Unit 5, Lesson 7

Graphs of Proportional Relationships

Goals

- Generalize (orally and in writing) that a proportional relationship can be represented in the coordinate plane by a line that includes the "origin" or by a collection of points that lie on such a line.
- Identify the constant of proportionality from the graph of a proportional relationship.
- Interpret (orally and in writing) points on the graph of a proportional relationship.

Learning Targets

- I can find the constant of proportionality from a graph.
- I know that the graph of a proportional relationship lies on a line through $(0, 0)$.

Lesson Narrative

This lesson introduces an important way of representing a proportional relationship: its graph. Students plot points on the graph from a table of values in a proportional relationship. They see that the graph of a proportional relationship always lies on a line that passes through the **origin**, $(0, 0)$.

Then, students interpret points on the graph of a proportional relationship. They make connections between the graph, the equation, and the context modeled by the relationship. Extra attention is given to the meaning of the point $(1, k)$ on the graph, both in terms of the constant of proportionality k in the equation $y = kx$ and in terms of a constant rate in the context. As they relate representations and situations, students practice reasoning quantitatively and abstractly (MP2).

Standards

Building On	**5.G.A, 6.NS.C.8**
Addressing	**7.RP.A.2, 7.RP.A.2.b, 7.RP.A.2.d**
Building Toward	**7.RP.A.2.a**

Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Rulers: Activity 2

Required Preparation

Activity 1:

For the digital version of the activity, acquire devices that can run the applet.

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's see how graphs of proportional relationships differ from graphs of other relationships.

7.1 Notice These Points

Warm-up

5 min

Activity Narrative

There is a digital version of this activity.

This *Warm-up* prepares students for graphing proportional relationships in the **coordinate plane**. They practice graphing coordinate points and notice that all points lie on a straight line.

In the digital version of the activity, students use an applet to plot points on the coordinate plane. The applet allows students to add, remove, adjust, and label points. The digital version may help students graph quickly and accurately so they can focus more on the mathematical analysis.

Standards

Building On **5.G.A**

Building Toward **7.RP.A.2.a**

Instructional Routines

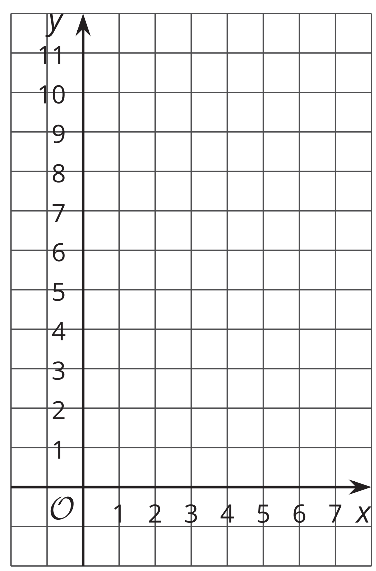
- MLR8: Discussion Supports

Launch

Give students 2–3 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

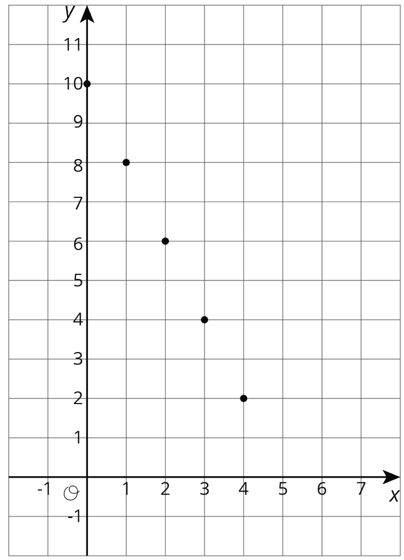
1. Plot the points $(0, 10), (1, 8), (2, 6), (3, 4), (4, 2)$.



2. What do you notice about the graph?

Student Response

1.



2. Sample responses:
    - The points line up so that they could all be connected with a single line.
    - The line goes down when reading left to right.
    - Every time the x-coordinate goes up 1, the y-coordinate goes down 2.

Building on Student Thinking

Some students may reverse the x- and y-coordinates when plotting points, such as plotting $(10, 0)$ and $(8, 1)$ instead of $(0, 10)$ and $(1, 8)$. Direct their attention to the x and y labels in the image and clarify that an ordered pair is written (x, y).

Activity Synthesis

The goal of this discussion is to review how to graph ordered pairs, (x, y), on the coordinate plane. Invite students to share their observations about the graph. Ask if other students agree. If some students do not agree that the points lie on a straight line, ask which points break the pattern and give students a chance to self-correct their work.

Access for English Language Learners

MLR8 Discussion Supports. Revoice student ideas to demonstrate and amplify mathematical language use. For example, revoice the student statement "The points are straight" as "The points line up on the coordinate plane so that they could all be connected with a single line."
Advances: Representing

7.2 T-shirts for Sale

10 min

Activity Narrative

There is a digital version of this activity.

