Unit 1, Lesson 16

Parallel Lines and the Angles in a Triangle

Goals

- Create diagrams using 180-degree rotations of triangles to justify (orally and in writing) that the measure of angles in a triangle sum up to 180 degrees.
- Generalize that the sum of angles in a triangle is 180 degrees using rigid transformations or the congruence of alternate interior angles of parallel lines cut by a transversal.

Learning Targets

- I can explain using pictures why the sum of the angles in any triangle is 180 degrees.

Lesson Narrative

In this lesson, students apply properties of rigid transformations to show that the sum of the interior angles of *any* triangle is 180°.

Students first use rigid transformations to explore the sum of the angles in a triangle drawn on a grid. They rotate the triangle so that angles corresponding to its three interior angles form a straight line. This shows that the three angles sum to 180°. However, this argument depends on the grid lines to show the three angles form a straight angle.

Then students complete the argument without using a grid. Instead, they use the structure of parallel lines cut by a transversal, corresponding parts of congruent triangles, and straight angles to generalize that the sum of the angles in any triangle is 180° (MP7).

In the optional activity, students have additional practice using the same structures to find angle measures in a composite figure.

Standards

Building On	**7.NS.A, 8.G.A.1.b**
Addressing	**8.G.A.5**
Building Toward	**8.G.B.6**

Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2

Student Facing Learning Goals

Let’s see why the angles in a triangle add to 180 degrees.

16.1 All the Angles

Warm-up

5 min

Activity Narrative

The purpose of this activity is for students to make sense of a diagram they will use in a later activity to justify that the sum of the interior angles of a triangle is 180 degrees.

Access for English Language Learners

This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of an image and practice generating mathematical questions.

Standards

Building On **7.NS.A**

Instructional Routines

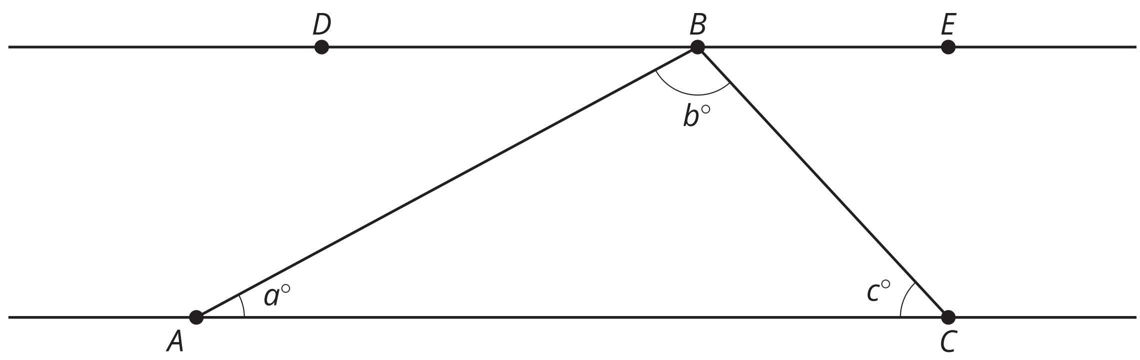
- MLR5: Co-Craft Questions

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2. Display the image of a triangle with two parallel lines for all to see. Use *Co-Craft Questions* to orient students to the image and elicit possible mathematical questions.
Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Student Task Statement

Here is triangle ABC. Line DE is parallel to line AC.



Student Response

Sample responses:

- What is the measure of angle DBA?
- What is the measure of angle EBC?

- What rigid transformation takes angle DBA to angle BAC?
- What is the sum of $a + b + c$?

Activity Synthesis

Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask, “What do these questions have in common? How are they different?” Listen for and amplify language related to the learning goal, such as using transformations to show the angle sum of triangles.

16.2 Angle Plus Two

15 min

Activity Narrative

The purpose of this activity is for students to use 180-degree rotations to create a diagram of 3 congruent triangles. As students create the diagram, they prepare informal arguments for congruent angles in the figure. Using these informal arguments, they find the sum of the interior angle measures of the original triangle.
A similar diagram will be used to generalize the sum of interior angles of a triangle in a later activity.

