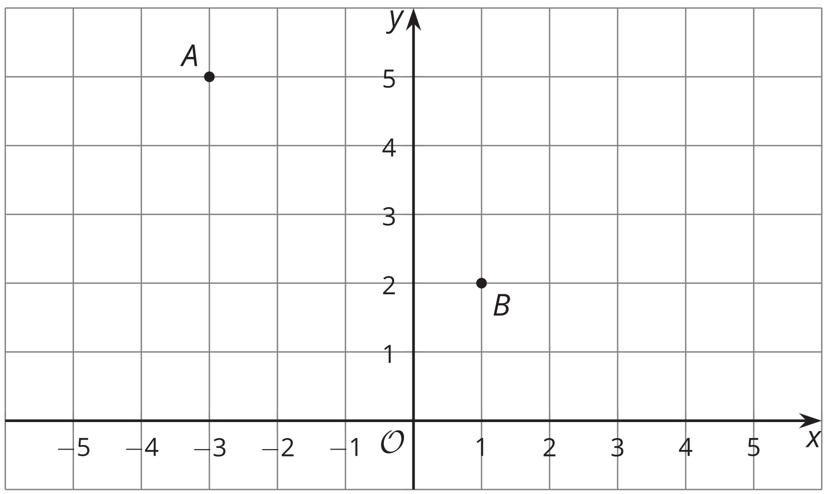
Rigid Transformations in a Plane

Let's try transformations with coordinates.

