Unit 5, Lesson 7

Using Diagrams to Represent Multiplication

Goals

- Calculate the product of two decimals by decomposing each factor by place value and finding partial products.
- Coordinate area diagrams and vertical calculations that represent the same decimal multiplication problem.
- Use an area diagram to represent and justify (orally and in writing) how to find the product of two decimals.

Learning Targets

- I can use area diagrams and partial products to represent and find products of decimals.

Lesson Narrative

In this lesson, students build on two previously learned methods for multiplying whole numbers—using an area diagram and an algorithm—to find products of decimals.

In earlier grades, students learned to represent two factors as side lengths of a rectangle, as well as to decompose multi-digit factors by place value to facilitate multiplication. For instance, in an 18 by 23 rectangle, the 18-unit side can be decomposed into 10 and 8 units, and the 23-unit side can be expressed as 20 and 3, creating four sub-rectangles whose areas constitute four partial products. The sum of these partial products is the product of 18 and 23. Students also learned to organize these partial products in a vertical calculation, which enabled them to multiply whole numbers without relying on a diagram. The first optional activity in the lesson allows students to revisit these ideas.

In the main activity, students extend both methods of reasoning to represent and find products such as $(1.8) \cdot (2.3)$. They see that decimal factors can likewise be decomposed by place value, the parts can be multiplied separately, and the results can be combined at the end. A second optional activity gives students additional practice with this line of reasoning.

In making connections—between multiplication of decimals and multiplication of whole numbers, between diagrams and calculations—students practice looking for and making use of structure (MP7).

Standards

Building On	**4.NBT.B.5, 5.NBT.B.7**
Addressing	**6.NS.B.3**

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Graph paper: Activity 3

Required Preparation

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Activity 3:

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's use area diagrams to find products.

7.1 Estimate the Product

Warm-up

5 min

Activity Narrative

In this *Warm-up*, students review multiplication of decimals and estimate the size of a product given the size of the decimal factors. They explain why their estimates are reasonable based on their understanding of place value and multiplication.

To make reasonable estimates, students need to look for and make use of structure (MP7). In explaining why their choice is the best estimate, students need to be precise in their word choice and use of language (MP6).

Standards

Building On **5.NBT.B.7**

Launch

Tell students to close their books or devices (or to keep them closed). Explain that they will see four multiplication expressions, displayed one at a time. For each expression, there will be three possible estimates of its value. Their job is to select the best estimate and to be able to explain why it is the best.

Reveal one expression at a time. For each expression:

- Give students 30 seconds of quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Ask others if they agree or disagree, and if they would like to offer alternative explanations.

Student Task Statement

For each multiplication expression, choose the best estimate of its value. Be prepared to explain your reasoning.

1. $(6.8) \cdot (2.3)$
    - 1.40
    - 14
    - 140
2. $74 \cdot (8.1)$
    - 5.6
    - 56

- 560

3. $166 \cdot (0.09)$

    - 1.66
    - 16.6
    - 166

4. $(3.4) \cdot (1.9)$

    - 6.5
    - 65
    - 650

Student Response

1. 14. Sample response: Round 6.8 to 7 and 2.3 to 2, and multiply. $7 \cdot 2 = 14$
2. 560. Sample response: Round 74 to 70 and 8.1 to 8, and multiply. $70 \cdot 8 = 560$
3. 16.6. Sample response: Round 0.09 to 0.1, and multiply. $166 \cdot (0.1) = 16.6$
4. 6.5. Sample response: Round 3.4 to 3 and 1.9 to 2, and multiply. $3 \cdot 2 = 6$, and 6.5 is closest to 6.

Building on Student Thinking

Students who know how to perform multiplication computation and use a "count the number of decimal places'' strategy might mix that method with estimation in placing the decimal point. For example, to estimate $74 \cdot (8.1)$, they might round the factors to 70 and 8 and find a product of 560. Seeing that there is a total of 1 place after the decimal point in the original factors, they place the decimal point to the left of the last digit and choose 56 as their answer. Prompt students to think about the reasonableness of their answer relative to the factors (for instance, ask if 56 is a reasonable product of 70 and 8).

Activity Synthesis

Focus the discussion on how the given factors can inform our estimate of each product. Emphasize that even if we were to calculate the products precisely, we can use estimation and our understanding of place value to check if our answers make sense.

7.2 Connecting Area Diagrams to Calculations with Whole Numbers

20 min

Optional

Activity Narrative

There is a digital version of this activity.

In this activity, students review two numerical methods for multiplication and use them to calculate products of two whole numbers. Students have had a chance to revisit area diagrams in the past couple of activities. Here they make explicit connections between numerical calculations and area diagrams. As in previous activities, place value plays a key role in how a rectangle can be partitioned to facilitate multiplication.

As students discuss and work, listen for the ways they describe the correspondences between numerical calculations and the numbers in the area diagrams. Identify students who can articulate their reasoning so they can share later.

In the digital version of the activity, students can use an applet to represent multiplication of two numbers with an area diagram and to check their calculations. The applet allows students to specify two-digit factors by place value, represent the tens and ones as side lengths of a rectangle, and see the area of the rectangle decomposed accordingly. It also displays the corresponding partial products and final product. The digital version may be helpful for students to verify their work and to better visualize the structure of multiplication by place value.

If students don't have individual access, displaying the applet for all to see would be helpful during the synthesis.

Standards

Building On **4.NBT.B.5**

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give groups 2–3 minutes to discuss the first set of questions. Pause for a brief whole-class discussion. Select a few students to share their responses.

Give groups another 2–3 minutes to discuss the second set of questions. Pause for another whole-class discussion. Invite groups to share their responses.

Highlight the relationship between the blue numbers in the calculations and the partial areas in the diagram. Discuss questions such as:

- "The numbers in blue—in the calculations and area diagrams—are called "partial products." Why might that be?" (They represent products of parts of the factors. They are parts of the final product.)
- "How are the partial products in Calculation A different from those in Calculation B?" (Calculation A lists the product of any two base-ten digits separately. Calculation B groups them.) "How are they similar?" (They both lead to the same final product.)
- "In Calculation B, what calculation would give 72?" (Multiplying 24 by 3) "What calculation would give 240?" (Multiplying 24 by 10.) "What about the 312?" (Adding 72 and 240.)