Standards

Building On **8.G.A.1.b**
Addressing **8.G.A.5**

Instructional Routines

- MLR8: Discussion Supports

Launch

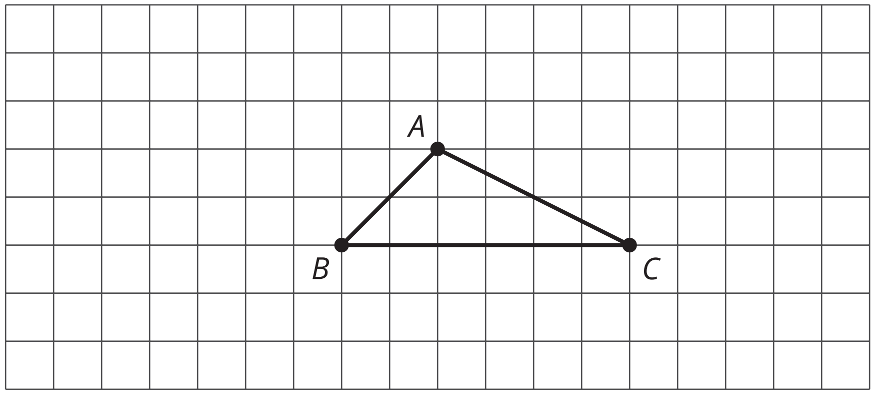
Arrange students in groups of 2–3. Provide access to geometry toolkits. Give 5 minutes individual work time, then 2–3 minutes to share their reasoning with their groups. Follow with a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present 2–3 questions at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization, Attention

Student Task Statement

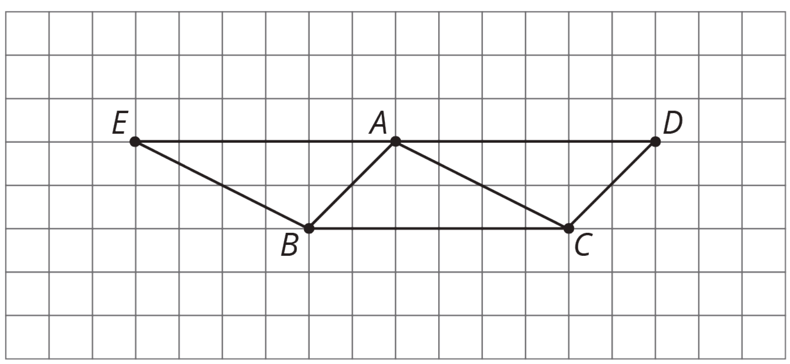
Here is triangle ABC.



1. Rotate triangle ABC 180° around the midpoint of side AC. Label the new vertex D.
2. Rotate triangle ABC 180° around the midpoint of side AB. Label the new vertex E.
3. Look at angles EAB, BAC, and CAD. Without measuring, write what you think is the sum of the measures of these angles. Explain or show your reasoning.
4. Is the measure of angle EAB equal to the measure of any angle in triangle ABC? If so, which one? Explain your reasoning.
5. Is the measure of angle CAD equal to the measure of any angle in triangle ABC? If so, which one? Explain your reasoning.
6. What is the sum of the measures of angles ABC, BAC, and ACB? Explain your reasoning.

Student Response

1.



2. See image
3. Sample response: They look like they will add to 180° because they appear to form a straight angle and there are 180° in a straight angle.
4. Yes, angle ABC. Sample reasoning: When triangle ABC is rotated 180 degrees with center the midpoint of segment AB, $\angle ABC$ goes to $\angle EAB$.
5. Yes, angle ACB. Sample reasoning: When triangle ABC is rotated 180 degrees with center the midpoint of segment AC, $\angle ACB$ goes to $\angle CAD$.
6. Sample response: The total measure of these angles should be 180° because it is the same as the total measure of angles EAB, BAC, and CAD and these angles add up to 180°.

Building on Student Thinking

Some students may have trouble with the rotations. If they struggle, remind them of similar work they did in a previous lesson. Help them with the first rotation, and allow them to do the second rotation on their own.

Activity Synthesis

The goal of this discussion is for students to explain how rigid transformations connect with the sum of interior angles of a triangle.
Here are some questions for discussion:

- "How can you tell that line DE is parallel to line BC?" (They were made using a 180-degree rotation, they are both on grid lines)
- "How can you tell that the 3 angles around point A make a straight angle?" (They all line up along a grid line, so they form a straight line.)
- If there were no grid lines, could we know that the three angles around point A would make a straight angle? (Since we used a 180-degree rotation of the same line segment but around 2 different points, EA and DA have to be parallel to each other. Since they share point A, they have to be on the same line.)
- How does this tell us about the sum of the angles for triangle ABC? (A straight angle is 180 degrees, and the three angles around point A that make a straight angle are congruent to the three angles in triangle ABC, so they must also total to 180 degrees.)

