



# Applying the Quadratic Formula (Part 2)

Let's use the quadratic formula and solve quadratic equations with care.

## 18.1 Bits and Pieces

Evaluate each expression for  $a = 9$ ,  $b = -5$ , and  $c = -2$

1.  $-b$
2.  $b^2$
3.  $b^2 - 4ac$
4.  $-b \pm \sqrt{a}$

## 18.2 Using the Formula with Care

Here are four equations, followed by attempts to solve them using the quadratic formula. Each attempt contains at least one error.

- Solve 1–2 equations by using the quadratic formula.
- Then, find and describe the error(s) in the worked solutions of the same equations as the ones you solved.

Equation 1:  $2x^2 + 3 = 8x$

Equation 2:  $x^2 + 3x = 10$

Equation 3:  $9x^2 - 2x - 1 = 0$

Equation 4:  $x^2 - 10x + 23 = 0$

Here are the worked solutions with errors:

Equation 1:  $2x^2 + 3 = 8x$

$a = 2, b = -8, c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{64 - 24}}{4}$$

$$x = \frac{8 \pm \sqrt{40}}{4}$$

$$x = 2 \pm \sqrt{10}$$

Equation 2:  $x^2 + 3x = 10$

$a = 1, b = 3, c = 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 - 40}}{2}$$

$$x = \frac{-3 \pm \sqrt{-31}}{2}$$

No solutions

Equation 3:  $9x^2 - 2x - 1 = 0$

$a = 9, b = -2, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(9)(-1)}}{2}$$

$$x = \frac{2 \pm \sqrt{4 + 36}}{2}$$

$$x = \frac{2 \pm \sqrt{40}}{2}$$

Equation 4:  $x^2 - 10x + 23 = 0$

$a = 1, b = -10, c = 23$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{(-10)^2 - 4(1)(23)}}{2}$$

$$x = \frac{-10 \pm \sqrt{100 - 92}}{2}$$

$$x = \frac{-10 \pm \sqrt{-192}}{2}$$

No solutions



## 18.3 Sure about That?

1. The equation  $h(t) = 2 + 30t - 5t^2$  represents the height, as a function of time, of a pumpkin that was catapulted up in the air. Height is measured in meters, and time is measured in seconds.
  - a. The pumpkin reached a maximum height of 47 meters. How many seconds after launch did that happen? Show your reasoning.
  - b. Suppose someone was unconvinced by your solution. Find another way (besides the steps you already took) to show your solution is correct.
2. The equation  $r(p) = 80p - p^2$  models the revenue a band expects to collect as a function of the price of one concert ticket. Ticket prices and revenues are in dollars.

A band member says that a ticket price of either \$15.50 or \$74.50 would generate approximately \$1,000 in revenue. Do you agree? Show your reasoning.

 **Are you ready for more?**

Function  $g$  is defined by the equation  $g(t) = 2 + 30t - 5t^2 - 47$ . Its graph opens downward.

1. Find the zeros of function  $g$  without graphing. Show your reasoning.
2. Explain or show how the zeros you found can tell us the vertex of the graph of  $g$ .
3. Study the expressions that define functions  $g$  and  $h$  (which defined the height of the pumpkin). Explain how the maximum of function  $h$ , once we know it, can tell us the maximum of  $g$ .

## Lesson 18 Summary

The quadratic formula has many parts in it. A small error in any one part can lead to incorrect solutions.

Suppose we are solving  $2x^2 - 6 = 11x$ . To use the formula, let's rewrite it in the form of  $ax^2 + bx + c = 0$ , which gives  $2x^2 - 11x - 6 = 0$ .

Here are some things to keep in mind:

- Use the correct values for  $a$ ,  $b$ , and  $c$  in the formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2(2)}$$

Nope!  $b$  is  $-11$ , so  $-b$  is  $-(-11)$ , which is  $11$ , not  $-11$ .

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2(2)}$$

That's better!

- Multiply  $2$  by  $a$  for the denominator in the formula.

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2}$$

Nope! The denominator is  $2a$ , which is  $2(2)$ , or  $4$ .

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2(2)}$$

That's better!

- Remember that squaring a negative number produces a positive number.

$$x = \frac{11 \pm \sqrt{-121 - 4(2)(-6)}}{4}$$

Nope!  $(-11)^2$  is  $121$ , not  $-121$ .

$$x = \frac{11 \pm \sqrt{121 - 4(2)(-6)}}{4}$$

That's better!

- Remember that a negative number times a positive number is a negative number.

$$x = \frac{11 \pm \sqrt{121 - 48}}{4}$$

Nope!  $4(2)(-6) = -48$ , and  $121 - (-48)$  is  $121 + 48$ .

$$x = \frac{11 \pm \sqrt{121 + 48}}{4}$$

That's better!

- Follow the properties of algebra.

$$x = \frac{11 \pm \sqrt{169}}{4}$$

$$x = 11 \pm \sqrt{42.25}$$

Nope! Both parts of the numerator, 11 and  $\sqrt{169}$ , get divided by 4. Also,  $\frac{\sqrt{169}}{4}$  is not  $\sqrt{42.25}$ .

$$x = \frac{11 \pm 13}{4}$$

That's better!

Let's finish by evaluating  $\frac{11 \pm 13}{4}$  correctly:

$$x = \frac{11 + 13}{4}$$

$$x = \frac{24}{4}$$

$$x = 6$$

$$\text{or } x = \frac{11 - 13}{4}$$

$$\text{or } x = -\frac{2}{4}$$

$$\text{or } x = -\frac{1}{2}$$

To make sure our solutions are correct, we can substitute each solution back into the original equation and see whether it results in a true equation.

Checking 6 as a solution:

$$\begin{aligned} 2x^2 - 6 &= 11x \\ 2(6)^2 - 6 &= 11(6) \\ 2(36) - 6 &= 66 \\ 72 - 6 &= 66 \\ 66 &= 66 \end{aligned}$$

Checking  $-\frac{1}{2}$  as a solution:

$$\begin{aligned} 2x^2 - 6 &= 11x \\ 2\left(-\frac{1}{2}\right)^2 - 6 &= 11\left(-\frac{1}{2}\right) \\ 2\left(\frac{1}{4}\right) - 6 &= -\frac{11}{2} \\ \frac{1}{2} - 6 &= -5\frac{1}{2} \\ -5\frac{1}{2} &= -5\frac{1}{2} \end{aligned}$$

We can also graph the equation  $y = 2x^2 - 11x - 6$  and find its  $x$ -intercepts to see whether our solutions to  $2x^2 - 11x - 6 = 0$  are accurate (or close to accurate).