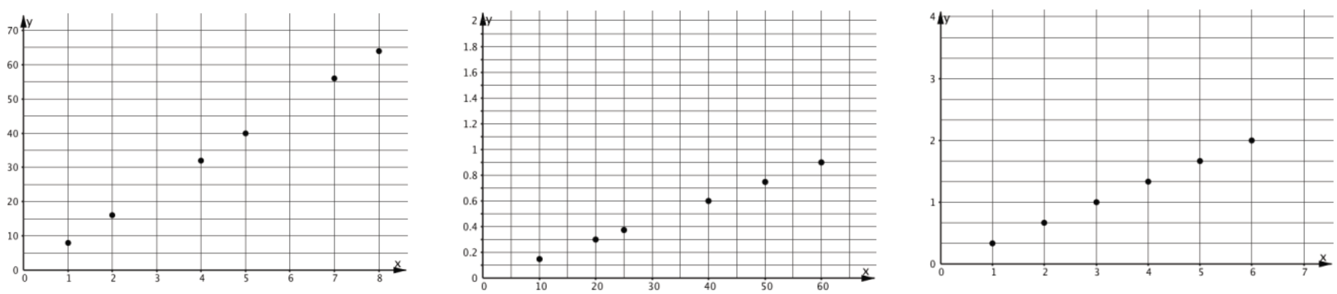
In this activity, students create a graph to represent the proportional relationship given in a table. The goal is for students to notice that the points lie on a straight line that goes through the origin. The class discussion also prompts students to consider whether it makes sense to connect the points with a line.

In the digital version of the activity, students use an applet to plot points from a table on the coordinate plane. The applet allows students to add, remove, adjust, and label points. The digital version may help students graph quickly and accurately so they can focus more on the mathematical analysis.

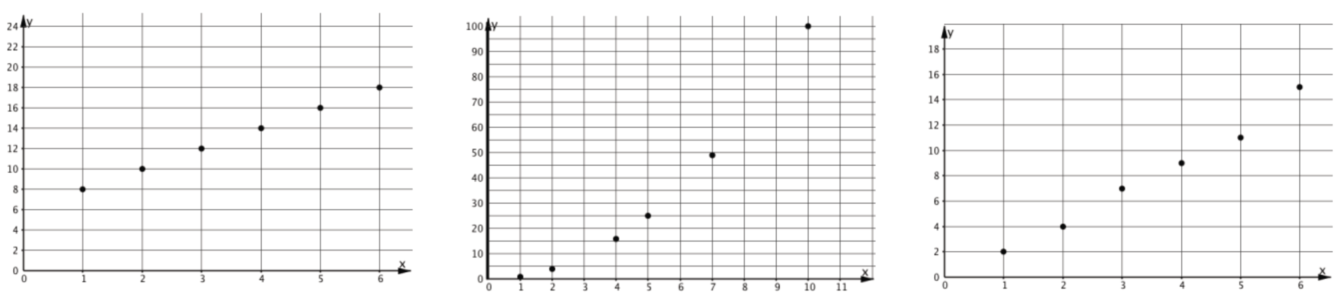
Teacher Notes for IM 6–8 Math Accelerated v.360
Adjust the timing of this activity to 15 minutes.

During the *Activity Synthesis*, display these graphs of proportional and nonproportional relationships for all to see.

Graphs of Proportional Relationships



Graphs of Nonproportional Relationships



Ask students, “What properties do the graphs representing proportional relationships have?” (The points are all in a line. The line goes through the point $(0, 0)$.) If necessary, use a straightedge to show that the points are all in a line.

Introduce the word **origin** to refer to the point $(0, 0)$ if students are unfamiliar with the term.

Standards

Building On **6.NS.C.8**

Addressing **7.RP.A.2**

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Provide access to rulers. Give students 5 minutes of quiet work time followed by partner and whole-class discussion.

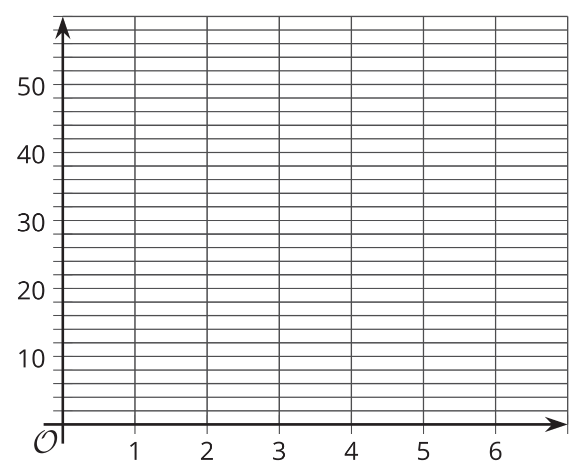
If students are unsure how to plot points from the table, consider rewriting the values in the first row of the table as an ordered pair, $(1, 8)$, and demonstrating how to plot this point.

Student Task Statement

Some T-shirts cost $8 each.

x	y
1	8
2	16
3	24
4	32
5	40
6	48

1. Use the table to answer these questions.
    a. What does x represent?
    b. What does y represent?
    c. Is there a proportional relationship between x and y?
2. Plot the pairs in the table on the coordinate plane.