Students will have more time to make the connection about parallel lines and a similar figure without grid lines, so it is okay if they cannot explain why they create a straight angle without using grid lines at this time.

Access for English Language Learners

MLR8 Discussion Supports. During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: “I heard you say . . . .” Original speakers can agree or clarify for their partner.
Advances: Listening, Speaking

16.3 Every Triangle in the World

15 min

Activity Narrative

In this activity, students will continue to explore the interior angles of a triangle. The purpose of this activity is to provide a complete argument, not depending on the grid, of why the sum of the three angles in a triangle is 180°.

Students may use the structure of parallel lines cut with a transversal to identify alternate interior angles on the figure or use a 180-degree rotation about the midpoint of triangle side lengths to identify corresponding parts of the figure (MP7). In either case, this argument works for any triangle in general, since no angle measures are given or needed.

Standards

Addressing **8.G.A.5**

Launch

Arrange students in groups of 2–3. Tell students they'll be working on this activity without the geometry toolkit.

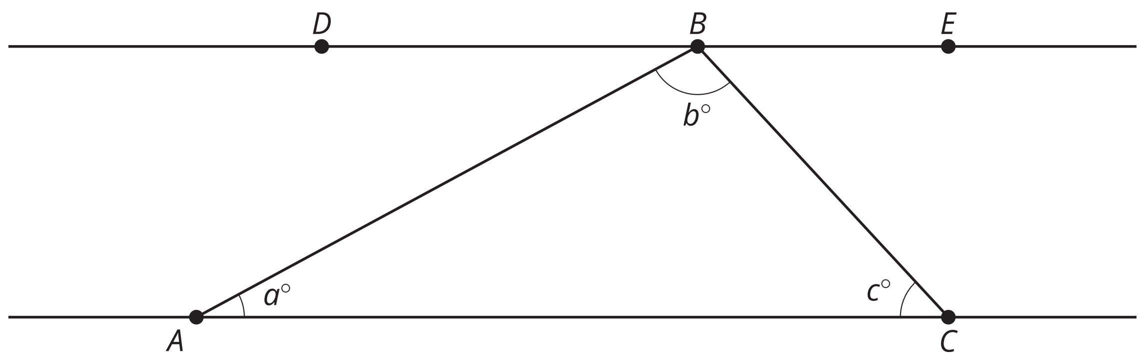
Tell students that this is the same image from the *Warm-up*. Display the question "What is the sum of the measures of angle DBA, angle ABC, and angle CBE?" Ask students how this question is similar and different to the questions they came up with. After students have shared the similarities and differences, give 1–2 minutes for groups to come up with a sum, then share with the whole class. Give another 3–5 minutes for groups to finish the rest of the questions.

Access for Students with Disabilities

*Action and Expression: Develop Expression and Communication.*Provide students with alternatives to writing on paper: students can share their learning orally or using a diagram.
Supports accessibility for: Language, Fine Motor Skills

Student Task Statement

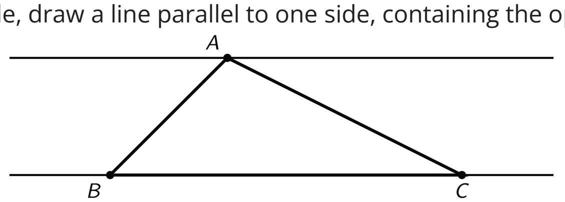
Here is triangle ABC. Line DE is parallel to line AC.



1. What is the sum of the measures of angle DBA, angle ABC, and angle CBE?
2. Use your answer to explain why $a + b + c = 180$.
3. Explain why your argument will work for any triangle: that is, explain why the sum of the angle measures in any triangle is 180°.

Student Response

1. 180°.
2. Sample response: Angles DBA and CAB are congruent because these are alternate interior angles for the parallel lines AC and DE with transversal AB. Angles EBC and BCA are congruent because these are alternate interior angles for the parallel lines AC and DE with transversal BC. Angles DBA, ABC, and CBE make a line, and so their angle measures add up to 180°. Then $a + b + c = 180$.
3. Sample response: For any triangle, draw a line parallel to one side, containing the opposite vertex.



With this picture, use the same argument to show that the sum of the three angles of the triangle is 180°. This works for *every* triangle.

Building on Student Thinking

Some students may say that a, b, and c are the three angles in a triangle, so they add up to 180. Make sure that these students understand that the goal of this activity is to explain why this must be true. Encourage them to use their answer to the first question and think about what they know about different angles in the diagram.

