Scope and Sequence for Accelerated 6

IM Accelerated 6 begins with an exploration of area and surface area—an invitation for students to engage with novel ideas that they can represent concretely and visually, and reason about in intuitive ways. Starting with geometry also creates opportunities to elicit close observation, sense- and connection-making, and the exchange of ideas—elements of a healthy learning community.

The next unit introduces ratios and rates, concepts that are also new. Students learn to represent, make sense of, and solve problems about equivalent ratios, rates, unit rates, and percentages. The mathematical reasoning here constitutes major work of the grade.

In the unit that follows, students expand and deepen their prior knowledge of numbers and operations. First, students explore division involving fractions, and work toward dividing a fraction by fraction. Next, they learn to multiply and divide multi-digit, base-ten numbers, including decimals, using the standard algorithm for each operation. Building fluency with algorithms takes time and continues beyond this unit.

Next, students further their understanding of equations and expressions, including those with variables. Students consider ways to represent, justify, and generate equivalent expressions. They also use expressions and equations to describe the relationship between quantities.

In the next two units, students work with proportional relationships represented by tables, equations, and graphs. Geometry and proportional relationships are interwoven when the important proportional relationship between a circle's circumference and its diameter is studied. Then students work with percent increase and percent decrease.

From there, students are introduced to rational numbers. Students learn about negative numbers, and represent negative numbers on the number line and on the coordinate plane. Then the emphasis becomes the role of the properties of operations in determining the rules for operating with negative numbers.

Toward the end of the course, students examine data sets and distributions. They learn about statistical questions, categorical data, and numerical data. They also explore ways to describe the center and the distribution of a data set. They use samples to make inferences about a population.

The final unit of the course is optional. The lessons provide students with additional opportunities to integrate and apply various ideas from the course to solve real-world and mathematical problems.

Unit 1: Areas

In this unit, students reason about areas of polygons and surface areas of polyhedra, building on geometric understandings developed in earlier grades.

In grade 3, students found the area of rectangles with whole-number side lengths. They also found the area of rectilinear figures by decomposing them into non-overlapping rectangles and adding those areas. Students used a formula for the area of rectangles in grade 4 and found the area of rectangles with fractional side lengths in grade 5.

In this unit, students extend their reasoning about area to include shapes that are not composed of rectangles. They use strategies such as decomposing and rearranging to find areas of parallelograms and generalize their process as a formula. Their work with parallelograms then becomes the basis for finding the area of triangles. Students see that other polygons can be decomposed into triangles and use this knowledge to find areas of polygons.

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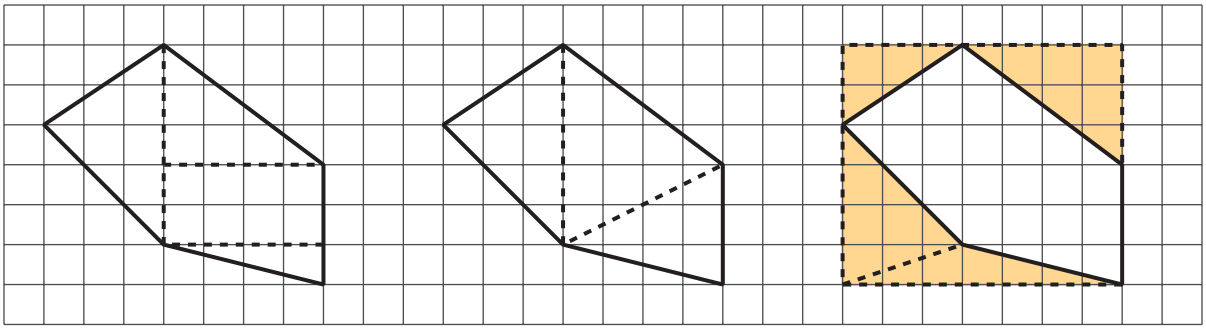

Next, students calculate the surface areas of polyhedra with triangular and rectangular faces. They study, assemble, and draw nets of prisms and pyramids and use nets to determine surface areas.