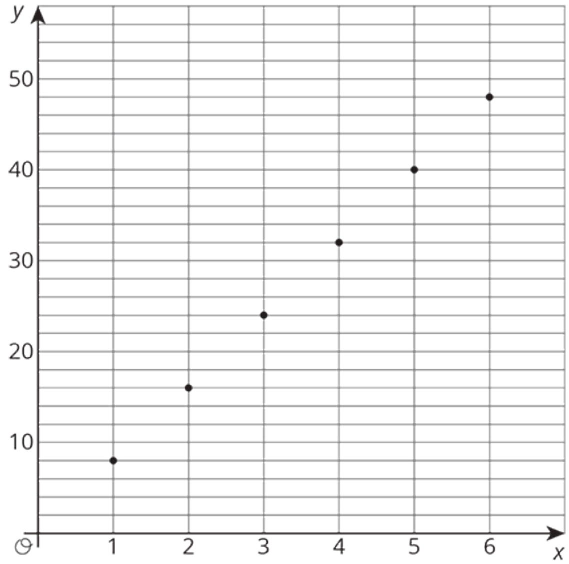


3. What do you notice about the graph?

Student Response

1. a. x is the number of T-shirts.
    b. y is the total cost of those T-shirts.

c. x is proportional to y. Students may identify 8 as the constant of proportionality.

2. 

3. Students may notice that the points lie on a line.

Activity Synthesis

The goal of this discussion is to highlight the fact that the graph of a proportional relationship makes a straight line through the origin. Display a graph with the points plotted correctly for all to see. Invite students to share how to label the axes. (The x-axis represents "number of T-shirts" and the y-axis represents "cost in dollars".)

Ask students to share their observations about the plotted points. If not mentioned by students, highlight that the points lie on a straight line and that the line goes through $(0, 0)$.

Direct students' attention to considering the meaning of other points that are also on this line but were not in the table. Ask, "Could we buy 0 shirts? 7 T-shirts? 10 T-shirts? Can we buy half of a T-shirt?" Note that the graph consists of discrete points because only whole numbers of T-shirts make sense in this context. However, people often connect discrete points with a line to make the relationship more clear, even when the in-between values don't make sense.

Ask the students, "Suppose instead of price per shirt, this graph displayed the cost of cherries that are $8 per pound. Given that context, how should we change the graph?" Weights need not have integer values, so the graph is not restricted to discrete points. If you haven't done so already, draw the ray starting at (0, 0) that passes through the points.

Access for English Language Learners

MLR8 Discussion Supports. During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say . . . ." Original speakers can agree or clarify for their partner.
Advances: Listening, Speaking

7.3 Tyler's Walk

20 min

Activity Narrative

In this activity students interpret points on the graph of a proportional relationship in terms of what they mean about the situation (MP2). This activity is intended to further students' understanding of the graphs of proportional relationships in the following respects:

- For any point on the graph except $(0, 0)$, the quotient of the coordinates, $\frac{y}{x}$, is the constant of proportionality.
- When the x-coordinate is 1, the corresponding y-coordinate is k, the constant of proportionality.

Students explain correspondences between parts of the table and parts of the graph. The graph is simple so that students can focus on what a point means in the situation represented. Students need to realize, however, that the axes are marked in 10-unit intervals.

Teacher Notes for IM 6–8 Math Accelerated v.360
Adjust the timing of this activity to 15 minutes. To move more quickly through the activity, consider instructing students to complete the last 2 questions as part of their partner discussion rather than during quiet work time.

Standards

Addressing **7.RP.A.2.b, 7.RP.A.2.d**

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time, followed by partner and whole-class discussion.

Access for English Language Learners

MLR5 Co-Craft Questions. Keep books or devices closed. Display only the problem stem and graph, without revealing the questions, and ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask, "What do these questions have in common? How are they different?" Reveal the intended questions for this task and invite additional connections.
Advances: Reading, Writing

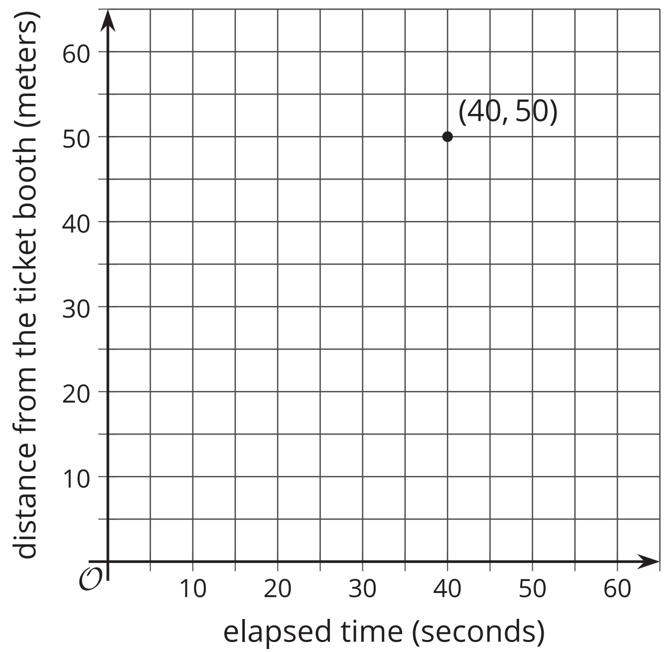
Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization, Attention

Student Task Statement

Tyler was at the amusement park. He walked at a steady pace from the ticket booth to the bumper cars.