For the last question students may not understand why their work in the previous question only shows $a + b + c = 180$ for one particular triangle. Consider drawing a different triangle (without the parallel line to one of the bases), labeling the three angle measures a, b, c, and asking the student why $a + b + c = 180$ for *this* triangle.

Are You Ready for More?

1. Using a ruler, create a few quadrilaterals. Use a protractor to measure the four angles inside the quadrilateral. What is the sum of these four angle measures?
2. Come up with an explanation for why anything you notice must be true. (Hint: draw one diagonal in each quadrilateral.)

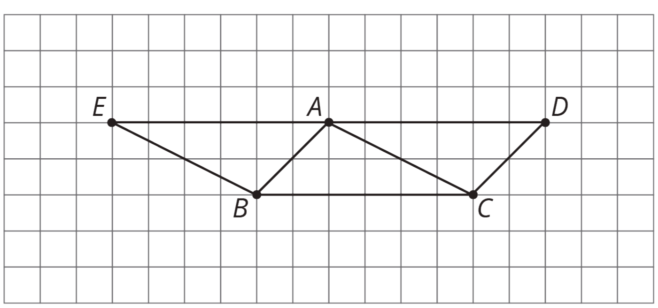
Extension Student Response

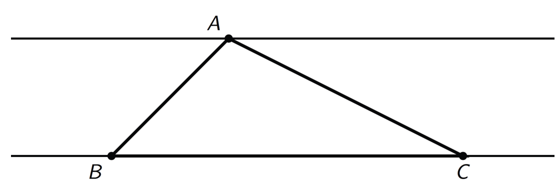
1. The sum of the interior angle measures in any quadrilateral is 360°. Note that since protractors are imprecise, physical measurements may range by a few degrees away from this.
2. In any quadrilateral, a diagonal that partitions the quadrilateral into 2 triangles can be drawn. The sum of the angle measures in each triangle is 180°, and the 6 angles in the two triangles comprise all of the angles in the quadrilateral.

Activity Synthesis

The goal of this activity is for students to explain that the sum of the interior angles of any triangle must be 180 degrees.

Display both of these images for all to see:





Ask students how they are similar and how they are different.

Similarities students may notice:

- Triangle ABC is in both images.
- Triangle ABC is between 2 parallel lines.
- The 3 angles around point A are a straight angle.

- The sum of the angles inside triangle ABC is 180 degrees.

Differences students may notice:

- There are grid lines on the first image but not the second.
- We can measure the height of the triangles in the first image.
- The first image has 3 triangles but the second only has 1 triangle.
- The first image has parallel segments but the second has parallel lines.

Tell students that the first image helps us show that the sum of the angles in that particular triangle is 180 degrees, but we can create a diagram like the second one for any triangle. That means we can know that the sum of the angles in any triangle is 180 degrees.

16.4 Four Triangles Revisited

Optional

5 min

Activity Narrative

This activity is optional because it provides additional opportunity for students to identify corresponding parts of figures using rigid transformations. This image will also be helpful in a later unit as students investigate the Pythagorean theorem.

In this activity, students use the structure of rigid transformations to find angle measures (MP7). Some strategies that students may use include:

- Rigid transformations take one triangle to a congruent triangle.
- Congruent triangles have congruent corresponding angles.
- The measures of the angles that make up a straight line add up to 180 degrees.
- Parallel lines cut by a transversal have congruent alternate interior angles.

Standards

Building On	**8.G.A.1.b**
Addressing	**8.G.A.5**
Building Toward	**8.G.B.6**

Instructional Routines

- MLR8: Discussion Supports

Launch

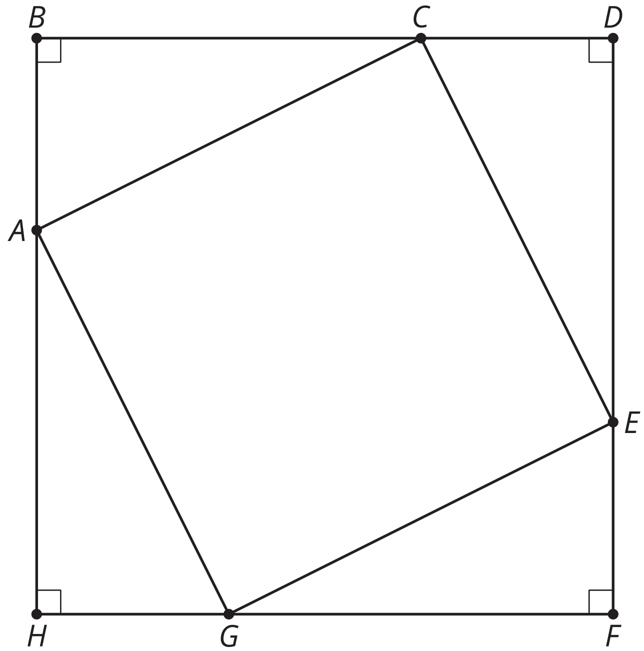
Provide 3 minutes of individual work time followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Maintain a display of important terms and information. Invite students to suggest language or diagrams to include that will support their understanding of transformations and congruence. Terms and properties may include the following: congruent triangles have congruent angles, a straight angle is equal to 180 degrees, and the sum of the angles in a triangle add up to 180 degrees.
Supports accessibility for: Conceptual Processing, Language

