



# Moving Functions

Let's represent vertical and horizontal translations using function notation.

## 2.1 What Happened to the Equation?

Graph each function using technology. Describe how to transform  $f(x) = x^2(x - 2)$  to get to the functions shown here, in terms of both the graph and the equation.

1.  $h(x) = x^2(x - 2) - 5$

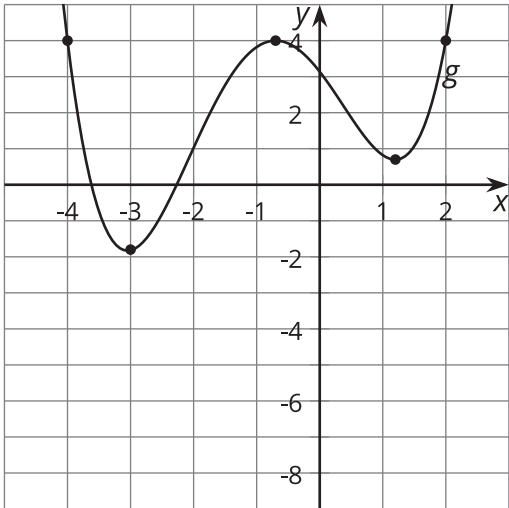
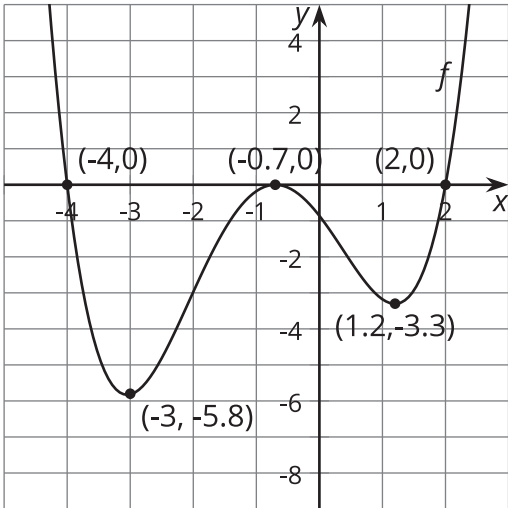
2.  $g(x) = (x - 4)^2(x - 6)$



2.2

Writing Equations for Vertical Translations

The graph of function  $g$  is a vertical translation of the graph of polynomial  $f$ .



1. Complete the  $g(x)$  column of the table.

| $x$  | $f(x)$ | $g(x)$ | $h(x) = f(x) - 2.5$ |
|------|--------|--------|---------------------|
| -4   | 0      |        |                     |
| -3   | -5.8   |        |                     |
| -0.7 | 0      |        |                     |
| 1.2  | -3.3   |        |                     |
| 2    | 0      |        |                     |

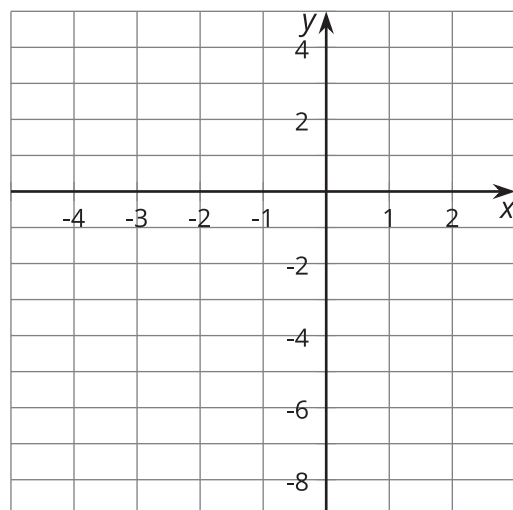
2. If  $f(0) = -0.86$ , what is  $g(0)$ ? Explain how you know.

3. Write an equation for  $g(x)$  in terms of  $f(x)$  for any input  $x$ .

4. The function  $h$  can be written in terms of  $f$  as  $h(x) = f(x) - 2.5$ . Complete the  $h(x)$  column of the table.



5. Sketch the graph of function  $h$ .



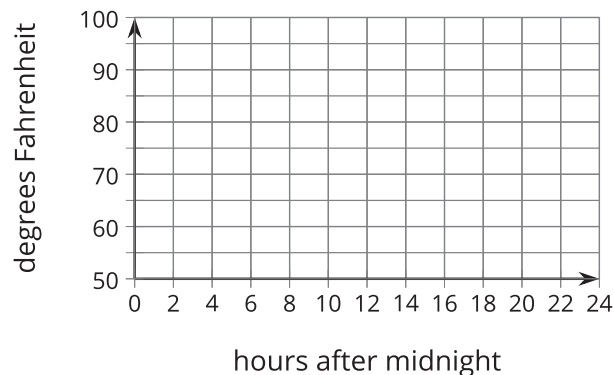
6. Write an equation for  $g(x)$  in terms of  $h(x)$  for any input  $x$ .