1. The point on the graph shows his arrival at the bumper cars. What do the coordinates of the point tell us about the situation?
2. The table representing Tyler's walk shows other values of time and distance. Complete the table. Next, plot the pairs of values on the grid.
3. What does the point $(0, 0)$ mean in this situation?
4. How far away from the ticket booth was Tyler after 1 second? Label the point on the graph that shows this information with its coordinates.
5. What is the constant of proportionality for the relationship between time and distance? What does it tell you about Tyler's walk? Where do you see it in the graph?



time (seconds)	distance (meters)
0	0
20	25
30	37.5
40	50
1	

Student Response

1. 40 seconds after Tyler started walking, he was 50 meters from the ticket booth. The 40 represents the elapsed time in seconds since Tyler started walking away from the ticket booth. The 50 represents Tyler's distance in meters from the ticket booth at that time.
2. Students should write 1.25 in the empty cell of the table and plot $(0, 0)$, $(1, 1.25)$, $(20, 25)$, and $(30, 37.5)$.
3. Before any time passed, there was no distance between Tyler and the ticket booth.
4. Tyler was 1.25 meters from the ticket booth after 1 second. The corresponding point is $(1, 1.25)$.
5. The constant of proportionality is 1.25. It expresses that Tyler is walking at a speed of 1.25 meters per second. It appears as the y-coordinate in $(1, 1.25)$.

Are You Ready for More?

If Tyler wanted to get to the bumper cars in half the time, how would the graph representing his walk change? How would the table change? What about the constant of proportionality?

Extension Student Response

The graph would be steeper. For the same first coordinate, the second coordinate would be twice as big as in the original situation. The table would include $(0, 0)$, $(20, 50)$, and $(1, 2.5)$. Tyler would already arrive at the bumper cars after 20 seconds and his speed would be 2.5 meters per second.

Activity Synthesis

The goal of the discussion is to make connections between the table and the graph and how they each represent the situation. First, ask students:

- "How far is the ticket booth from the bumper cars? How do you know?" (50 meters, because the point $(40, 50)$ represents his arrival at the bumper cars. 40 is the elapsed time in seconds and 50 is the distance in meters.)

Consider clarifying for students that this is assuming that Tyler walked in a straight line. This is an opportunity for attention to precision (MP6) and making explicit assumptions about a situation (MP4).

For each of the following questions, ask students to share how they can tell the answer from the table and how they can tell from the graph.

- “What quantities are shown?” (distance in meters that Tyler is from the ticket booth and elapsed time in seconds since he started walking)
  - “How can you tell from the table?” (by looking at the column headers)
  - “How can you tell from the graph?” (by looking at the axes labels)
- “What does each pair of values mean? For example $(20, 25)$ and $(0, 0)$?” (Tyler was 25 meters from the ticket booth after 20 seconds. Tyler was 0 meters away from the ticket booth after 0 seconds.)
  - “How can you tell from the table?” (The value in the first column gives the elapsed time in seconds since Tyler started walking. The value in the second column tells how many meters away from the ticket booth Tyler was at that time.)
  - “How can you tell from the graph?” (The x-coordinate gives the elapsed time in seconds since Tyler started walking. The corresponding y-coordinate shows how many meters away from the ticket booth Tyler was at that time.)
- “Is the relationship proportional?” (yes)
  - “How can you tell from the table?” (Dividing the values on any row, except $(0, 0)$, gives the same unit rate.)
  - “How can you tell from the graph?” (The points lie on a straight line through $(0, 0)$.)
- “What is the constant of proportionality?” (1.25)
  - “How can you tell from the table?” (Find the value for the second column that goes next to the 1 in the first column.)
  - “How can you tell from the graph?” (Find the point $(1, k)$ that lies on the same line as the other points or find the quotient $\frac{y}{x}$ for any point on the graph besides $(0, 0)$.)

After students have seen how the different representations show the same information, consider asking students, “Are there any benefits or drawbacks to one representation compared to the other? Which representation do you prefer?”

Lastly, ask students to write an equation for this proportional relationship. (Sample responses: $y = 1.25x$ or $d = 1.25t$)