Student Task Statement

This diagram shows a square $BDFH$ that has been made by images of triangle ABC under rigid transformations.



Given that angle BAC measures 53°, find as many other angle measures as you can.

Student Response

All other angles can be determined from the one given measure.

Angles ECD, GEF, and AGH all measure 53° because the measure of $\angle BAC$ is 53°. These three angles correspond to $\angle BAC$ under a rigid motion of triangle ABC. To find angles ACB, CED, EGF, and GAH, notice that these are all congruent because they are all correspond to $\angle ACB$ of triangle ABC under a rigid motion. Angle ACB measures 37° because the angles in a triangle add to 180°. One angle in triangle ABC measures 53°, and another measures 90°, so the third angle measures $180^\circ - 53^\circ - 90^\circ = 37^\circ$.

Angles ACE, CEG, EGA, and GAC all measure 90°. Here is an argument for $\angle ACE$: We know that angle ACB measures 37° and angle ECD measures 53°. So angle ACE must measure 90° because it makes a line together with $\angle ACB$ and $\angle ECD$.

Activity Synthesis

Display the image of the 4 triangles for all to see. Invite students to share how they calculated one of the other unknown angles in the image, adding to the image until all the unknown angles are filled in.

If no student points it out, highlight that angles ACE, CEG, EGA, and GAC are all right angles.

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to help students produce statements about finding the unknown angles in the diagram: "Knowing ____ , helps me find____ because . . ." or "Angle _____ corresponds to angle _____ because . . . ."

Advances: Speaking, Conversing

Lesson Synthesis

Display a triangle for all to see. Invite students to explain step by step the process to explain that the sum of the angles in the triangle is 180 degrees.
Steps students may describe:

- Add a line through one vertex parallel to the opposite side of the triangle.
- The three angles on the new line form a straight angle, so they sum to 180 degrees.
- Since the lines are parallel, we can use alternate interior angles to show that one angle on the line is congruent to one angle in the triangle, and another angle on the line is congruent to the other angle in the triangle.
- Since the three angles in the triangle are congruent to the three angles that make a straight angle, the angles in the triangle must add up to 180 degrees.

Consider adding the illustration to the display for the unit with the statement: "The sum of the angles in a triangle is 180 degrees."

16.5 Angle Sum

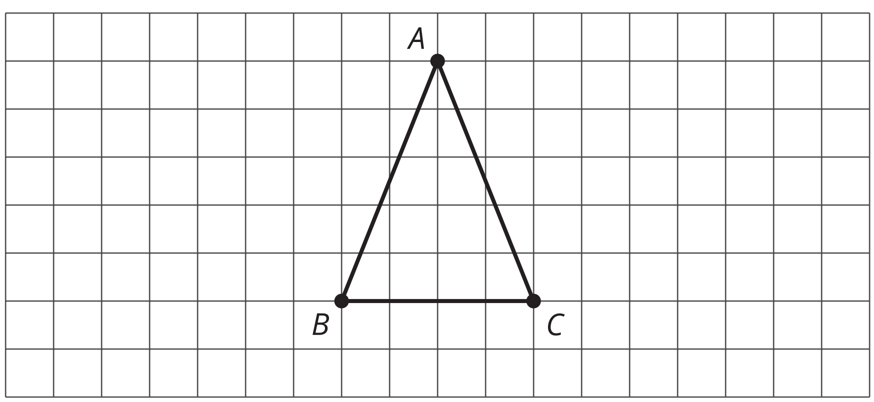
Cool-down

5 min

Standards

Addressing **8.G.A.5**

Student Task Statement



What is the sum of the angle measures of triangle ABC? How do you know?

Student Response

180°. Sample response: Since the base and vertex lie on grid lines, we can see the line parallel to BC through A is a straight line. The three angles around point A add up to a straight angle. Using alternate interior angles, two angles are congruent to angle B and angle C, and the third angle is the same as angle A. So angles A, B, and C add up to 180°.