1.1 Traversing the Plane

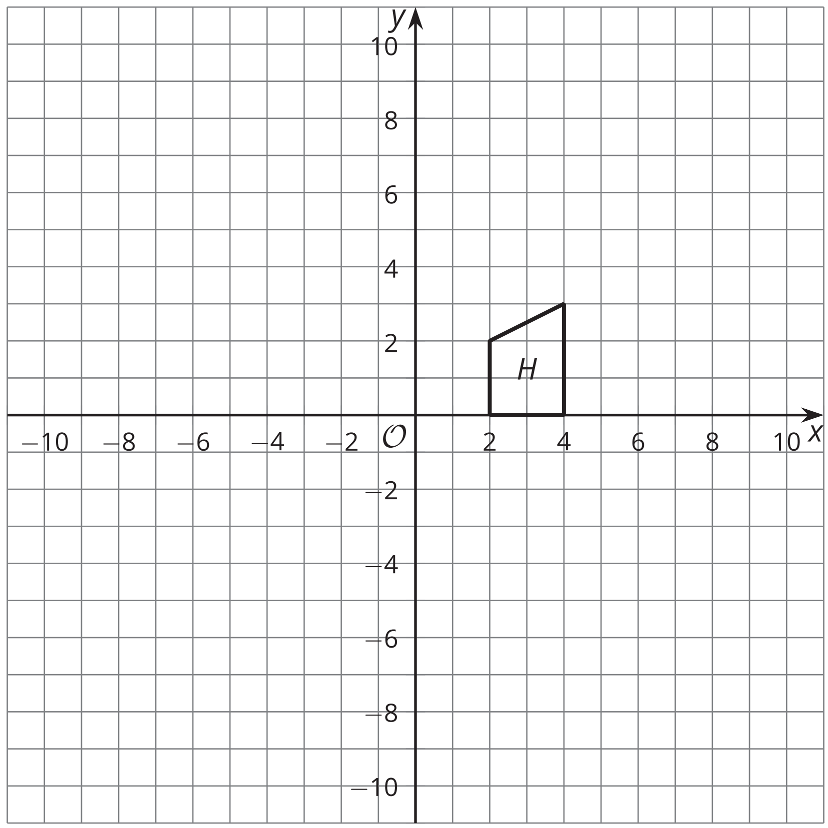


1. How far is point A from point B?

2. What transformations will take point A to point B?

1.2 Transforming with Coordinates

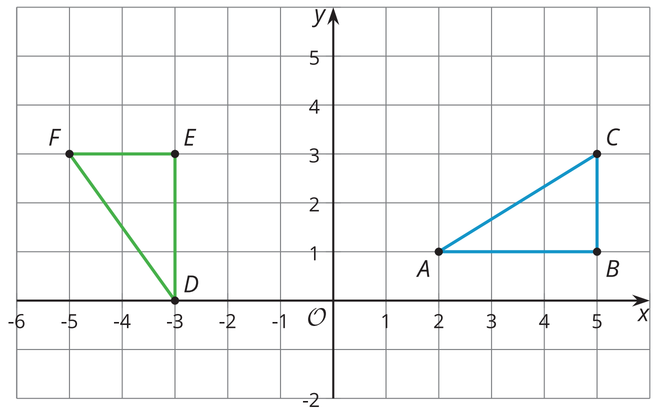
First, predict where each transformation will land. Next, carry out the transformation.



1. Rotate Figure H clockwise around center $(2, 0)$ by 90 degrees.
   Translate the image by the directed line segment from $(2, 0)$ to $(3, -4)$.
   Label the result R.

2. Reflect Figure H across the y-axis.
   Rotate the image counterclockwise around center $(0, 0)$ by 90 degrees.
   Label the result L.

1.3 Congruent by Coordinates



1. Calculate the length of each side in triangles ABC and DEF.

2. Calculate the measure of each angle in triangles ABC and DEF.

3. The triangles are congruent. How do you know this is true?

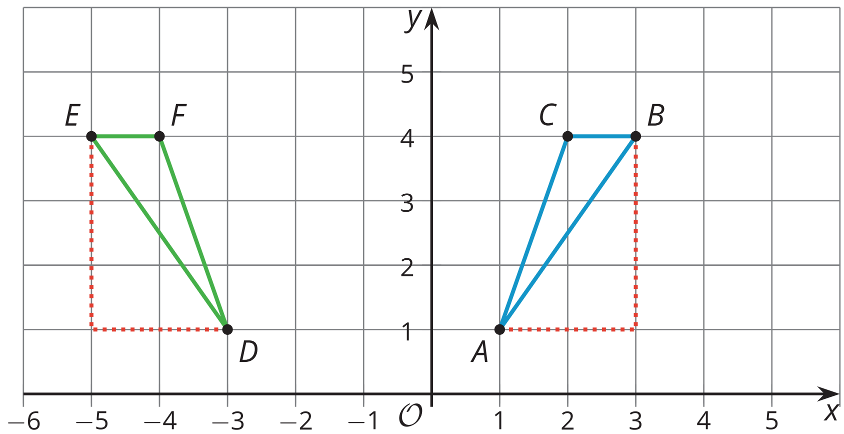
4. Because the triangles are congruent, there must be a sequence of rigid motions that takes one to the other. Find a sequence of rigid motions that takes triangle ABC to triangle DEF.

Are you ready for more?

What single transformation would take triangle ABC to triangle DEF?

Lesson 1 Summary

The triangles shown here look like they might be congruent. Since we know the coordinates of all the vertices, we can compare side lengths using the Pythagorean Theorem—if we draw line segments (see the red dotted lines) that create two right triangles that have segments AB and DE as their respective hypotenuses. The length of segment AB is $\sqrt{13}$ units because this segment is the hypotenuse of a right triangle with vertical side length of 3 units and horizontal side length of 2 units. The length of segment DE is $\sqrt{13}$ units as well, because this segment is also the hypotenuse of a right triangle with leg lengths of 3 and 2 units.



The other sides of the triangles are congruent as well: The lengths of segments BC and FE are 1 unit each, and the lengths of segments AC and DF are each $\sqrt{10}$ units, because they are both hypotenuses of right triangles with leg lengths 1 and 3 units (those lines are not shown, but could be drawn). So triangle ABC is congruent to triangle DEF by the Side-Side-Side Triangle Congruence Theorem.

Since triangle ABC is congruent to triangle DEF, there is a sequence of rigid motions that takes triangle ABC to triangle DEF. Here is one possible sequence: First, reflect triangle ABC across the y-axis. Then translate the image by the directed line segment from $(-1, 1)$ to $(-3, 1)$.