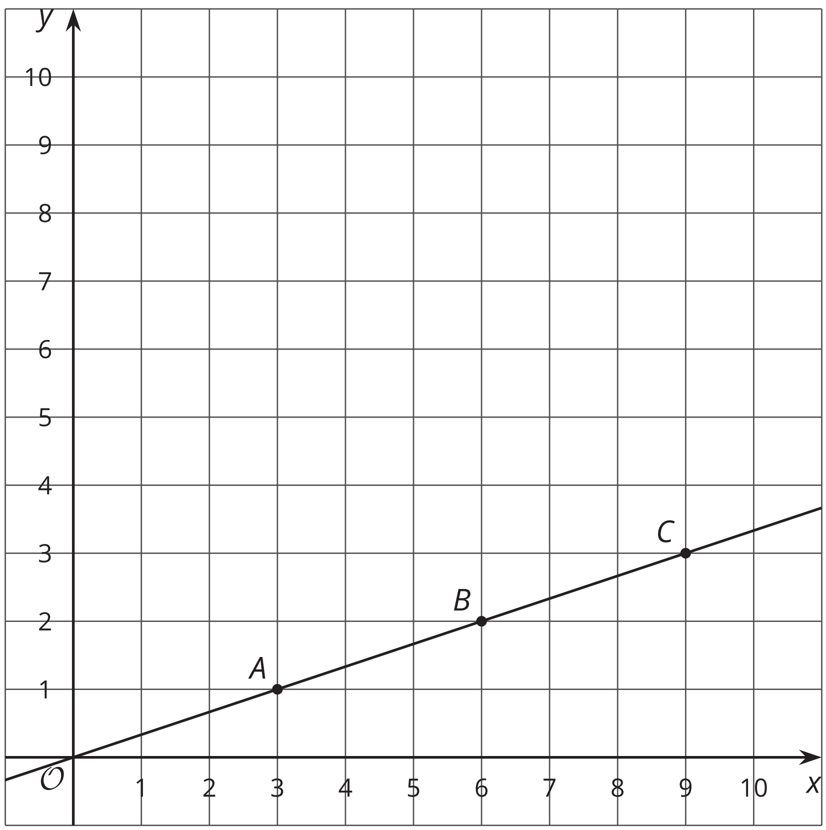
Lesson Synthesis

Share with students "Today we examined graphs of relationships. Some were proportional and some were not."

To help students generalize about graphs of proportional relationships, consider asking students:

- "What characteristics were shared by all the graphs of proportional relationships that we saw?" (The points were arranged in a straight line. The point $(0, 0)$ lines up with the other points.)
- "What characteristics might you see on a graph that would let you know that the relationship is not proportional?" (The points are not arranged in a straight line. The point $(0, 0)$ does not line up with the other points.)

If desired, use this example to review interpreting the points on a graph in terms of the context it represents.



- "What is the constant of proportionality for the relationship shown on this graph?" ($\frac{1}{3}$, which is the y-value that goes with the x-value of 1.)
- "Let's say the graph represents the cost of stickers that come in packs of 3."
  - "What does point B represent in this situation?" (You can buy 6 stickers for $2.)
  - "Does it make sense to connect the points with a line for this situation? Why or why not?" (No, because it is not possible to buy a fraction of a sticker.)
- "Instead let's say this graph shows the unit conversion between feet and yards."
  - "What does point B represent in this situation?" (There are 2 yards in 6 feet.)
  - "Does it make sense to connect the points with a line for this situation? Why or why not?" (Yes, because it is possible for a measurement to be a fraction of a foot or a fraction of a yard.)

7.4 Filling a Bucket

Cool-down

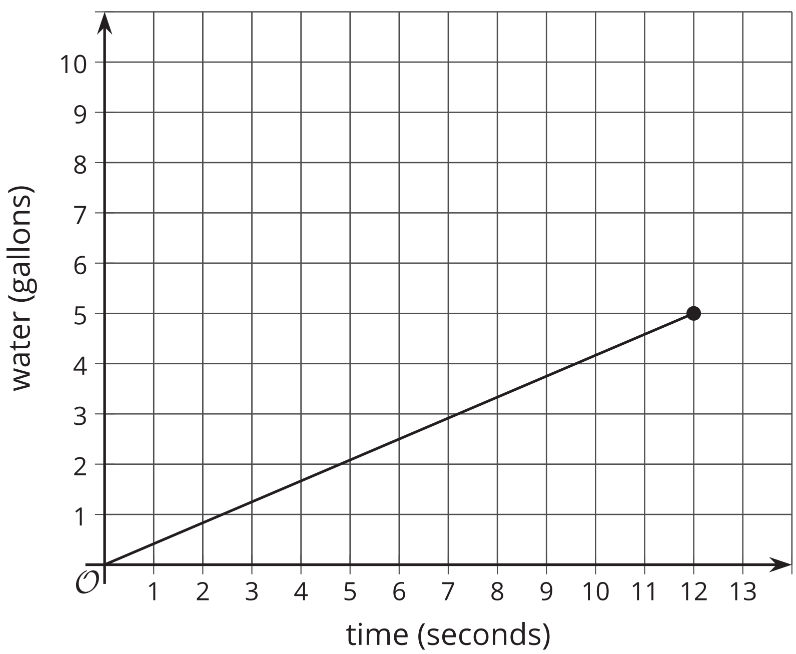
5 min

Standards

Addressing **7.RP.A.2.d**

Student Task Statement

Water runs from a hose into a bucket at a steady rate. The amount of water in the bucket for the time it is being filled is shown in the graph.



1. The point $(12, 5)$ is on the graph. What do the coordinates tell you about the water in the bucket?
2. How many gallons of water are in the bucket after 1 second? Label the point on the graph that shows this information.

Student Response

1. After 12 seconds, there were 5 gallons of water in the bucket.
2. $\frac{5}{12}$ (or equivalent). The point $\left(1, \frac{5}{12}\right)$ should be labeled.