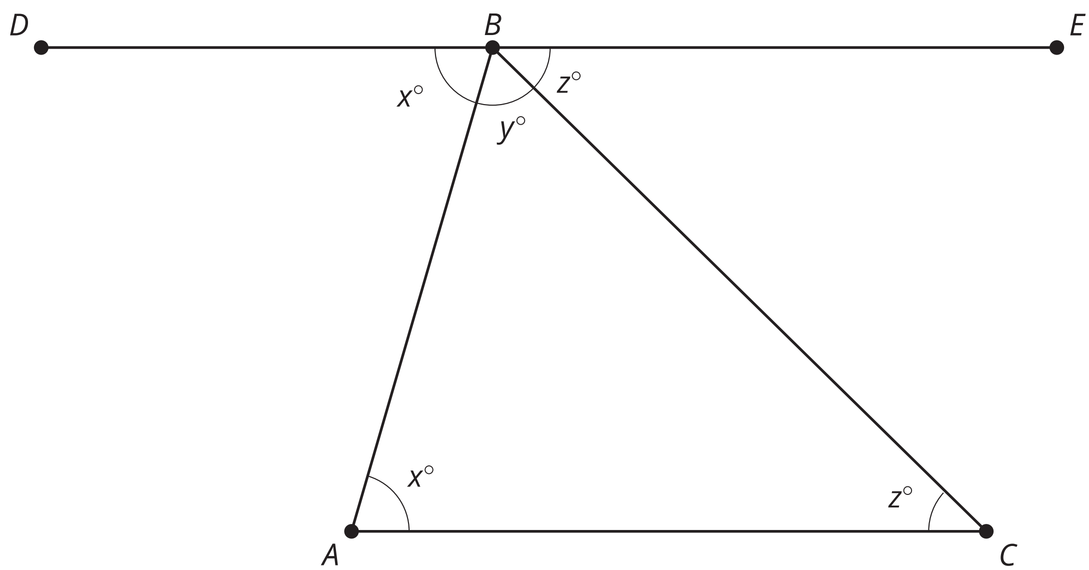
Responding to Student Thinking

Press Pause
If students struggle with the reasoning about the sum of interior angles in triangles, revisit the activity referred to here to emphasize the argument for why the sum of the three angles in a triangle is always 180 degrees.

Grade 8, Unit 1, Lesson 16, Activity 3 Every Triangle in the World

Lesson 16 Summary

Using parallel lines and rotations, we can understand why the angles in a triangle always add to 180°. Here is triangle ABC. Line DE is parallel to AC and contains B.



A 180° rotation of triangle ABC around the midpoint of AB interchanges angles A and DBA so they have the same measure (in the picture these angles are marked as x°).

A 180° rotation of triangle ABC around the midpoint of BC interchanges angles C and CBE so they have the same measure (in the picture, these angles are marked as z°).

Also, DBE is a straight line because 180° rotations take lines to parallel lines.

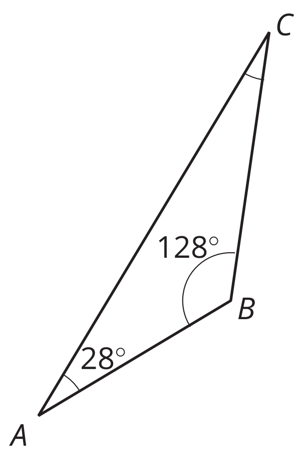
So the three angles with vertex B make a line and they add up to 180° ($x + y + z = 180$). But x, y, z are the measures of the three angles in triangle ABC so the sum of the angles in a triangle is always 180°!

Lesson 16 Practice Problems

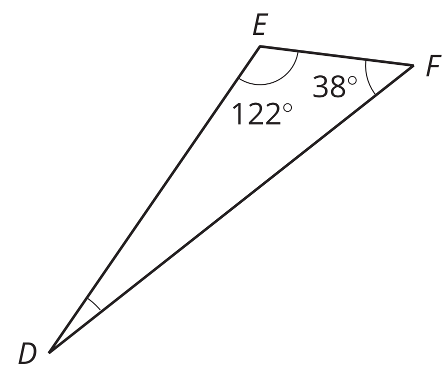
1 Student Task Statement

For each triangle, find the measure of the missing angle.