In many lessons, students engage in geometric work without a context. This design choice is made in recognition of the significant intellectual work of reasoning about area. Later in the unit, students have opportunities to apply their learning in context.

A note about multiplication notation:

Students in grade 6 will be writing algebraic expressions and equations involving the letter x. Because x is easily confused with the "cross" notation for multiplication, $\times$, these materials use the "dot" notation for multiplication. Starting a few lessons into the unit, students will see, for instance, $2 \cdot 3$ instead of 2×3. The notation will be new to many students, so they will need explicit guidance in using it.

A note about tools:

Students are likely to need physical tools to support their reasoning. For instance, they may find that tracing paper is an excellent tool for verifying that figures "match up exactly." At all times in the unit, each student should have access to a geometry toolkit, which contains tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles. Access to the toolkit also enables students to practice selecting appropriate tools and using them strategically (MP5). In a digitally enhanced classroom, apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, explaining, and describing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Explain

- How to find areas by composing (Lesson 3).
- Strategies used to find areas of parallelograms (Lesson 4) and triangles (Lesson 7).
- How to determine the area of a triangle using its base and height (Lesson 8).
- Strategies to find surface areas of polyhedra (Lesson 11).

Describe

- Observations about decomposition of parallelograms (Lesson 6).
- Information needed to find the surface area of rectangular prisms (Lesson 10).
- The features of polyhedra and their nets (Lesson 11).
- The features of polyhedra (Lesson 12).
- Relationships among features of a tent and the amount of fabric needed for the tent (Lesson 13).

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Justify

- Claims about the area of shapes (Lesson 2).
- Claims about the area of parallelograms (Lesson 5).
- Claims about the bases, heights, and areas of triangles (Lesson 8).
- The relationships among the features of the tent and the amount of fabric required (Lesson 13).

In addition, students are expected to use language to compare strategies for finding the areas of shapes and polygons and characteristics of prisms and pyramids; generalize about the features of parallelograms and polygons; and interpret relevant information for finding the surface area of rectangular prisms. Over the course of the unit, teachers can support students' mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

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lesson	new terminology	
	receptive	productive
Acc6.1.1	area **region** plane gap overlap	
Acc6.1.2	**area** compose decompose rearrange two-dimensional	
Acc6.1.3	shaded strategy	
Acc6.1.4	parallelogram opposite (sides or angles)	quadrilateral
Acc6.1.5	**base (of a parallelogram or triangle)** **height** corresponding expression represent	
Acc6.1.6	horizontal vertical	
Acc6.1.7	identical	parallelogram
Acc6.1.8	diagram	**base (of a parallelogram or triangle)** **height** **compose** **decompose** rearrange
Acc6.1.9	**opposite vertex**	
Acc6.1.10	**vertex** **edge**	
Acc6.1.11	**polygon**	horizontal vertical
Acc6.1.12	**face** **surface area**	**area** **region**

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lesson	new terminology	
	receptive	productive
Acc6.1.13	**polyhedron** **net** **prism** **pyramid** **base (of a prism or pyramid)** three-dimensional	**polygon** **vertex** **edge** **face**
Acc6.1.12		**prism** **pyramid**
Acc6.1.19	estimate description	**surface area** **volume**

Section A: Reasoning to Find Area

- Lesson 1: Tiling the Plane
- Lesson 2: Finding Area by Decomposing and Rearranging
- Lesson 3: Reasoning to Find Area

Section B: Parallelograms

- Lesson 4: Parallelograms
- Lesson 5: Area of Parallelograms
- Lesson 6: From Parallelograms to Triangles

Section C: Triangles and Other Polygons

- Lesson 7: Area of Triangles
- Lesson 8: Formula for the Area of a Triangle
- Lesson 9: Polygons

Section D: Surface Area

- Lesson 10: What Is Surface Area?
- Lesson 11: Polyhedra and Nets
- Lesson 12: More Nets, More Surface Area