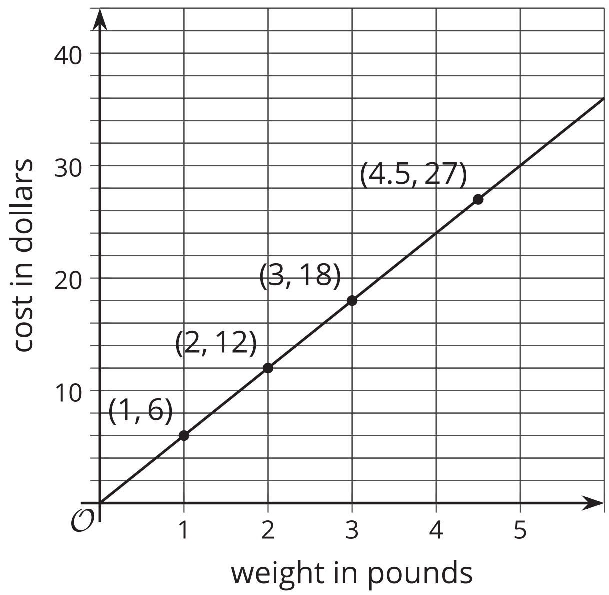
Responding to Student Thinking

Points to Emphasize

If students struggle with interpreting points on the graph of a proportional relationship, focus on this as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about the meaning of various points on the graph.

Lesson 7 Summary

One way to represent a proportional relationship is with a graph. Here is a graph that represents different amounts that fit the situation, "Blueberries cost $6 per pound."



Different points on the graph tell us, for example, that 2 pounds of blueberries cost $12, and 4.5 pounds of blueberries cost $27.

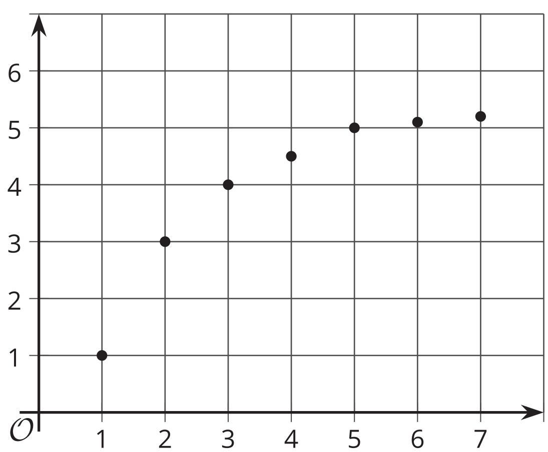
Sometimes it makes sense to connect the points with a line, and sometimes it doesn't. For example, we could buy 4.5 pounds of blueberries or 1.875 pounds, or any other part of a whole pound. So all the points between the whole numbers make sense in the situation, and any point on the line is meaningful.

If the graph represented the cost for different *numbers of sandwiches* (instead of pounds of blueberries), it might not make sense to connect the points with a line, because it is often not possible to buy 4.5 sandwiches or 1.875 sandwiches. Even if only points make sense in the situation, though, sometimes we connect them with a line anyway to make the relationship easier to see.

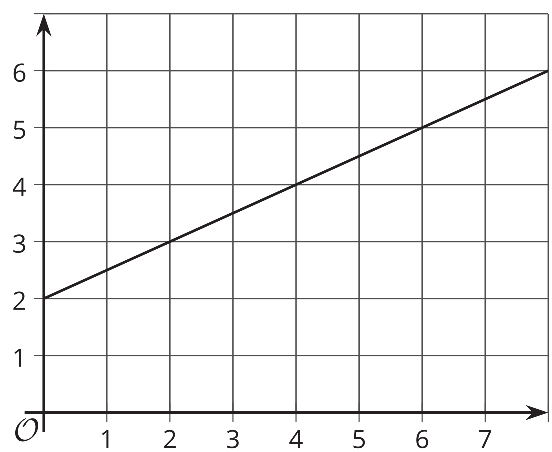
Graphs that represent proportional relationships all have a few things in common:

- Points that satisfy the relationship lie on a straight line.
- The line that they lie on passes through the **origin**, $(0, 0)$.

Here are some graphs that do *not* represent proportional relationships:



These points do not lie on a line.



This is a line, but it doesn't go through the origin.

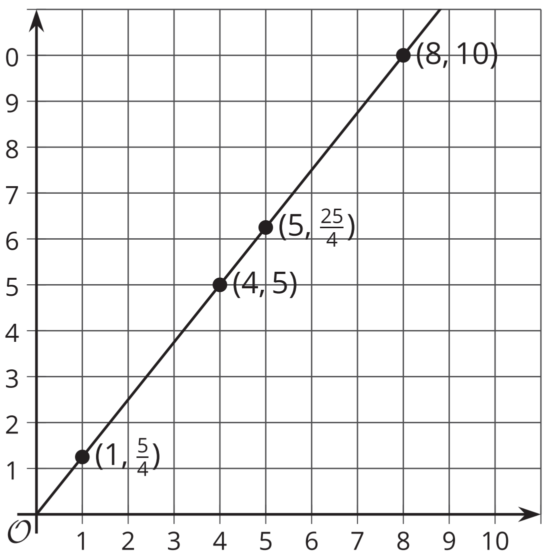
Here is a different example.

For the relationship represented in this table, y is proportional to x. We can see in this table that $\frac{5}{4}$ is the constant of proportionality because it's the y value when x is 1.