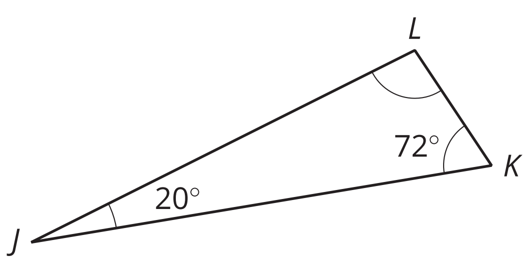
a.



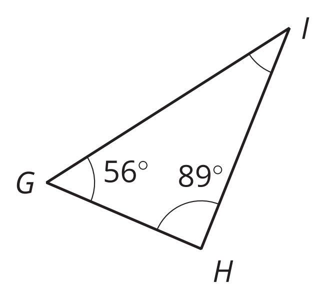
b.



c.



d.



Solution

a. 24°

b. 20°

c. 88°

d. 35°

2 Student Task Statement

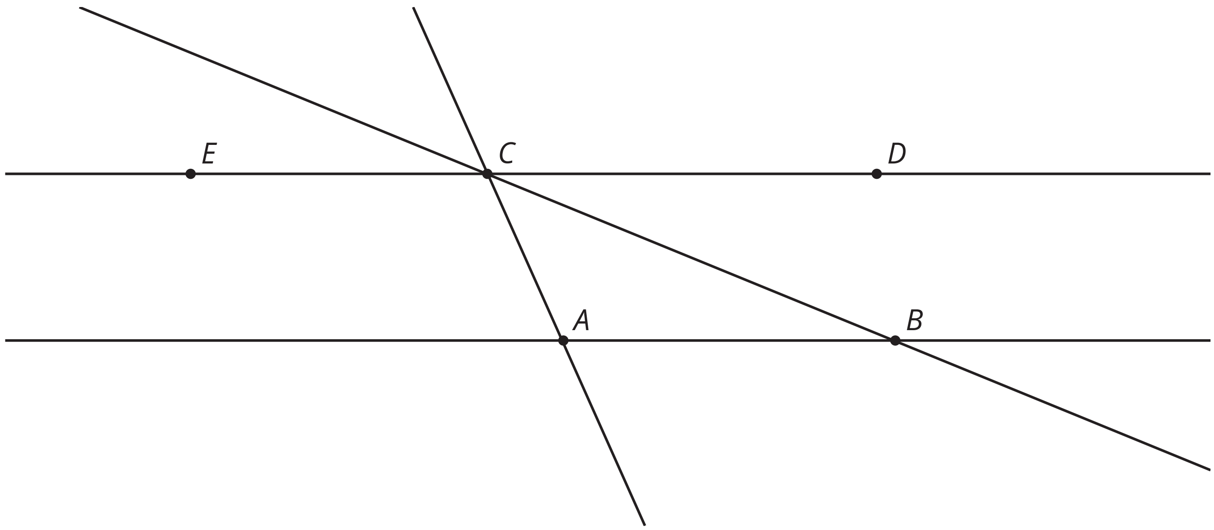
Is there a triangle with *two* right angles? Explain your reasoning.

Solution

No. Sample reasoning: The three angles in a triangle add up to 180°. Two right angles would already make 180°, and so the third angle of the triangle would have to be 0°, which is not possible.

3 Student Task Statement

In this diagram, lines AB and CD are parallel.



Angle ABC measures 35° and angle BAC measures 115°.

a. What is the measure of angle ACE?

b. What is the measure of angle DCB?

c. What is the measure of angle ACB?

Solution

a. 115°

b. 35°

c. 30°

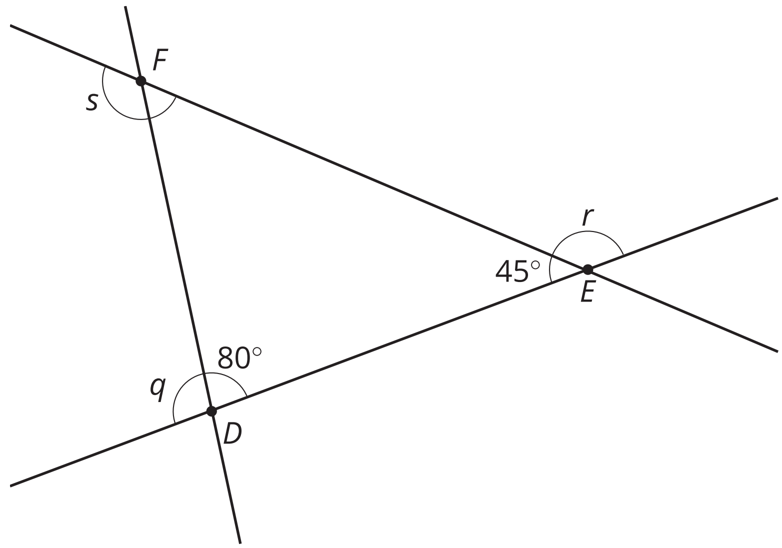
4 Student Task Statement

Here is a diagram of triangle DEF.

a. Find the measures of angles q, r, and s.

b. Find the sum of the measures of angles q, r, and s.

c. What do you notice about these three angles?