Next, give students a few minutes of quiet time to complete the remaining questions.

Access for English Language Learners

MLR8 Discussion Supports. Before students share with the whole class, provide them with the opportunity to rehearse with a partner what they will say.
Advances: Speaking Action and Expression: Provide Access for Physical Action. Support effective and efficient use of tools and assistive technologies. To use the digital applet, some students may benefit from a demonstration on the use of the sliders or access to step-by-step instructions.
Supports accessibility for: Organization, Memory, Attention

Student Task Statement

1. Here are three ways of finding the area of a rectangle that is 24 units by 13 units.

Diagram 1

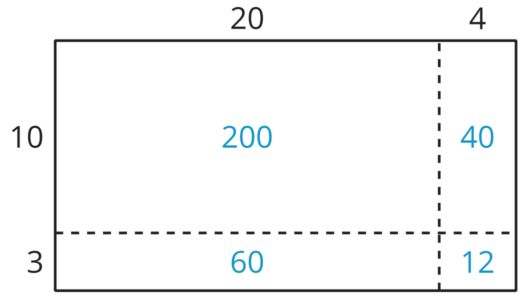


Diagram 2

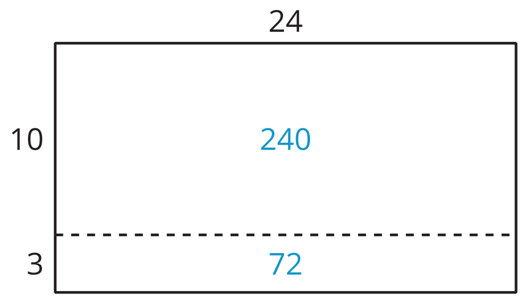
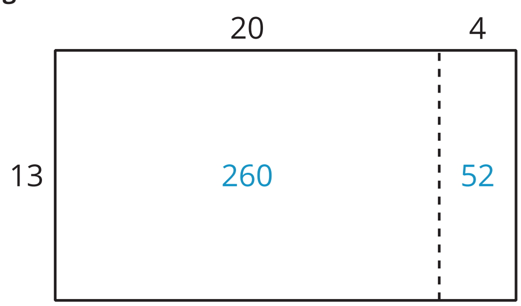


Diagram 3



Discuss with your partner:

a. How are the diagrams the same?

b. How are the diagrams different?

c. If you were to find the area of a rectangle that is 37 units by 19 units, which of the three ways of decomposing the rectangle would you use? Why?

2. Here are two ways to calculate 24 times 13.

$$\begin{array}{r} 24 \\ \times\quad 13 \\ \hline 12 \\ 60 \\ 40 \\ +\quad 200 \\ \hline 312 \end{array}\quad \text{partial products}$$

Calculation A

$$\begin{array}{r} 24 \\ \times\quad 13 \\ \hline 72 \\ +\quad 240 \\ \hline 312 \end{array}$$

Calculation B

Discuss with your partner:

a. In Calculation A, where does each partial product—the 12, 60, 40, and 200—come from?

b. In Calculation B, where do 72 and 240 come from?

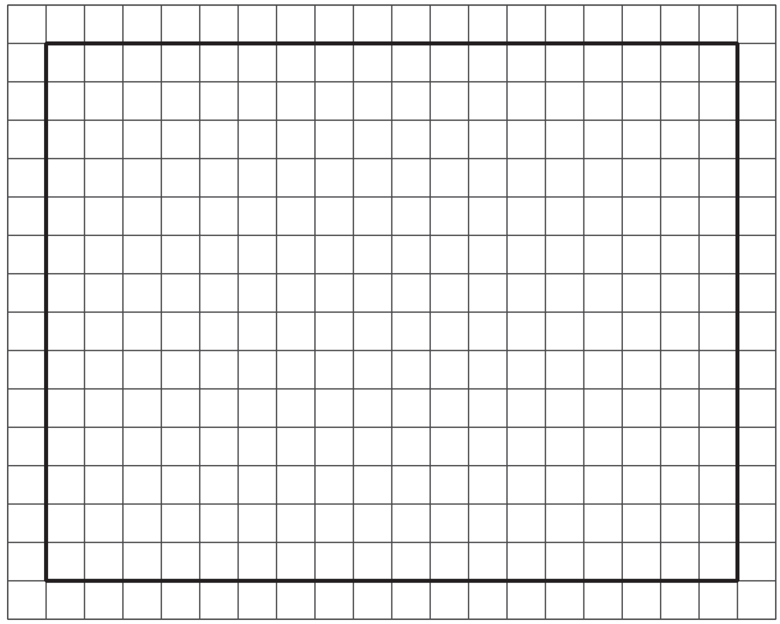
c. Which diagram in the first question corresponds to Calculation A? Which one corresponds to Calculation B? How do you know?

3. Find the product of 18 and 14 in two ways:

   a. Calculate numerically.

$$\begin{array}{r} 18 \\ \times\ 14 \\ \hline \end{array}$$

   b. Find the area, in square units, of this 18-by-14 rectangle. Show your reasoning.