The equation $y = \frac{5}{4}x$ also represents this relationship.

x	y
4	5
5	$\frac{25}{4}$
8	10
1	$\frac{5}{4}$

Here is the graph of this relationship.



If y represents the distance in feet that a snail crawls in x minutes, then the point $(4, 5)$ tells us that the snail can crawl 5 feet in 4 minutes.

If y represents the cups of yogurt and x represents the teaspoons of cinnamon in a recipe for fruit dip, then the point $(4, 5)$ tells us that you can mix 4 teaspoons of cinnamon with 5 cups of yogurt to make this fruit dip.

We can find the constant of proportionality by looking at the graph: $\frac{5}{4}$ is the y-coordinate of the point on the graph where the x-coordinate is 1. This could mean the snail is traveling $\frac{5}{4}$ feet per minute or that the recipe calls for $1\frac{1}{4}$ cups of yogurt for every teaspoon of cinnamon.

In general, when y is proportional to x, the corresponding constant of proportionality is the y-value when $x = 1$.

Glossary

- coordinate plane
- origin

Lesson 7 Practice Problems

1 Student Task Statement

A lemonade recipe calls for $\frac{1}{4}$ cup of lemon juice for every 1 cup of water.

a. Use the table to answer these questions.
   i. What does x represent?
   ii. What does y represent?
   iii. Is there a proportional relationship between x and y?

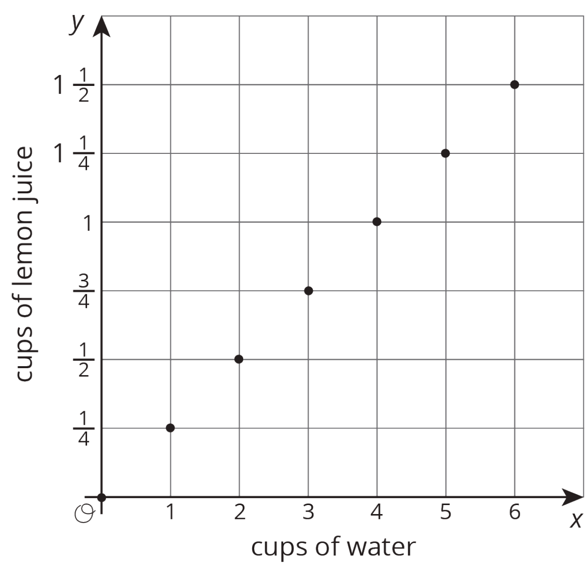
b. Plot the pairs in the table in a coordinate plane.

x	y
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

Solution

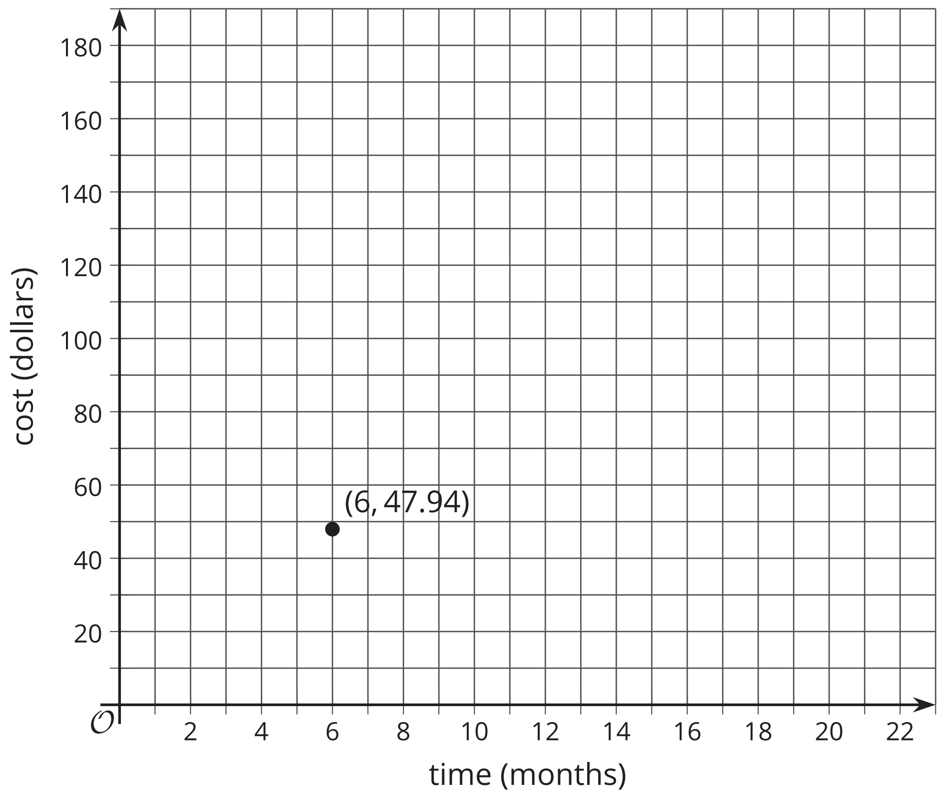
a. i. x represents the cups of water
   ii. y represents the cups of lemon juice
   iii. Yes

b.



