



Using Graphs to Find Average Rate of Change

Let's measure how quickly the output of a function changes.

7.1 Temperature Drop

Here are the recorded temperatures at three different times on a winter evening.

time	4 p.m.	6 p.m.	10 p.m.
temperature	25°F	17°F	8°F

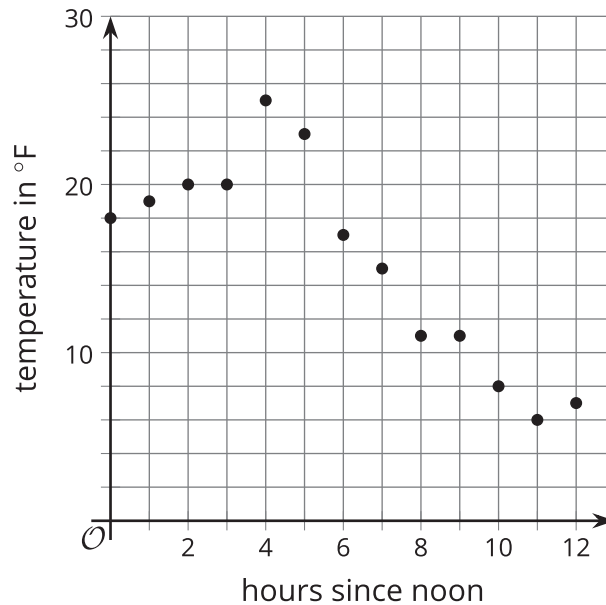
- Tyler says the temperature dropped faster between 4 p.m. and 6 p.m.
- Mai says the temperature dropped faster between 6 p.m. and 10 p.m.

Who do you agree with? Explain your reasoning.

7.2 Drop Some More

The table and graph show a more complete picture of the temperature changes on the same winter day. The function T gives the temperature in degrees Fahrenheit, h hours since noon.

h	$T(h)$
0	18
1	19
2	20
3	20
4	25
5	23
6	17
7	15
8	11
9	11
10	8
11	6
12	7



- Find the **average rate of change** for the following intervals. Explain or show your reasoning.
 - between noon and 1 p.m.
 - between noon and 4 p.m.
 - between noon and midnight
- Remember Mai and Tyler's disagreement? Use average rate of change to show which time period—4 p.m. to 6 p.m. or 6 p.m. to 10 p.m.—experienced a faster temperature drop.



- ### 7.3 Populations of Two States

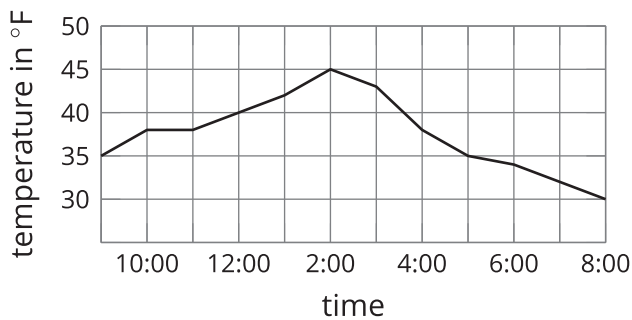
The graph displays the population growth of California and Texas over a century. The y-axis represents population in millions, ranging from 0 to 40. The x-axis represents the year, from 1910 to 2010 in 10-year increments. California's population, shown by a solid blue line, starts at approximately 1.5 million in 1910 and grows steadily, crossing Texas' population around 1935. By 2010, California's population reaches nearly 40 million. Texas' population, shown by a dashed red line, starts at approximately 3 million in 1910 and grows more slowly, reaching about 25 million by 2010.

Year	California (millions)	Texas (millions)
1910	1.5	3.0
1920	3.0	4.0
1930	5.0	5.5
1940	7.0	6.5
1950	10.0	8.0
1960	15.0	10.0
1970	20.0	11.0
1980	24.0	14.0
1990	30.0	17.0
2000	34.0	21.0
2010	38.0	25.0

1.
 - a. Estimate the average rate of change in the population in each state between 1970 and 2010. Show your reasoning.
 - b. In this situation, what does each rate of change mean?
2. Which state's population grew more quickly between 1900 and 2000? Show your reasoning.

Lesson 7 Summary

Here is a graph of one day's temperature as a function of time.



The temperature was 35°F at 9 a.m. and 45°F at 2 p.m., an increase of 10°F over those 5 hours.

The increase wasn't constant, however. The temperature rose from 9 a.m. and 10 a.m., stayed steady for an hour, then rose again.

- On average, how fast was the temperature rising between 9 a.m. and 2 p.m.?

Let's calculate the **average rate of change** and measure the temperature change per hour. We do that by finding the difference in the temperature between 9 a.m. and 2 p.m. and dividing it by the number of hours in that interval.

$$\text{average rate of change} = \frac{45 - 35}{5} = \frac{10}{5} = 2$$

On average, the temperature between 9 a.m. and 2 p.m. increased 2°F per hour.

- How quickly was the temperature falling between 2 p.m. and 8 p.m.?

$$\text{average rate of change} = \frac{30 - 45}{6} = \frac{-15}{6} = -2.5$$

On average, the temperature between 2 p.m. and 8 p.m. dropped by 2.5°F per hour.

In general, we can calculate the average rate of change of a function f between input values a and b by dividing the difference in the outputs by the difference in the inputs.

$$\text{average rate of change} = \frac{f(b) - f(a)}{b - a}$$

If the two points on the graph of the function are $(a, f(a))$ and $(b, f(b))$, the average rate of change is the slope of the line that connects the two points.