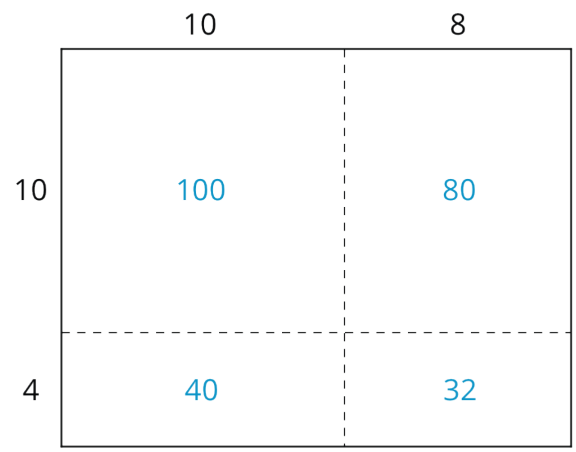
Student Response

1. Sample responses:
   a. Similarities: They show the same rectangle with the same area. The sum of all the partial products is the same in all cases.
   b. Differences: Some side lengths are broken up by place value, others are not. Some have two sub-rectangles, others have four sub-rectangles.
   c. I would use the method as in Diagram 1. I would break both numbers into tens and ones because it is easier to multiply tens by tens and ones by ones. If I broke up only one number into tens and ones; the other number is still not friendly enough to multiply quickly.
2. Sample responses:
   a. In Calculation A, 12 is $3 \cdot 4$, 60 is $3 \cdot 20$, 40 is $10 \cdot 4$, and 200 is $10 \cdot 20$.
   b. In Calculation B, 72 is $12 + 60$, and 240 is $40 + 200$. We can also see 72 as $3 \cdot 24$ and 240 as $10 \cdot 24$.
   c. Calculation A corresponds to Diagram 1. Calculation B corresponds to Diagram 2. Sample reasoning: The areas of the four sub-rectangles in Diagram 1 (12, 60, 40, 200) are the four partial products in Calculation A. Diagram 2 shows 2 sub-rectangles; their areas are the partial products of 72 and 240. The area of the bottom rectangle is the sum of $12 + 60$, and the area of the top rectangle is the sum of $40 + 200$.

3. a.

$$\begin{array}{rr} & 18 \\ \times & 14 \\ \hline & 72 \\ + & 18 \\ \hline & 252 \end{array} \qquad \begin{array}{rr} & 18 \\ \times & 14 \\ \hline & 72 \\ + & 180 \\ \hline & 252 \end{array}$$

b.



100 + 80 + 40 + 32 = 252

Building on Student Thinking

If the products students find by numerical calculation and by using a diagram don't match, urge students to examine the partial products and their sum in both the calculation and the diagram.

Activity Synthesis

Focus the discussion on the connections between the numbers in Calculation B and the multiplication algorithm students learned in grade 5.

Display the following computation of $24 \cdot 13$, done using the standard algorithm:

$$\begin{array}{rr} & 24 \\ \times & 13 \\ \hline & 72 \\ + & 24 \\ \hline & 312 \end{array}$$

Remind students that they have used this method to find products of two-digit numbers in an earlier grade. Ask students:

- "How is this method like and unlike Calculation B?" (The 0 for 240 is not written but the 2 and 4 are in the hundreds and tens place, just as in Calculation B.)
- "Why might it be fine to write "24" without the 0 at the end?" (The 24 is understood to mean "24 tens" because it is written with 4 in the tens place.)

- “What would the calculation for $18 \cdot 14$ look like if we use this algorithm?” (We’ll see “10” as a partial product, but it represents 100. The 1 will be in the hundreds place and the 0 in the tens place.)

Consider using the applet to represent $18 \cdot 14$ and, if time allows, exploring other multiplications of other pairs of numbers.

The Geogebra applet 'Connecting Area Diagrams to Calculations with Whole Numbers' is available here: https://www.geogebra.org/m/K9B6Eg4H.

7.3 Connecting Area Diagrams to Calculations with Decimals

20 min

Activity Narrative

There is a digital version of this activity.

In this activity, students find the product of two decimals using an area diagram, and they analyze two ways to multiply the same pair of decimals numerically, making connections between the strategies along the way. Then they apply the strategies to multiply another pair of decimals.

The key ideas here are:

- We can find products of decimals by decomposing the factors by place value, finding partial products, and adding them, just as we have done when multiplying whole numbers.
- We can represent the multiplication and the partial products using a diagram or with numerical calculations.

In an upcoming lesson, students will generalize the process and use the algorithm to compute products of decimals.

In the digital version of the activity, students can use an applet to represent multiplication of two decimals with an area diagram and to check their calculations. The applet allows students to specify the factors by place value, represent the ones and tenths as side lengths of a rectangle, and see the area of the rectangle decomposed accordingly. It also displays the corresponding partial products and final product. The digital version may be helpful for students to verify their work and to better visualize the structure of multiplication by place value.

If students don't have individual access, displaying the applet for all to see would be helpful during the synthesis.

Standards

Addressing **6.NS.B.3**

Launch

Arrange students in groups of 2. Give students 1–2 minutes of quiet work time for the first problem. Pause for a brief whole-class discussion, making sure that all students label each region correctly.

Then give partners 2–3 minutes to analyze the given calculations and discuss the questions. Monitor student discussions to check for understanding. If necessary, pause to have a whole-class discussion on the interpretation of these calculations.

Next, give students 5–6 minutes of quiet time to complete the remaining questions. Tell students that their diagram need not be drawn exactly and that it is fine to estimate appropriate side lengths, but that the labels should reflect the numbers being multiplied.

Some students might find it helpful to use a grid to align the digits in vertical calculations. Provide access to graph paper.

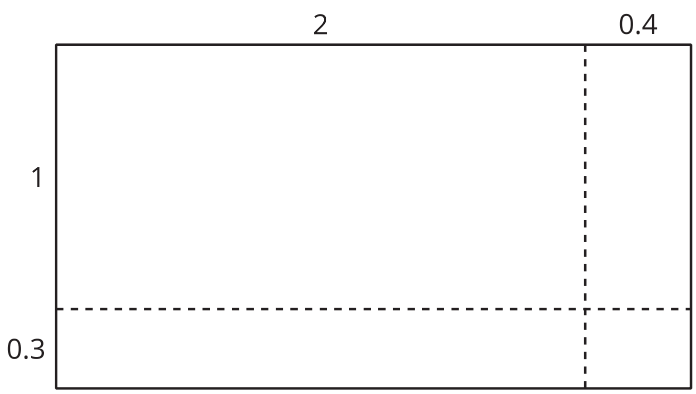
If access to digital devices is available, consider allowing students to use the applet to check their calculations and to explore the products of other decimals.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color coding and annotations to highlight connections between representations. For example, use color coding to highlight connections between the partial products in the calculations and in the area diagrams.
Supports accessibility for: Visual-Spatial Processing

Student Task Statement

1. Here is an area diagram that represents $(2.4) \cdot (1.3)$.



   a. Find the region that represents $(0.4) \cdot (0.3)$. Label it with its area of 0.12.

   b. Label the other regions with their areas.

   c. Find the value of $(2.4) \cdot (1.3)$. Show your reasoning.