## 2.3

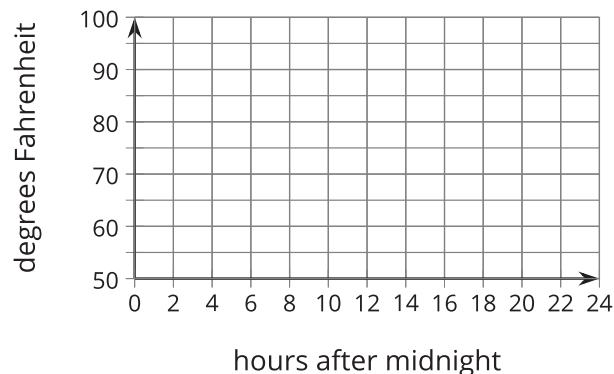
## Heating the Kitchen

A bakery kitchen has a thermostat set to  $65^{\circ}\text{F}$ . Starting at 5:00 a.m., the temperature in the kitchen rises to  $85^{\circ}\text{F}$  when the ovens and other kitchen equipment are turned on to bake the daily breads and pastries. The ovens are turned off at 10:00 a.m. when the baking finishes.

1. Sketch a graph of the function  $H$  that gives the temperature in the kitchen  $H(x)$ , in degrees Fahrenheit,  $x$  hours after midnight.



2. The bakery owner decides to change the shop hours to start and end 2 hours earlier. This means the daily baking schedule will also start and end two hours earlier. Sketch a graph of the new function  $G$ , which gives the temperature in the kitchen as a function of time.



3. Explain what  $H(10.25) = 80$  means in this situation. Why is this reasonable?
4. If  $H(10.25) = 80$ , then what would the corresponding point on the graph of  $G$  be? Use function notation to describe the point on the graph of  $G$ .
5. Write an equation for  $G$  in terms of  $H$ . Explain why your equation makes sense.

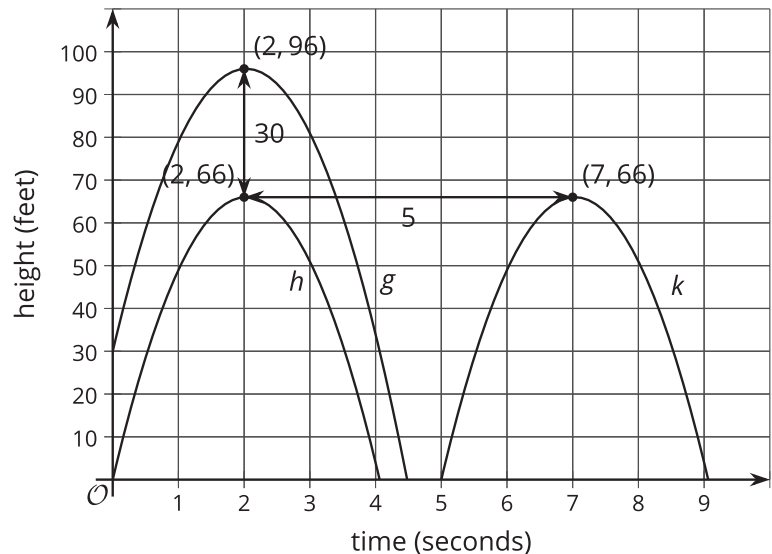
## Are you ready for more?

Write an equation that defines your piecewise function,  $H$ , algebraically.

### Lesson 2 Summary

A pumpkin catapult is used to launch a pumpkin vertically into the air. The function  $h$  gives the height  $h(t)$ , in feet, of this pumpkin above the ground  $t$  seconds after launch.

Now consider what happens if the pumpkin had been launched at the same time, but from a platform 30 feet above the ground. Let function  $g$  represent the height  $g(t)$ , in feet, of this pumpkin. How would the graphs of  $h$  and  $g$  compare?



Since the height of the second pumpkin is 30 feet greater than the first pumpkin at all times  $t$ , the graph of function  $g$  is translated up 30 feet from the graph of function  $h$ . For example, the point  $(2, 66)$  on the graph of  $h$  tells us that  $h(2) = 66$ , so the original pumpkin was 66 feet high after 2 seconds. The new pumpkin would be 30 feet higher than that, so  $g(2) = 96$ . Since all the outputs of  $g$  are 30 more than the corresponding outputs of  $h$ , we can express  $g(t)$  in terms of  $h(t)$ , using function notation as  $g(t) = h(t) + 30$ .

Now suppose instead the pumpkin launched 5 seconds later. Let function  $k$  represent the height  $k(t)$ , in feet of this pumpkin. The graph of  $k$  is translated right 5 seconds from the graph of  $h$ . We can also say that the output values of  $k$  are the same as the output values of  $h$  5 seconds earlier. For example,  $k(7) = 66$  and  $h(7 - 5) = h(2) = 66$ . This means we can express  $k(t)$  in terms of  $h(t)$  as  $k(t) = h(t - 5)$ .

