Unit 6, Lesson 20

Different Compounding Intervals

Let's find out what happens when we repeatedly apply the same percent increase at different intervals of time.

20.1 Returns over Three Years

$1,000 is deposited into a bank account that earns 1% interest each month. Each month, the interest is added to the account, and no other deposits or withdrawals are made.

To calculate the account balance in dollars after 3 years, Elena wrote: $1{,}000 \cdot (1.01)^{36}$ and Tyler wrote: $1{,}000 \cdot \left((1.01)^{12}\right)^3$.

Discuss with a partner:

1. Why do Elena's expression and Tyler's expression both represent the account balance correctly?

2. Kiran said, "The account balance is about $1{,}000 \cdot (1.1268)^3$." Do you agree? Why or why not?

20.2 Contemplating Credit Cards

A credit card company lists a nominal APR (annual percentage rate) of 24% but compounds interest monthly, so it calculates 2% per month.



Suppose a cardholder made $1,000 worth of purchases using his credit card and made no payments or other purchases. Assume the credit card company does not charge any additional fees other than the interest.

1. Write expressions for the balance on the card after 1 month, 2 months, 6 months, and 1 year.

2. Write an expression for the balance on the card, in dollars, after m months without payment.

3. How much does the cardholder owe after 1 year without payment? What is the *effective* APR of this credit card?

4. Write an expression for the balance on the card, in dollars, after t years without payment. Be prepared to explain your expression.

Are you ready for more?

A bank account has an annual interest rate of 12% and an initial balance of $800. Any earned interest is added to the account, but no other deposits or withdrawals are made. Write an expression for the account balance:

1. After 5 years, if interest is compounded n times per year.
2. After t years, if interest is compounded n times per year.
3. After t years, with an initial deposit of P dollars and an annual interest percentage rate of r, compounded n times per year.

20.3 Which One Would You Choose?

Suppose you have $500 to invest and can choose between two investment options.

- Option 1: Every 3 months 3% interest is applied to the balance
- Option 2: Every 4 months 4% interest is applied to the balance

Which option would you choose? Build a mathematical model for each investment option and use them to support your investment decision. Remember to state your assumptions about the situation.

Are you ready for more?

Is there a period of time during which the first option (3% interest rate, compounded quarterly) will *always* be the better option? If so, when might it be? If not, why might that be?

20.4 Changes over the Years

1. The function, f, defined by $f(x) = 15 \cdot (1.07)^x$ models the cost of tuition, in thousands of dollars, at a local college x years since 2017.
   a. What is the cost of tuition at the college in 2017?
   b. At what annual percentage rate does the tuition grow?
   c. Assume that the model function holds for the years before 2017. What was the tuition in 2000? Show your reasoning.
   d. What was the tuition in 2010?
   e. What will the tuition be when you graduate from high school?

2. Between 2000 and 2010 the tuition nearly doubled.
   a. By what factor will the tuition grow between 2017 and 2027? Show your reasoning.
   b. Choose another 10-year period, and find the factor by which the tuition grows. Show your reasoning.
   c. What can you say about how the tuition changes over any 10-year period (assuming that the function, f, continues to be an accurate model)? Explain or show how you know that this will always be the case.

Lesson 20 Summary

In many situations, interest is calculated more frequently than once a year. How often the interest is compounded and calculated and added to the previous amount affects the overall amount of interest earned (or owed) over time.

Suppose a bank account has a balance of $1,000 and a nominal annual interest rate of 6% per year. No additional deposits or withdrawals are made.

If the bank compounds interest annually, the account will have one interest calculation in one year, at a 6% rate. If it compounds interest every 6 months, the account will see two interest calculations in one year, at a 3% rate each time (because $6 \div 2 = 3$). If it is compounded every 3 months, there will be 4 calculations at 1.5% each time, and so on.

This table shows the nominal interest rates used for different compounding intervals, as well as the corresponding expressions for the account balance in one year.

compounding interval	compounding frequency per year	nominal interest rate	account balance in one year
annually (12 months)	1 time	6%	$1{,}000 \cdot (1 + 0.06)$
semi-annually (6 months)	2 times	3%	$1{,}000 \cdot (1 + 0.03)^2$
quarterly (3 months)	4 times	1.5%	$1{,}000 \cdot (1 + 0.015)^4$
monthly (1 month)	12 times	0.5%	$1{,}000 \cdot (1 + 0.005)^{12}$

If we evaluate the expressions, we find these account balances:

- annually: $1{,}000 \cdot (1.06) = 1{,}060$
- semi-annually: $1{,}000 \cdot (1.03)^2 = 1{,}060.90$
- quarterly: $1{,}000 \cdot (1.015)^4 \approx 1{,}061.36$
- monthly: $1{,}000 \cdot (1.005)^{12} \approx 1{,}061.68$

Notice that the more frequently interest is calculated, the greater the balance is.