Section E: Let's Put It to Work

- Lesson 13: All about Tents

Unit 2: Ratios, Rates, and Percentages

This unit introduces students to ratios, rates, and percentages. It builds on previous experiences students had with relating two quantities, such as converting measurements and finding equivalent fractions starting in grade 3, multiplicative comparison in grade 4, and interpreting multiplication as scaling in grade 5. The work prepares students

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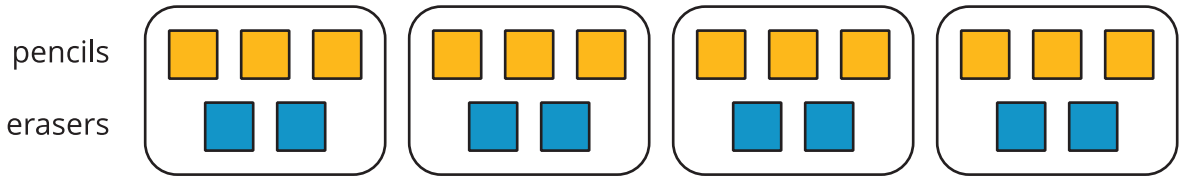
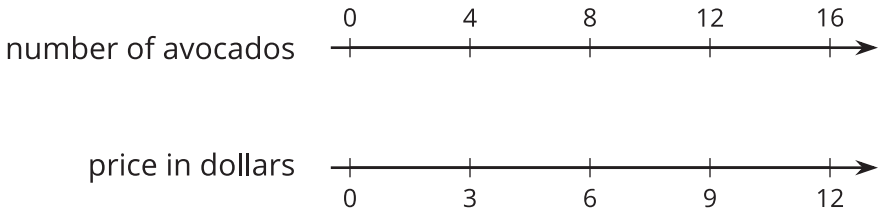
to reason about proportional relationships, constants of proportionality, and percent increase and decrease later in this course.

The first half of the unit focuses on ratios and equivalent ratios. Students learn that a ratio is an association between two quantities, for instance, “There are 3 pencils for every 2 erasers.” They use sentences, drawings, or discrete diagrams to represent ratios that describe collections of objects and recipes.



Next, students encounter equivalent ratios in terms of multiple batches of a recipe. They learn to use double number line diagrams and tables to represent and reason about equivalent ratios. These representations are more abstract than are discrete diagrams and offer greater flexibility. Use of tables here is a stepping stone toward use of tables to represent functional relationships in future courses. Students explore equivalent ratios in contexts, such as constant speed and uniform pricing.



The second half of the unit focuses on rates and percentages. Students begin by recalling what they know about standard units of measurement—the attributes that they measure and their relative sizes. They use ratios and rates to reason about measurements and to convert between units of measurement.

Next, students learn about unit rates. They see that there are two unit rates—$\frac{a}{b}$ and $\frac{b}{a}$—associated with any ratio $a : b$ and interpret them in context. Students practice finding unit rates and using them to solve various problems.

Students then use their understanding of ratios and rates to make sense of percentages. Just as a unit rate can be interpreted in context as a rate per 1, a percentage can be interpreted in context as a rate per 100. Students see that tables and double number line diagrams are also helpful for reasoning about percentages.

time (minutes)	percentage
90	125
18	25
72	100

$\cdot \frac{1}{5}$ $\cdot 4$ $\cdot \frac{1}{5}$ $\cdot 4$

A note on using the terms "quantity," "ratio," "rate," and "proportion":

In these materials, a “quantity” is a measurement that can be specified by a number and a unit, for instance, 4 oranges, 4 centimeters, “my height in feet,” or “my height” (with the understanding that a unit of measurement will need to be chosen).

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The term “ratio” is used to mean an association between two or more quantities. In this unit, the fractions $\frac{a}{b}$ and $\frac{b}{a}$ are never called “ratios,” and the meanings of these fractions in contexts are very carefully developed before they are identified as “unit rates” for the ratio $a : b$. For example, the word “per” is used with students in interpreting a unit rate in context, as in “$3 per ounce,” and the phrase “at the same rate” is used to signify a situation characterized by equivalent ratios. Later in the unit, students learn then that if two ratios $a : b$ and $c : d$ are equivalent, then the unit rates $\frac{a}{b}$ and $\frac{c}{d}$ are equal.

