Logarithm Power Rule

Let's rewrite the product of a logarithm and a rational number.

17.1 A Pattern with a Logarithm Product

For each equation, find the value of the missing terms by finding the value of the logarithms and comparing the values on each side of the equation.

The first one is done for you. Discuss with your partner why it is true.

1. $\log_2(8^2) = \underline{6}$

   $\log_2(8^2) = \underline{2} \cdot \log_2(8)$

2. $\log_2(8^3) = \underline{\qquad}$

   $\log_2(8^3) = \underline{\qquad} \cdot \log_2(8)$

3. $\log_3\left(\left(\frac{1}{3}\right)^2\right) = \underline{\qquad}$

   $\log_3\left(\left(\frac{1}{3}\right)^2\right) = \underline{\qquad} \cdot \log_3\left(\frac{1}{3}\right)$

4. $\log_6(\sqrt{36}) = \underline{\qquad}$

   $\log_6(\sqrt{36}) = \underline{\qquad} \cdot \log_6(36)$

5. $\log(100^5) = \underline{\qquad}$

   $\log(100^5) = \underline{\qquad} \cdot \log(100)$

17.2 Making a Conjecture about a Logarithm Product

1. Use the pattern you noticed about logarithms of expressions with an exponent to write a conjecture.

$$\log_b(A^c) = \underline{\hspace{3cm}}$$

2. Assume the conjecture is true. Rewrite each expression using your conjecture, then find the value of the expression.

   a. $\log(\sqrt{1000})$

   b. $\log_9(81^{20})$

   c. $\log_2\left(\sqrt[5]{\frac{1}{2}}\right)$

3. If $\log(3) = 0.4771$ and $\log(7) = 0.8451$, find the values of each logarithm. Explain or show your reasoning.

   a. $\log(7^4)$

   b. $\log\left(\frac{1}{9}\right)$

   c. $\log(\sqrt{7})$

17.3 Proving the Conjecture about a Logarithm Product

Let's work through some steps of a proof for your conjecture.

Start with the equation: $b^x = A$

1. Rewrite the equation as a logarithm, and circle your answer to use later. $\log \quad (\qquad)$
2. Raise each side of the original equation to the power of c. $\underline{\qquad\qquad} = A^c$
3. Combine the exponents on the left side of the equation so that the left side is written with a single base. $\underline{\qquad\qquad} = A^c$
4. Rewrite the last equation as a logarithm with a base of b. $\log_b(\qquad) =$
5. Use your circled equation to replace any x in that equation with an equivalent logarithm.

Are you ready for more?

To see that the graph of $f(x) = \log(x)$ goes to infinity as x goes to infinity, we can show that it will eventually be greater than any large number we can think of after passing a certain value of x.

For each of these M values, find a value for n so $f(x) > M$ for every x-value greater than n.

1. $M = 10^{100}$
2. $M = 10^{1000}$
3. M is the number that can be written as a 1 followed by one trillion zeros.

Lesson 17 Summary

The **power rule** for logarithms allows us to rewrite logarithms with values raised to powers. The power rule states that

$$\log_a(b^c) = c \cdot \log_a(b)$$

For example, $\log(6^5) = 5 \cdot \log(6)$.

Thinking about logarithms in relation to exponents, this may make more sense. We learned in an earlier course that

$$\left(a^b\right)^c = a^{b \cdot c}$$

By rewriting parts of that equation into their logarithm form, we can combine the pieces to prove the power rule.