2. Here are two ways of calculating $(2.4) \cdot (1.3)$.

$$\begin{array}{r} 2.4 \\ \times \quad 1.3 \\ \hline 0.12 \\ 0.6 \\ 0.4 \\ + \quad 2 \\ \hline 3.12 \end{array} \left.\vphantom{\begin{array}{c}0\\0\\0\\0\end{array}}\right\} \text{partial products}$$

Calculation A

$$\begin{array}{r} 2.4 \\ \times \quad 1.3 \\ \hline 0.72 \\ + \quad 2.4 \\ \hline 3.12 \end{array}$$

Calculation B

   Analyze the calculations and discuss these questions with a partner:

   a. In Calculation A, where do the 0.12 and other partial products come from?

   b. In Calculation B, where do the 0.72 and 2.4 come from?

   c. In each calculation, why are the numbers below the horizontal line aligned vertically the way they are?

3. Find the value of $(3.1) \cdot (1.5)$ in two ways:

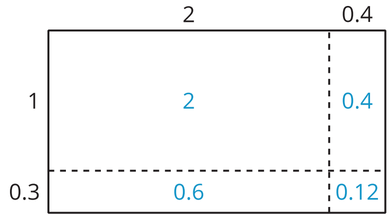
   a. Draw and label a diagram. Show your reasoning.

   b. Calculate numerically, without using a diagram. Be prepared to explain your reasoning.

Student Response

1. a. See labeled diagram.

   b.



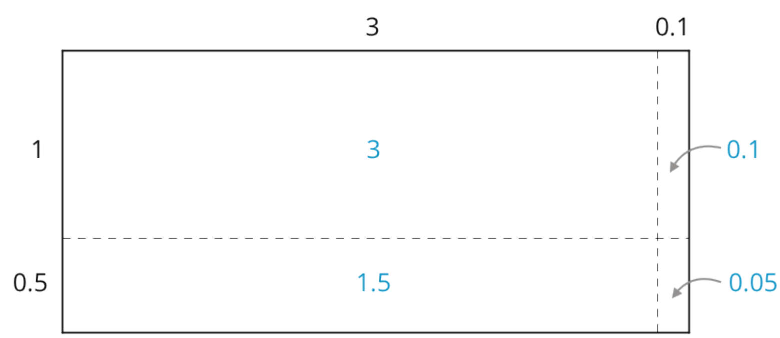
   c. The area of the rectangle is the sum of the sub-rectangles, which is 3.12: $2 + 0.4 + 0.6 + 0.12 = 3.12$.

2. a. In Calculation A:
   - $0.12 = (0.4) \cdot (0.3)$
   - $0.6 = 2 \cdot (0.3)$
   - $0.4 = (0.4) \cdot 1$
   - $2 = 2 \cdot 1$

   b. In Calculation B: $0.72 = (0.3) \cdot (2.4)$ and $2.4 = 1 \cdot (2.4)$.

   c. Sample responses:
   - The numbers are lined up so that digits in the same place value are aligned (for example, the 1 in 0.12, the 6 in 0.6, and the 4 in 0.4 are all in the tenths place).
   - The numbers are lined up this way so that like base-ten units can be easily added.

3. a. The sum of the sub-rectangles is 4.65, so that is the value of the product $(3.1) \cdot (1.5)$.
   $3 + 0.1 + 1.5 + 0.05 = 4.65$.



   b.

$$
\begin{array}{r}
3.1 \\
\times \quad 1.5 \\
\hline
0.05 \\
1.5 \\
0.1 \\
+ \quad 3 \\
\hline
4.65
\end{array}
$$

Building on Student Thinking

When using vertical calculations, students might find the correct partial products but not align them by place value (for example, they might align the rightmost digit of all partial products), resulting in an incorrect sum. Ask them what values the digits in each partial product represent and to consider how they should be added.

Are You Ready for More?

Zhang (JAHNG), or "Chinese yard," and *li* (LEE), or "Chinese mile," are two units of length used in China.

1. If 1 li is equal to 150 zhangs, and 1 zhang is approximately 3.645 yards (as used in the United States), about how many yards are in 1 li?
2. There are 1,760 yards in 1 mile (as used in the United States). Estimate how many lis are in 1 mile. Explain your reasoning.

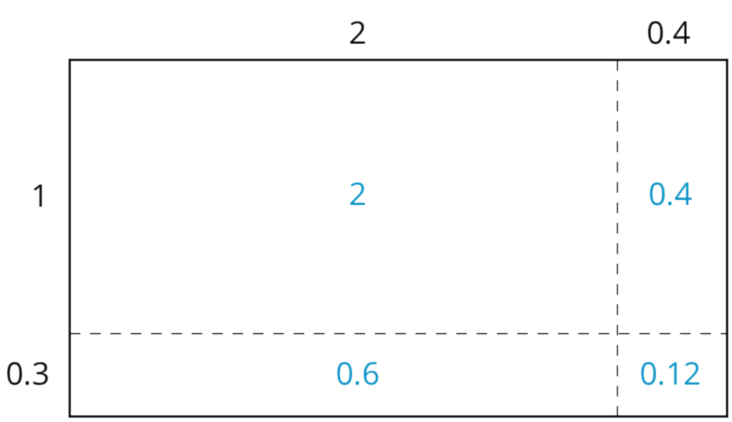
Extension Student Response

1. About 546.75 yards, because $150 \cdot (3.645) = 546.75$
2. There are a little more than 3.2 lis in 1 mile. Sample reasoning: I rounded 547.75 yards to 550 yards as an estimate for 1 li. Multiplying 550 by 3.2 gives 1,760.

Activity Synthesis

The goal of the discussion is to highlight the connections between an area diagram, which can help us visualize partial products, and numerical calculations, which can help us record partial products concisely and calculate more efficiently.