The terms “proportion” and “proportional” are not used in this unit. A “proportional relationship” is a collection of equivalent ratios, which will be studied in later units. In high school—after their study of ratios, rates, and proportional relationships—students can discard the term “unit rate” and refer to a to b, $a : b$, and $\frac{a}{b}$ all as “ratios.”

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, explaining, and representing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Different representations of ratios (Lessons 1 and 4).
- Situations involving equivalent ratios (Lessons 6 and 7).
- Tables of equivalent ratios (Lessons 9 and 10).
- Unit rates in different contexts (Lesson 18).
- A context in which identifying a unit rate is helpful (Lesson 20).
- Diagrams used to represent percentages (Lessons 21 and 22).
- Situations involving measurement, rates, and cost (Lesson 28).

Explain

- Reasoning about equivalence (Lesson 2).
- Reasoning about equivalent rates (Lesson 8).
- Reasoning with reference to tables (Lessons 9 and 10).
- Reasoning with reference to tape diagrams (Lessons 12 and 13).
- Reasoning for estimating and sorting measurements (Lesson 14).
- Reasoning about relative sizes of units of measurement (Lesson 15).
- Reasoning for comparing rates (Lesson 17).
- Reasoning about percentages (Lessons 21 and 22).

Represent

- Ratio associations (Lesson 1).
- Doubling and tripling of quantities in a ratio (Lesson 2).
- Equivalent ratios (Lessons 5 and 6).
- Ratios and total amounts (Lessons 12 and 13).
- Measurement unit conversions as equivalent ratios (Lesson 15).
- Percentages using diagrams (Lessons 21 and 22).

In addition, students are expected to justify whether ratios are or aren't equivalent and why information is needed to

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solve a ratio problem. Students also have opportunities to generalize about equivalent ratios, unit rates, and percentages from multiple contexts and with reference to benchmark percentages, tape diagrams, and other mathematical representations.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

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lesson	new terminology	
	receptive	productive
Acc6.2.1	**ratio** diagram ___ to ___ ___ for every ___	
Acc6.2.2	recipe batch mixture same taste same color equivalent	**ratio** ___ to ___ ___ for every ___
Acc6.2.3	**equivalent ratios**	batch
Acc6.2.4	**double number line diagram** tick marks representation	diagram
Acc6.2.5	**per**	
Acc6.2.6	**unit price** how much for 1 at this rate	**double number line diagram**
Acc6.2.7	**constant speed** meters per second	
Acc6.2.8	**same rate**	**equivalent ratios**
Acc6.2.10	table row column	
Acc6.2.11	calculation	**per** table
Acc6.2.12	**tape diagram** parts suppose	
Acc6.2.13		**tape diagram**
Acc6.2.15	order	
Acc6.2.17	(good / better / best) deal rate per 1	**unit price** same speed
Acc6.2.18	**unit rate**	
Acc6.2.19	result	**unit rate**

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lesson	new terminology	
	receptive	productive
Acc6.2.20		at this rate
Acc6.2.21	**percentage** ___ % of	tick marks
Acc6.2.22	___ % as much	___ % of
Acc 6.2.25		**percentage**

Section A: What Are Ratios?

- Lesson 1: Representing Ratios with Diagrams
- Lesson 2: Mixtures
- Lesson 3: Defining Equivalent Ratios

Section B: Representing Equivalent Ratios

- Lesson 4: Introducing Double Number Line Diagrams
- Lesson 5: Creating Double Number Line Diagrams
- Lesson 6: How Much for One?
- Lesson 7: Constant Speed
- Lesson 8: Comparing Situations by Examining Ratios
- Lesson 9: Representing Ratios with Tables
- Lesson 10: Navigating a Table of Equivalent Ratios
- Lesson 11: Solving Equivalent Ratio Problems

