Explaining Steps for Rewriting Equations

Let’s think about why some steps for rewriting equations are valid but other steps are not.

7.1 Math Talk: Could It Be Zero?

Is 0 a solution to each equation?

- $4(x+2)=10$
- $12-8x=3(x+4)$
- $5x=\frac{1}{2}x$
- $\frac{6}{x}+1=8$

7.2 Explaining Acceptable Moves

Here are some pairs of equations. While one partner listens, the other partner should:

- Choose a pair of equations from column A. Explain why, if x is a number that makes the first equation true, then it also makes the second equation true.
- Choose a pair of equations from column B. Explain why the second equation is no longer true for a value of x that makes the first equation true.

Then switch roles until you run out of time or you run out of pairs of equations.

	A	B
1.	$16 = 4(9 - x)$ $16 = 36 - 4x$	$9x = 5x + 4$ $14x = 4$
2.	$5x = 24 + 2x$ $3x = 24$	$\frac{1}{2}x - 8 = 9$ $x - 8 = 18$
3.	$-3(2x + 9) = 12$ $2x + 9 = -4$	$6x - 6 = 3x$ $x - 1 = 3x$
4.	$5x = 3 - x$ $5x = -x + 3$	$-11(x - 2) = 8$ $x - 2 = 8 + 11$
5.	$18 = 3x - 6 + x$ $18 = 4x - 6$	$4 - 5x = 24$ $5x = 20$

7.3 It Doesn't Work!

Noah is having trouble solving two equations. In each case, he takes steps that he thinks are acceptable but ends up with statements that are clearly not true.

Analyze Noah's work on each equation and the moves he made. Are they acceptable moves? Why do you think he ends up with a false equation?

Discuss your observations with your group and be prepared to share your conclusions. If you get stuck, consider solving each equation.

1.
$$\begin{array}{rl} x + 6 = 4x + 1 - 3x & \text{original equation} \\ x + 6 = 4x - 3x + 1 & \text{apply the commutative property} \\ x + 6 = x + 1 & \text{combine like terms} \\ 6 = 1 & \text{subtract } x \text{ from each side} \end{array}$$

2.

$2(5+x)-1=3x+9$	original equation
$10+2x-1=3x+9$	apply the distributive property
$2x-1=3x-1$	subtract 10 from each side
$2x=3x$	add 1 to each side
$2=3$	divide each side by x

Are you ready for more?

1. We can’t divide the number 100 by zero because dividing by zero is undefined. Let's reason about why.
   a. Divide 100 by 10, then 1, then 0.1, then 0.01. What happens as you divide by smaller numbers?
   b. Now divide the number -100 by 10, by 1, by 0.1, 0.01. What is the same and what is different?
2. Tape diagrams can be used to represent division. This tape diagram shows that $6 \div 2 = 3$

2	2	2

   a. Draw a tape diagram that shows why $6 \div \frac{1}{2} = 12$.
   b. Try to draw a tape diagram that represents $6 \div 0$. Explain why this is so difficult.

Lesson 7 Summary

When solving an equation, sometimes we end up with a false equation instead of a solution. Let's look at two examples.

Example 1: $4(x+1) = 4x$

Here are two attempts to solve it.

$4(x+1) = 4x$	original equation	$4(x+1) = 4x$	original equation
$x+1 = x$	divide each side by 4	$4x+4 = 4x$	apply the distributive prop
$1 = 0$	subtract x from each side	$4 = 0$	subtract $4x$ from each side

Each attempt shows acceptable moves, but the final equation is a false statement. Why is that?

When solving an equation, we usually start by assuming that there is at least one value that makes the equation true. The equation $4(x+1) = 4x$ can be interpreted as: 4 groups of $(x+1)$ are equal to 4 groups of x. There are no values of x that can make this true.

For instance, if $x = 10$, then $x + 1 = 11$. It's not possible that 4 times 11 is equal to 4 times 10. Likewise, 1.5 is 1 more than 0.5, but 4 groups of 1.5 cannot be equal to 4 groups of 0.5.

Because of this, the moves made to solve the equation would not lead to a solution. The equation $4(x+1) = 4$ has no solutions.

Example 2: $2x - 5 = \frac{x-20}{4}$

$2x - 5 = \frac{x-20}{4}$	original equation
$8x - 20 = x - 20$	multiply each side by 4
$8x = x$	add 20 to each side
$8 = 1$	divide each side by x

Each step in the process seems acceptable, but the last equation is a false statement.

It is not easy to tell from the original equation whether it has a solution, but if we look at the equivalent equation $8x = x$, we can see that 0 could be a solution. When x is 0, the equation is $0 = 0$, which is a true statement. What is going on here?

The last move in the solving process was division by x. Because 0 could be the value of x and dividing by 0 gives an undefined number, we don't usually divide by the variable we're solving for. Doing this might make us miss a solution, namely $x = 0$.

Here are two ways to solve the equation once we get to $8x = x$:

$8x = x$		$8x = x$	
$7x = 0$	subtract x from each side	$0 = -7x$	subtract $8x$ from each side
$x = 0$	divide each side by 7	$0 = x$	divide each side by -7