Applying the Quadratic Formula (Part 2)

Let's use the quadratic formula and solve quadratic equations with care.

18.1 Bits and Pieces

Evaluate each expression for $a = 9$, $b = -5$, and $c = -2$

1. $-b$
2. b^2
3. $b^2 - 4ac$
4. $-b \pm \sqrt{a}$

18.2 Using the Formula with Care

Here are four equations, followed by attempts to solve them using the quadratic formula. Each attempt contains at least one error.

- Solve 1–2 equations by using the quadratic formula.
- Then, find and describe the error(s) in the worked solutions of the same equations as the ones you solved.

Equation 1: $2x^2 + 3 = 8x$

Equation 2: $x^2 + 3x = 10$

Equation 3: $9x^2 - 2x - 1 = 0$

Equation 4: $x^2 - 10x + 23 = 0$

Here are the worked solutions with errors:

Equation 1: $2x^2 + 3 = 8x$

$a = 2,\ b = \text{-}8,\ c = 3$

$$x = \frac{\text{-}b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{\text{-}(\text{-}8) \pm \sqrt{(\text{-}8)^2 - 4(2)(3)}}{2(2)}$$
$$x = \frac{8 \pm \sqrt{64 - 24}}{4}$$
$$x = \frac{8 \pm \sqrt{40}}{4}$$
$$x = 2 \pm \sqrt{10}$$

Equation 2: $x^2 + 3x = 10$

$a = 1,\ b = 3,\ c = 10$

$$x = \frac{\text{-}b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{\text{-}3 \pm \sqrt{3^2 - 4(1)(10)}}{2(1)}$$
$$x = \frac{\text{-}3 \pm \sqrt{9 - 40}}{2}$$
$$x = \frac{\text{-}3 \pm \sqrt{\text{-}31}}{2}$$

No solutions

Equation 3: $9x^2 - 2x - 1 = 0$

$a = 9,\ b = \text{-}2,\ c = \text{-}1$

$$x = \frac{\text{-}b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{2 \pm \sqrt{(\text{-}2)^2 - 4(9)(\text{-}1)}}{2}$$
$$x = \frac{2 \pm \sqrt{4 + 36}}{2}$$
$$x = \frac{2 \pm \sqrt{40}}{2}$$

Equation 4: $x^2 - 10x + 23 = 0$

$a = 1,\ b = \text{-}10,\ c = 23$

$$x = \frac{\text{-}b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{\text{-}10 \pm \sqrt{(\text{-}10)^2 - 4(1)(23)}}{2}$$
$$x = \frac{\text{-}10 \pm \sqrt{\text{-}100 - 92}}{2}$$
$$x = \frac{\text{-}10 \pm \sqrt{\text{-}192}}{2}$$

No solutions

18.3 Sure about That?

1. The equation $h(t) = 2 + 30t - 5t^2$ represents the height, as a function of time, of a pumpkin that was catapulted up in the air. Height is measured in meters, and time is measured in seconds.

   a. The pumpkin reached a maximum height of 47 meters. How many seconds after launch did that happen? Show your reasoning.

   b. Suppose someone was unconvinced by your solution. Find another way (besides the steps you already took) to show your solution is correct.

2. The equation $r(p) = 80p - p^2$ models the revenue a band expects to collect as a function of the price of one concert ticket. Ticket prices and revenues are in dollars.

   A band member says that a ticket price of either \$15.50 or \$74.50 would generate approximately \$1,000 in revenue. Do you agree? Show your reasoning.

Are you ready for more?

Function g is defined by the equation $g(t) = 2 + 30t - 5t^2 - 47$. Its graph opens downward.

1. Find the zeros of function g without graphing. Show your reasoning.

2. Explain or show how the zeros you found can tell us the vertex of the graph of g.

3. Study the expressions that define functions g and h (which defined the height of the pumpkin). Explain how the maximum of function h, once we know it, can tell us the maximum of g.

Lesson 18 Summary

The quadratic formula has many parts in it. A small error in any one part can lead to incorrect solutions.

Suppose we are solving $2x^2 - 6 = 11x$. To use the formula, let's rewrite it in the form of $ax^2 + bx + c = 0$, which gives $2x^2 - 11x - 6 = 0$.

Here are some things to keep in mind:

- Use the correct values for a, b, and c in the formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2(2)}$$

Nope! b is -11, so -b is -(-11), which is 11, not -11.

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2(2)}$$

That's better!

- Multiply 2 by a for the denominator in the formula.

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2}$$

Nope! The denominator is $2a$, which is $2(2)$, or 4.

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2(2)}$$

That's better!

- Remember that squaring a negative number produces a positive number.

$$x = \frac{11 \pm \sqrt{-121 - 4(2)(-6)}}{4}$$

Nope! $(-11)^2$ is 121, not -121.

$$x = \frac{11 \pm \sqrt{121 - 4(2)(-6)}}{4}$$

That's better!

- Remember that a negative number times a positive number is a negative number.

$$x = \frac{11 \pm \sqrt{121 - 48}}{4}$$

Nope! $4(2)(-6) = -48$, and $121 - (-48)$ is $121 + 48$.

$$x = \frac{11 \pm \sqrt{121 + 48}}{4}$$

That's better!

- Follow the properties of algebra.

$$x = \frac{11 \pm \sqrt{169}}{4}$$

$$x = 11 \pm \sqrt{42.25}$$

Nope! Both parts of the numerator, 11 and $\sqrt{169}$, get divided by 4. Also, $\frac{\sqrt{169}}{4}$ is not $\sqrt{42.25}$.

$$x = \frac{11 \pm 13}{4}$$

That's better!

Let's finish by evaluating $\frac{11 \pm 13}{4}$ correctly:

$x = \frac{11+13}{4}$ or $x = \frac{11-13}{4}$

$x = \frac{24}{4}$ or $x = -\frac{2}{4}$

$x = 6$ or $x = -\frac{1}{2}$

To make sure our solutions are correct, we can substitute each solution back into the original equation and see whether it results in a true equation.

Checking 6 as a solution:

$$\begin{aligned} 2x^2 - 6 &= 11x \\ 2(6)^2 - 6 &= 11(6) \\ 2(36) - 6 &= 66 \\ 72 - 6 &= 66 \\ 66 &= 66 \end{aligned}$$

Checking $-\frac{1}{2}$ as a solution:

$$\begin{aligned} 2x^2 - 6 &= 11x \\ 2\left(-\frac{1}{2}\right)^2 - 6 &= 11\left(-\frac{1}{2}\right) \\ 2\left(\frac{1}{4}\right) - 6 &= -\frac{11}{2} \\ \frac{1}{2} - 6 &= -5\frac{1}{2} \\ -5\frac{1}{2} &= -5\frac{1}{2} \end{aligned}$$

We can also graph the equation $y = 2x^2 - 11x - 6$ and find its x-intercepts to see whether our solutions to $2x^2 - 11x - 6 = 0$ are accurate (or close to accurate).