Display the diagram and calculations for $(2.4) \cdot (1.3)$ for all to see.



$$\begin{array}{r} 2.4 \\ \times\ 1.3 \\ \hline 0.12 \\ 0.6 \\ 0.4 \\ +\ 2 \\ \hline 3.12 \end{array}$$

Ask questions such as:

- "What do you notice about the labels of side lengths?" (Each one is decomposed into a whole number and a decimal, or ones and tenths.)
- "In the diagram, what do the numbers in each sub-rectangle represent?" (The area of the sub-rectangle) "How do we find them?" (Multiply the length and width of each sub-rectangle.)
- "In Calculation A, what do 0.12, 0.6, 0.4, and 2 represent?" (Partial products, or the areas of those sub-rectangles)
- "If we didn't draw a diagram, how could we find those partial products?" (We can multiply each digit in one factor by each digit in the other factor, keeping in mind the place value of each digit.)

Consider annotating each partial product in Calculation A with its factors:

	2 . 4	
×	1 . 3	
	0 . 1 2	0.3 × 0.4
	0 . 6	0.3 × 2
	0 . 4	1 × 0.4
+	2	1 × 2
	3 . 1 2	

Alternatively, consider separating each step and using color coding to show the digits being multiplied, as shown here:

	2 . 4	
×	1 . 3	
	0 . 1 2	0.3 × 0.4

	2 . 4	
×	1 . 3	
	0 . 6	0.3 × 2

	2 . 4	
×	1 . 3	
	0 . 4	1 × 0.4

	2 . 4	
×	1 . 3	
	2	1 × 2

7.4 Using Partial Products

Optional

10 min

Activity Narrative

This activity gives students another opportunity to use partial products to multiply decimals and to connect area diagrams and vertical calculations. More attention is given to how to record the partial products more concisely (as shown in Calculation B in this activity and earlier ones), paving the way to multiplying decimals using the standard algorithm.

In these problems, the side lengths and areas of the rectangle are left blank. Students fill in these fields by decomposing the factors by place value and multiplying them. Like many of the activities in this unit, the work involves understanding the structure of the base-ten system used in multiplication calculations (MP7) and making sense of it from arithmetic and geometric perspectives.

Students also make another key connection about several previously developed ideas here. They see that we can write the non-zero digits of decimal factors as whole numbers, use vertical calculations to multiply them, and then attend to the decimal point in the product afterward. In other words, they notice the algorithm for multiplication can streamline the reasoning processes that they have used up to this point.

Standards

Addressing **6.NS.B.3**

Launch

Keep students in groups of 2. Give groups 1–2 minutes to make sense of the first set of questions and the area diagram.

Before students begin labeling and computing, discuss questions such as:

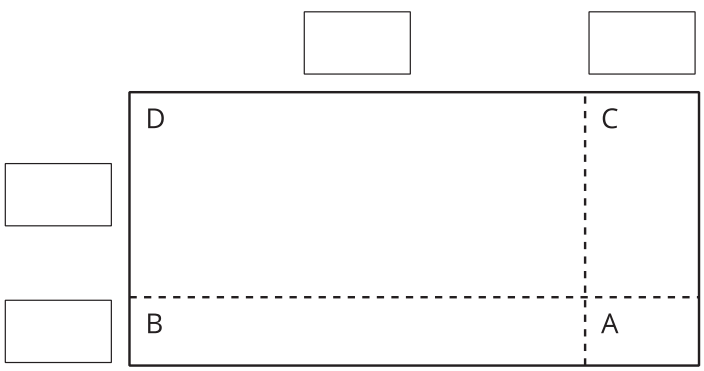
- "How might you decompose each factor?"
- "How would you decide what number to use for each part of a side length?"
- "How would you know what value to record for the regions A, B, C, and D?"

If students propose decomposing each factor or side length in ways other than by place value, consider using an example to discuss the practicality of doing so. (For instance, suppose 2.5 is decomposed into 1.5 and 1, and 1.2 is decomposed into 0.7 and 0.5, what computations would give us the values of A, B, C, and D?)

Give students 3–4 minutes of quiet work time and then time to share their responses with their partner. Follow with a whole-class discussion.

Student Task Statement

1. Label the area diagram to represent $(2.5) \cdot (1.2)$ and to find that product.

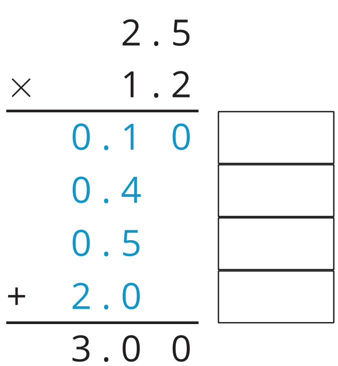


   a. Decompose each factor by place value and write the ones and tenths in the blank boxes.

   b. Label Regions A, B, C, and D with their areas. Show your reasoning.

   c. Find the product that the area diagram represents. Show your reasoning.

2. Here are two ways to calculate $(2.5) \cdot (1.2)$. Each number with a box gives the area of one or more regions in the area diagram.



Calculation A

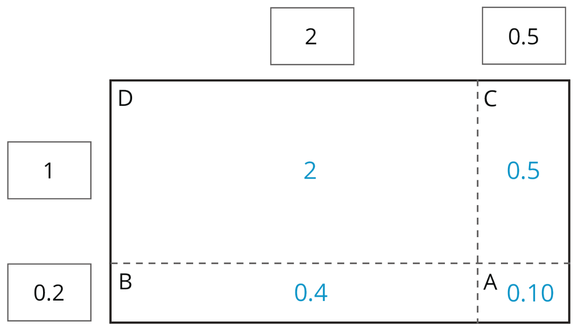
Calculation B

a. In the boxes next to each number, write the letter(s) of the corresponding region(s).

b. In Calculation B, which two numbers are being multiplied to give 0.5? Which numbers are being multiplied to give 2.5?