Solution

a. $q = 100$, $r = 135$, $s = 125$

b. $q + r + s = 360$

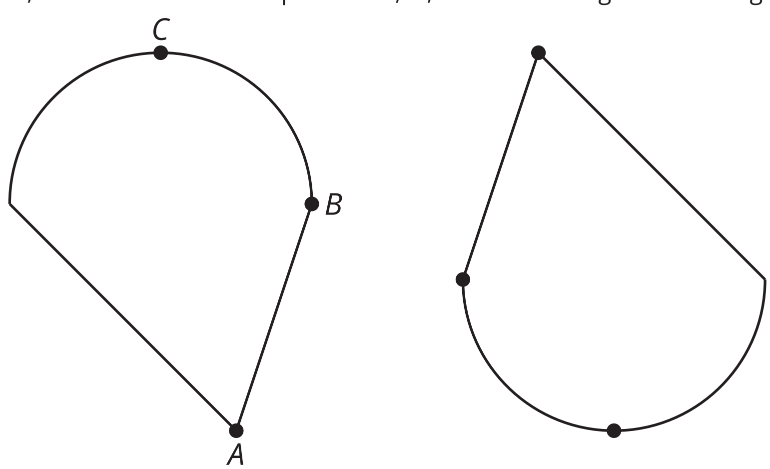
c. Sample response: The three angles together make one full revolution of a circle, or 360 degrees.

5 from Unit 1, Lesson 13

Student Task Statement

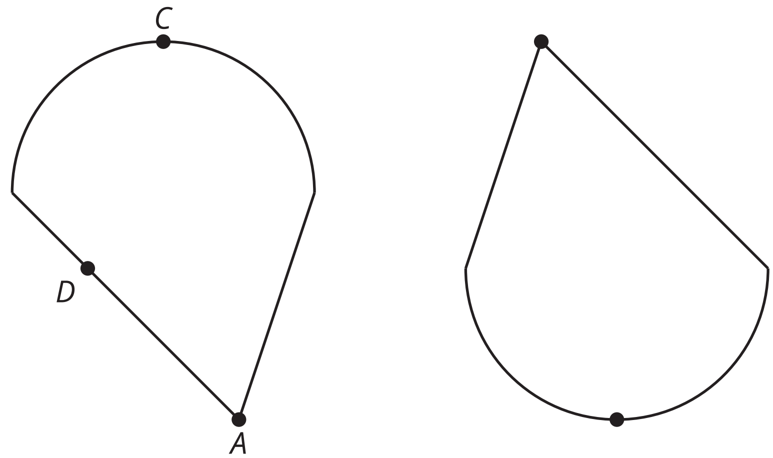
The two figures are congruent.

a. Label the points A', B' and C' that correspond to A, B, and C in the figure on the right.



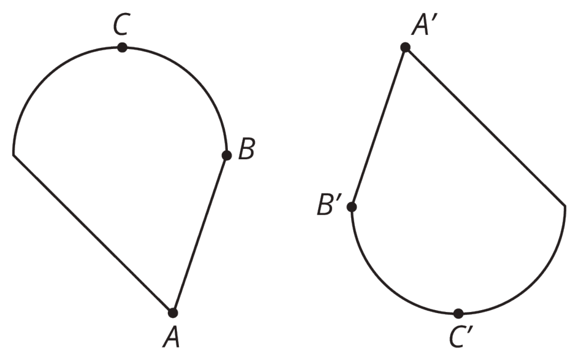
b. If segment AB measures 2 cm, how long is segment $A'B'$? Explain.

c. The point D is shown in addition to A and C. How can you find the point D' that corresponds to D? Explain your reasoning.



Solution

a.



b. 2 cm. Sample reasoning: The shapes are congruent, and the corresponding segments of congruent shapes are congruent.

c. Sample reasoning: As the figures are congruent, the point D' will be on the corresponding side and will be the same distance from C' that D is from C. D can be found by looking for the point on the segment going down and to the right from A' that is the appropriate distance from C'.