Unit 1, Lesson 6

Scaling and Unscaling

Let’s examine the relationships between areas of dilated figures and scale factors.

6.1 Transamerica Building

The image shows the Transamerica Building in San Francisco. It’s shaped like a pyramid.

The bottom floor of the building is a rectangle measuring approximately 53 meters by 44 meters. The top floor of the building is a dilation of the base by scale factor $k = 0.32$.

Ignoring the triangular “wings” on the sides, what is the area of the top floor? Explain or show your reasoning.

6.2 Two Viewpoints

A triangle has an area of 100 square inches. It’s dilated by a factor of $k = 0.25$.

Mai says, “The dilated triangle’s area is 25 square inches.”

Lin says, “The dilated triangle’s area is 6.25 square inches.”

1. For each student, decide whether you agree with their student's statement. If you agree, explain why. If you disagree, explain what the student may have done to arrive at their answer.

2. Calculate the area of the image if the original triangle is dilated by each of these scale factors:
   a. $k = 9$
   b. $k = \frac{3}{4}$

6.3 Graphing Areas and Scale Factors

An artist created a painting on a canvas with an area of 1 square foot. Now she wants to create more paintings of different sizes that are all scaled copies of her original painting. The primer she uses to prepare the canvas is expensive, so she wants to know the sizes she can create using different amounts of primer.

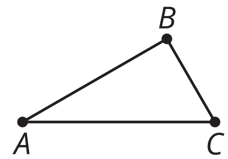
1. Suppose the artist has enough primer to cover 9 square feet. If she uses all her primer, by what scale factor can she dilate her original painting?
2. Complete the table that shows the relationship between the dilated area (x) and the scale factor (y). Round values to the nearest tenth, if needed.

dilated area in square feet	scale factor
1	
4	
9	
16	
x	

3. On graph paper, plot the points from the table and connect them with a smooth curve.
4. Use your graph to estimate the scale factor the artist could use if she had enough primer to cover 12 square feet.

Are you ready for more?

The image shows triangle ABC.



1. Sketch the result of dilating triangle ABC using a scale factor of 2 and a center of A. Label it $AB'C'$.
2. Sketch the result of dilating triangle ABC using a scale factor of -2 and a center of A. Label it $AB''C''$.
3. Find a transformation that would take triangle $AB'C'$ to $AB''C''$.

6.4 Prepping Canvas

One bottle of primer covers about 10 square feet of canvas.

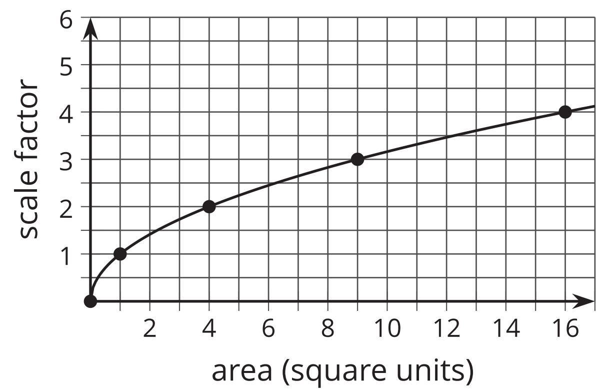
1. Suppose the artist has enough primer to cover 1 square foot, and she buys another bottle.
   a. What is the scale factor for a 1 square foot painting?
   b. What is the scale factor for the 11 square foot painting?
   c. Find the rate of change between the original amount of paint and the new total.
2. Suppose the artist has enough primer to cover 16 square feet, and she buys another bottle. Find the rate of change between the original amount of paint and the new total.
3. The artist also needs to stretch the canvas onto a frame. The original painting had a perimeter of 4 feet. The function representing the scale factor for a given length of frame, x is $g(x) = \frac{1}{4}x$. Graph the function. Verify that the graph gives the correct scale factors for a perimeter of 4 feet and a perimeter of 8 feet.
4. Compare the graph of square feet vs. scale factor to the graph of perimeter vs. scale factor.

Lesson 6 Summary

If we know the area of an original figure and its dilation, we can work backward to find the scale factor. For example, suppose we have a circle with area of 1 square unit, and a dilation of the circle with area of 64 square units. We know that the circle must have been dilated by a factor of 8, because $8^2 = 64$. Another way to say this is $\sqrt{64} = 8$.

A graph can help us understand the relationship between dilated areas and scale factors. We can make a table of values for the dilated circle, plot the points on a graph, and connect them with a smooth curve. In this table, the dilated area is the input or x-value, and the scale factor is the output or y-value. Remember that the area of the original circle is 1 square unit, so the square root of the dilated area is the same as the scale factor.

dilated area in square units	**scale factor**
0	0
1	1
4	2
9	3
16	4



This graph represents the equation that describes the relationship between area and scale factor: $y = \sqrt{x}$. Note that the rate of change isn't constant. On the left side, the graph is fairly steep. As the area increases, the scale factor increases quickly. But on the right side, the graph flattens out. As the area continues to increase, the scale factor still increases, but not as quickly.