Student Response

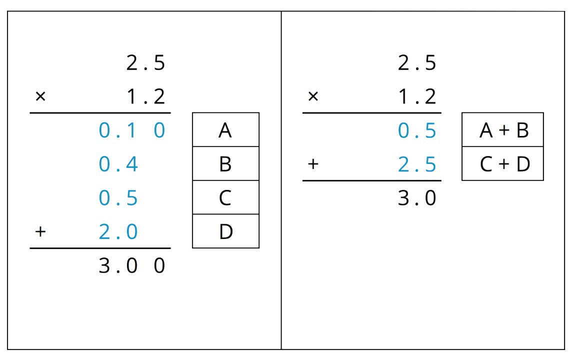
1. a.



b. A: $(0.5) \cdot (0.2) = 0.1$, B: $(2) \cdot (0.2) = 0.4$, C: $(0.5) \cdot 1 = 0.5$, D: $2 \cdot 1 = 2$

c. $0.1 + 0.4 + 0.5 + 2 = 3$, so $(2.5) \cdot (1.2) = 3$

2. a.



Calculation A   Calculation B

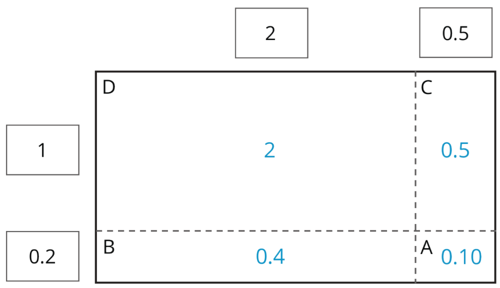
b. $0.5 = (0.2) \cdot (2.5)$ and $2.5 = (1.0) \cdot (2.5)$

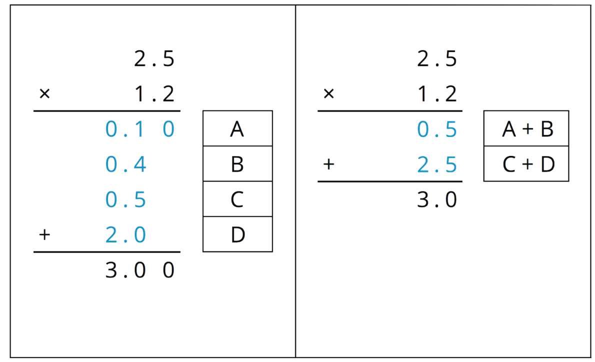
Building on Student Thinking

Some students might think that the partial products in Calculation A must be listed in a particular order for it to be correct. Invite students to list the partial products in any way that makes sense to them and compare their results with those of others who list them a different way.

Activity Synthesis

Display the solution shown here for all to see. Give students a minute to make comparisons and to share their observations or questions.





Calculation A

Calculation B

If no students mentioned listing the partial products in Calculation A in a different order from what's shown, ask students to discuss this idea. Clarify that the order doesn't affect the result, but it is conventional to start with the smallest place value in each factor and work toward the larger ones. It means starting with $(0.2) \cdot (0.5)$ (the area of A) and ending with $1 \cdot 2$ (area of D).

Consider annotating each partial product with the two values being multiplied, as shown:

$$
\begin{array}{rr}
 & 2.5 \\
\times & 1.2 \\
\hline
 & 0.10 \\
 & 0.4 \\
 & 0.5 \\
+ & 2 \\
\hline
 & 3.00
\end{array}
\qquad
\begin{array}{lcl}
0.2 & \times & 0.5 \\
0.2 & \times & 2 \\
1 & \times & 0.5 \\
1 & \times & 2
\end{array}
$$

Next, draw students' attention to the partial products in Calculation B. Discuss the calculations that produce the partial products 0.5 and 2.5. If not mentioned by students, point out that each partial product comes from multiplying each digit in the second factor by the entirety of the first factor.

- The 0.5, represented by the area of A + B in the diagram, is the product of 0.2 and 2.5.
- The 2.5, represented by the area of C + D in the diagram, is the product of 1 and 2.5

$$
\begin{array}{rr}
 & 2.5 \\
\times & 1.2 \\
\hline
 & 0.5
\end{array}
\qquad 0.2 \times 2.5
$$

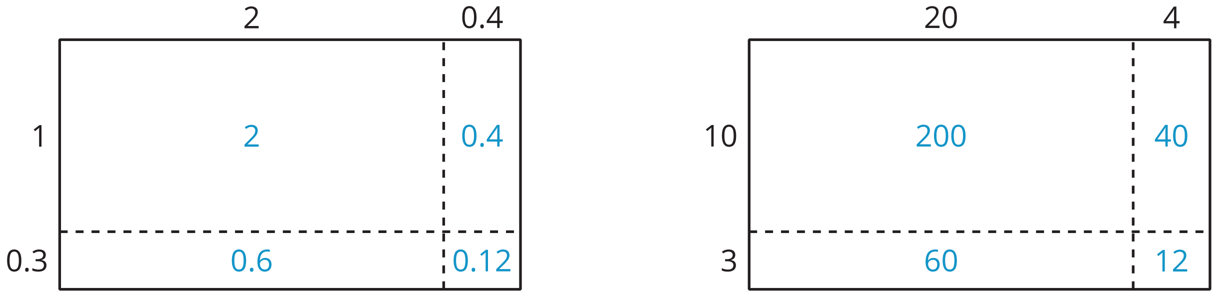
$$
\begin{array}{rr}
 & 2.5 \\
\times & 1.2 \\
\hline
 & 2.5
\end{array}
\qquad 1 \times 2.5
$$

To highlight this connection, consider shading or coloring the corresponding areas in the diagram and circling these values in the calculations.

Lesson Synthesis

The goal of this discussion is to highlight that we can multiply decimals using the same reasoning used with whole numbers: by decomposing the factors by place value, finding partial products, and adding them. An area diagram can help us visualize the parts, and a numerical calculation is a way to record them more efficiently.