2 Student Task Statement

There is a proportional relationship between the number of months a person has had a streaming movie subscription and the total amount of money they have paid for the subscription. The cost for 6 months is \$47.94. The point $(6, 47.94)$ is shown on this graph:



a. What is the constant of proportionality in this relationship?

b. What does the constant of proportionality tell us about the situation?

c. Add at least three more points to the graph and label them with their coordinates.

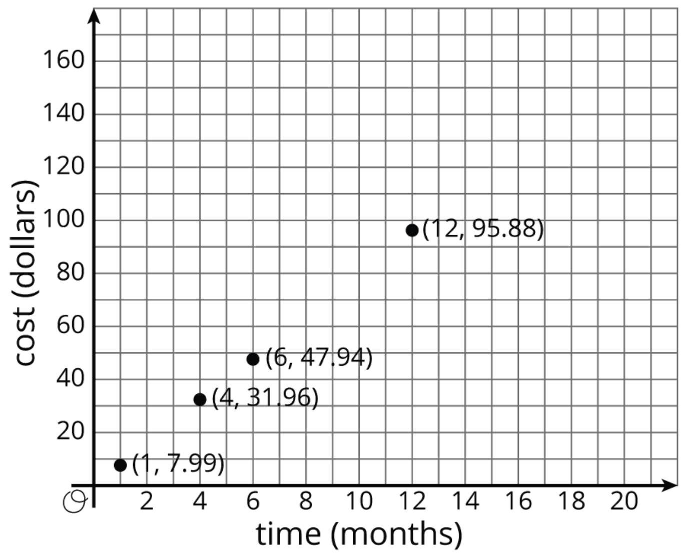
d. Write an equation that represents the relationship between C, the total cost of the subscription, and m, the number of months.

Solution

a. $7.99

b. The movie streaming service costs $7.99 for one month of service.

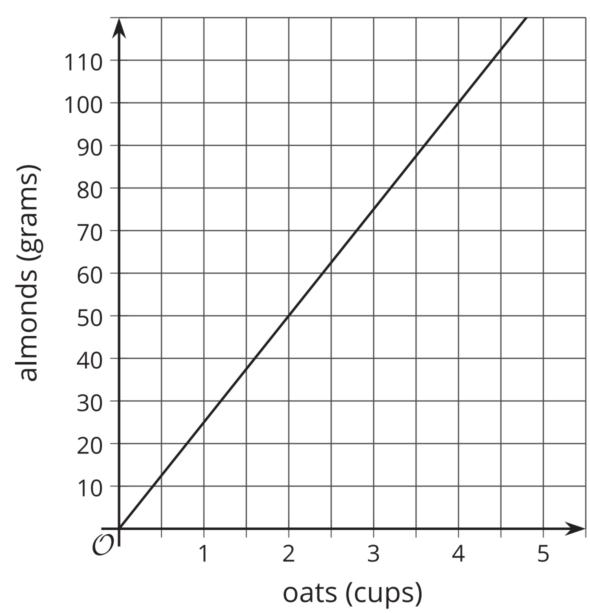
c. Sample response:



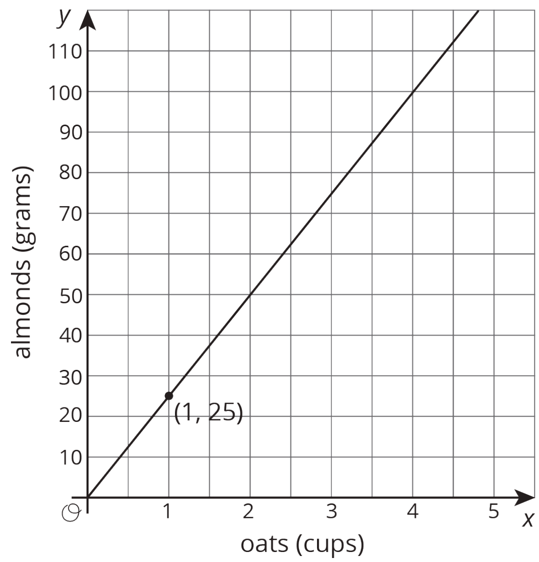
d. $C = 7.99m$

3 Student Task Statement

The graph shows the amounts of almonds, in grams, for different amounts of oats, in cups, in a granola mix. Label the point $(1, k)$ on the graph, find the value of k, and explain its meaning.



Solution



The point $(1, 25)$ is on the graph. It means that for each cup of oats there are 25 grams of almonds in the granola mix.

4 from Unit 5, Lesson 6

Student Task Statement

Select **all** the pieces of information that would tell you x and y have a proportional relationship. Let y represent the distance in meters between a rock and a turtle's current position and x represent the time in minutes the turtle has been moving.

A. $y = 3x$

B. After 4 minutes, the turtle has walked 12 feet away from the rock.

C. The turtle walks for a bit, then stops for a minute before walking again.

D. The turtle walks away from the rock at a constant rate.

E. The turtle starts out walking slowly and speeds up as it gets farther away from the rock.

Solution

A, D

5 from Unit 5, Lesson 6

Student Task Statement

What information do you need to know to write an equation relating two quantities that have a proportional relationship?

Solution

A constant of proportionality and variables for the quantities.

