** is a number that is not rational, meaning it cannot be expressed as a positive or negative fraction. For example, $\sqrt{2}$ has a location on the number line (it's a tiny bit to the right of $\frac{7}{5}$), but its location can not be found by dividing the segment from 0 to 1 into b equal parts and going a of those parts away from 0.



$\frac{17}{12}$ is close to $\sqrt{2}$ because $\left(\frac{17}{12}\right)^2 = \frac{289}{144}$, which is very close to 2 since $\frac{288}{144} = 2$. We could keep looking forever for rational numbers that are solutions to $x^2 = 2$, and we would not find any since $\sqrt{2}$ is an irrational number.

The square root of any whole number is either a whole number, like $\sqrt{36} = 6$ or $\sqrt{64} = 8$, or an irrational number. Here are some examples of irrational numbers: $\sqrt{10}, \ -\sqrt{3}, \ \frac{\sqrt{5}}{2}, \ \pi$.