Display the diagrams and calculations for $(2.4) \cdot (1.3)$ and $24 \cdot 13$ for all to see.



$$\begin{array}{r} 24 \\ \times\ 13 \\ \hline 12 \\ 60 \\ 40 \\ +\ 200 \\ \hline 312 \end{array} \qquad \begin{array}{l} \ \ 2.4 \\ \times\ \ \ 1.3 \\ \hline 0.12 \\ 0.6 \\ 0.4 \\ +\ 2 \\ \hline 3.12 \end{array}$$

Invite students to make connections by asking questions such as:

- “How are the two diagrams alike?” (The side lengths are decomposed by place value. The rectangles are partitioned the same way.)
- “How are the diagrams different?” (The side lengths are whole numbers in one diagram and decimals in the other. The decimal lengths are 1 tenth of the whole-number ones.)
- “How are the two calculations alike?” (The structure, setup, and non-zero digits are the same. There are four partial products, found using a similar process.)
- “How are the two calculations different?” (The partial products in the whole-number calculation have no decimals. The partial products in the decimal calculation are 1 hundredth of the whole-number calculations.)
- “Why might it help to decompose each factor by place value (such as 20 and 4 for 24, or 2 and 0.4 for 2.4)? Could we decompose each factor into any two numbers (such as 13 and 11 for 24, or 1.3 and 1.1 for 2.4)?” (It is easier to multiply two numbers if there is only 1 non-zero digit in each.)

If students do not recall how to use the standard algorithm to multiply whole numbers and did not complete the first optional activity, consider reviewing the algorithm before the next lesson.

7.5 Find the Product

Cool-down

5 min

Standards

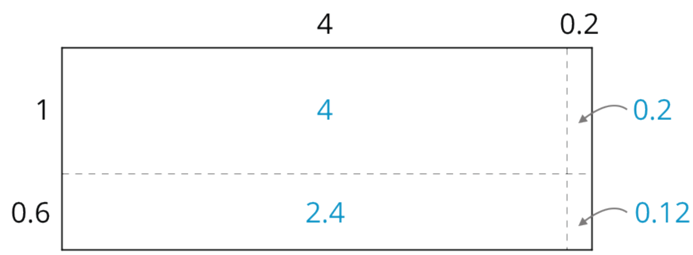
Addressing 6.NS.B.3

Student Task Statement

Find the value of $(4.2) \cdot (1.6)$ by drawing an area diagram or using another method. Show your reasoning.

Student Response

6.72. Sample reasoning:



The sum of the areas of the sub-rectangles is $4 + 0.2 + 2.4 + 0.12 = 6.72$.

Responding to Student Thinking

Points to Emphasize
If most students struggle with finding the product of two decimals in tenths, reiterate that we can multiply numbers such as 0.6 and 0.2 using their fraction form. For example, when working on the practice problem referred to here, after students have decomposed the factors by place value, ask them to also write each decomposed number as a fraction to help them compute the partial products.

Grade 6, Unit 5, Lesson 7, Practice Problem 3

Lesson 7 Summary

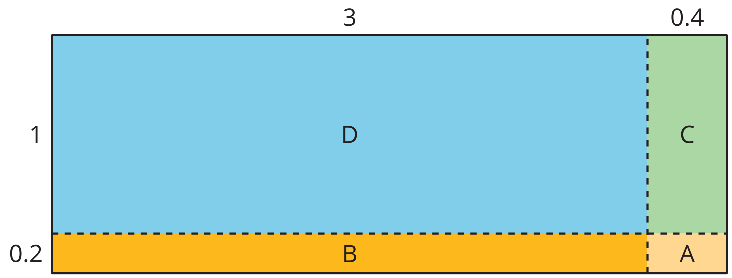
To find the product of two numbers, such as $(3.4) \cdot (1.2)$, we can think of finding the area of a rectangle with those numbers, 3.4 units and 1.2 units, as side lengths.

First, we draw a rectangle and partition each side length by place value, into ones and tenths:

$3.4 = 3 + 0.4$

$1.2 = 1 + 0.2$

Then, we decompose the rectangle into four smaller sub-rectangles and find their areas.



A: $(0.4) \cdot (0.2) = 0.08$

B: $3 \cdot (0.2) = 0.6$

C: $(0.4) \cdot 1 = 0.4$

D: $3 \cdot 1 = 3$

$0.08 + 0.6 + 0.4 + 3 = 4.08$

Each multiplication gives a *partial product* that represents the area of a sub-rectangle. The sum of the four partial products gives the area of the entire rectangle, 4.08 square units.

We can show the same partial-product calculations vertically. Here are two ways:

$$\begin{array}{rrl} & 3.4 & \\ \times & 1.2 & \\ \hline & {}^{1} & \\ & 0.08 & \text{A} \\ & 0.6 & \text{B} \\ & 0.4 & \text{C} \\ + & 3 & \text{D} \\ \hline & 4.08 & \end{array} \qquad \begin{array}{rrl} & 3.4 & \\ \times & 1.2 & \\ \hline & {}^{1} & \\ & 0.68 & \text{A} + \text{B} \\ + & 3.4 & \text{C} + \text{D} \\ \hline & 4.08 & \end{array}$$

The calculation on the left shows four partial products, one for the area of each sub-rectangle.

The calculation on the right shows two partial products:

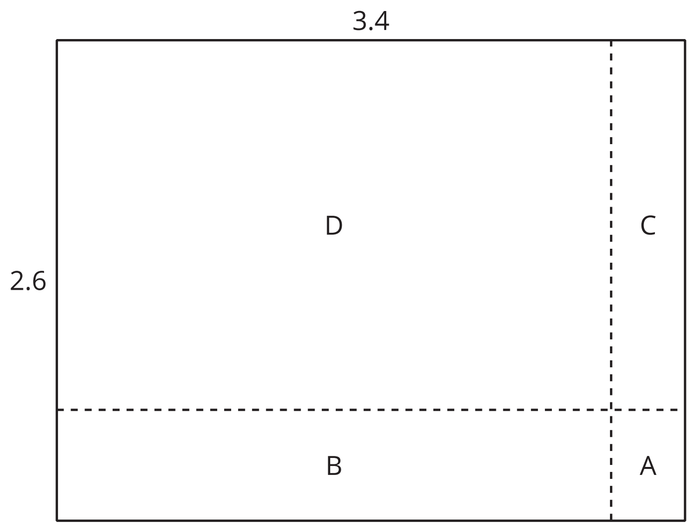
- 0.68 is the value of $(3.4) \cdot (0.2)$, or the combined area of A and B.
- 3.4 is the value of $(3.4) \cdot 1$, or the combined area of C and D.

In both calculations, adding the partial products gives a total of 4.08, which is the product of $(3.4) \cdot (1.2)$ and the area (in square units) of the entire rectangle.

Lesson 7 Practice Problems

1 Student Task Statement

Here is a rectangle that has been partitioned into four smaller rectangles.



For each expression, choose the sub-rectangle whose area, in square units, matches the expression.

a. $3 \cdot (0.6)$

b. $(0.4) \cdot 2$

c. $(0.4) \cdot (0.6)$

d. $3 \cdot 2$

Solution

a. B

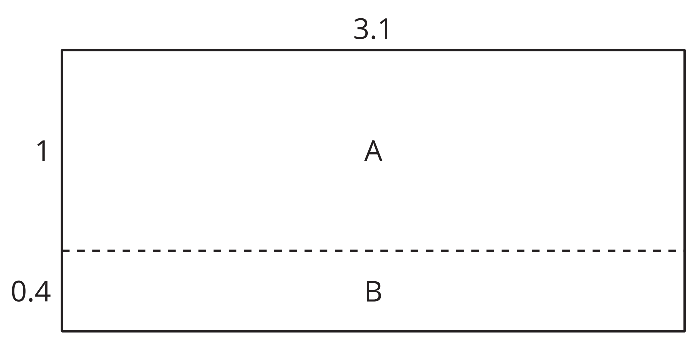
b. C

c. A

d. D

2 Student Task Statement

Here is an area diagram that represents $(3.1) \cdot (1.4)$.



a. Find the areas of sub-rectangles A and B.

b. What is the area of the 3.1 by 1.4 rectangle?

Solution

a. Rectangle A: 3.1 square units, Rectangle B: 1.24 square units

b. 4.34 square units $(3.1 + 1.24 = 4.34)$

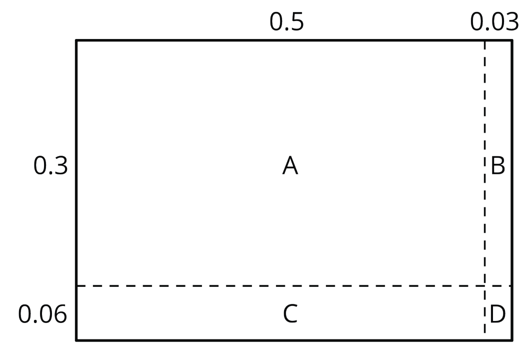
3 Student Task Statement

Draw an area diagram to find $(0.36) \cdot (0.53)$. Label and organize your work so that it can be followed by others.

Solution

0.1908. Sample reasoning:

- Area of A is $(0.5)(0.3) = 0.15$.
- Area of B is $(0.03)(0.3) = 0.009$.
- Area of C is $(0.5)(0.06) = 0.03$.
- Area of D is $(0.03)(0.06) = 0.0018$.
- The area of the rectangle, in square units, is $0.15 + 0.009 + 0.03 + 0.0018 = 0.1908$.



4 Student Task Statement

Find each product. Show your reasoning.

a. $(2.5) \cdot (1.4)$

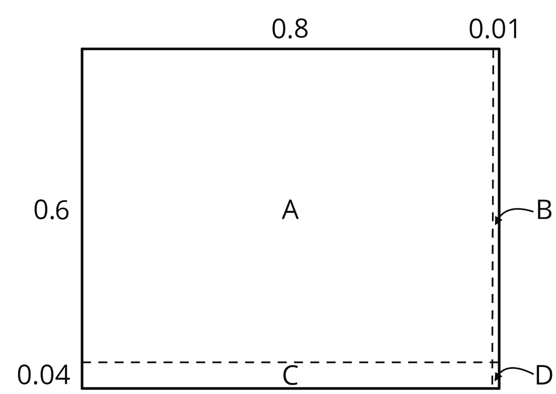
b. $(0.64) \cdot (0.81)$

Solution

a. 3.5. Sample reasoning: $2 \cdot (1.4) = 2.8$ and $(0.5) \cdot (1.4) = 0.7$. The product is $2.8 + 0.7$ or 3.5.

b. 0.5184. Sample reasoning:

- Area of A is $(0.8)(0.6) = 0.48$.
- Area of B is $(0.01)(0.6) = 0.006$.
- Area of C is $(0.8)(0.04) = 0.032$.
- Area of D is $(0.01)(0.04) = 0.0004$.
- The area of the rectangle, in square units, is $0.48 + 0.006 + 0.032 + 0.0004 = 0.5184$.



5

from Unit 5, Lesson 3

Student Task Statement

Complete the calculations so that each shows the correct sum.

$$\begin{array}{r} 2.3\square \\ +\ \square.64 \\ \hline 9.\square 5 \end{array} \qquad \begin{array}{r} 2.3\square \\ +\ \square.64 \\ \hline 9.\square 2 \end{array} \qquad \begin{array}{r} 4.3\square \\ +\ \square.15 \\ \hline 6.\square 2 \end{array} \qquad \begin{array}{r} 1.5\square \\ +\ \square.38 \\ \hline 1.\square 4 \end{array}$$

Solution

$$\begin{array}{r} 2.3\boxed{1} \\ +\ \boxed{7}.64 \\ \hline 9.\boxed{9}5 \end{array} \qquad \begin{array}{r} 2.3\boxed{8} \\ +\ \boxed{6}.64 \\ \hline 9.\boxed{0}2 \end{array} \qquad \begin{array}{r} 4.3\boxed{7} \\ +\ \boxed{2}.15 \\ \hline 6.\boxed{5}2 \end{array} \qquad \begin{array}{r} 1.5\boxed{6} \\ +\ \boxed{0}.38 \\ \hline 1.\boxed{9}4 \end{array}$$

6 from Unit 2, Lesson 12

Student Task Statement

Diego bought 12 paint brushes for $4.20.

a. At this rate, how much would Diego pay for 4 paint brushes?

b. How many paint brushes could Diego buy with $3.00? Explain or show your reasoning. If you get stuck, consider using the table.

number of paint brushes	price in dollars
12	4.20

Solution

a. $1.40.

b. 8 paint brushes. Sample reasoning: 8 paint brushes would cost $2.80. He does not have enough money for 9 paint brushes, because that would cost $3.15.

number of paint brushes	price in dollars
12	4.20
1	0.35
8	2.80
9	3.15