Section C: Part-Part-Whole Ratios

- Lesson 12: Part-Part-Whole Ratios
- Lesson 13: Solving More Ratio Problems

Section D: Units of Measurement and Unit Conversion

- Lesson 14: Anchoring Units of Measurement
- Lesson 15: Measuring with Different-Size Units
- Lesson 16: Converting Units

Section E: Rates

- Lesson 17: Comparing Speeds and Prices
- Lesson 18: Interpreting Rates
- Lesson 19: Equivalent Ratios Have the Same Unit Rates
- Lesson 20: Solving Rate Problems

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Section F: Percentages

- Lesson 21: What Are Percentages?
- Lesson 22: Representing Percentages in Different Ways
- Lesson 23: Benchmark Percentages
- Lesson 24: Solving Percentage Problems
- Lesson 25: Finding This Percent of That
- Lesson 26: Finding the Percentage

Section G: Let's Put It to Work

- Lesson 27: A Fermi Problem
- Lesson 28: Painting a Room

Unit 3: Fractions and Decimals

This unit develops students' understanding of division of fractions by fractions and operations on whole numbers and decimals. This work draws on students' prior knowledge of multiplication, division, and the relationship between the two. It also builds on place value, properties of operations, and the relationship between addition and subtraction.

Students begin by exploring meanings of division and the relationship between the quantities in division situations. They recall that we can think of dividing as finding an unknown factor in a multiplication equation. In situations involving equal-size groups, division can be used to answer two questions: "How many groups?" and "How much in each group?" Students investigate ways to answer those two questions.

Students then apply their insights to generalize the process of finding quotients. They notice regularity: Dividing a number by a fraction $\frac{a}{b}$ is the same as multiplying that number by $\frac{b}{a}$. Students go on to use this algorithm to solve problems about geometric figures that have fractional length, area, or volume measurements and to compute unit rates of fractions.

Next, students revisit addition and subtraction of decimals, using both concrete representations and numerical calculations. They also investigate various ways to find the product of two decimals: using decimal fractions, using diagrams and partial products, and reasoning about the relationship between a decimal and a related whole number.

The next section focuses on division. Students have an opportunity to use base-ten blocks or diagrams to represent division of multi-digit numbers before exploring other numerical methods, such as using partial quotients and long division. Students progress through calculations of increasing complexity. They first divide whole numbers that give a whole-number quotient, and then divide whole numbers with a (terminating) decimal quotient. Next, they divide a decimal by a whole number, and finally a decimal by a decimal.

Mai's diagram for $62 \div 5$

Lin's calculation for $62 \div 5$

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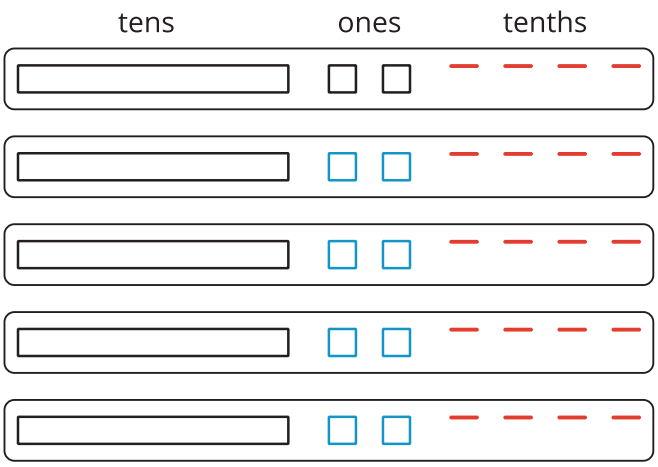

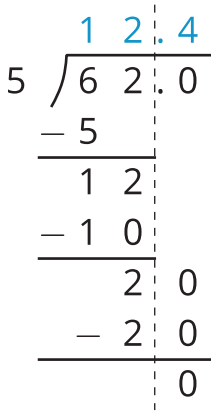

A deeper understanding of multiplication, division, and ways to represent them will support students in reasoning about writing and solving variable equations later in the course.

A note about diagrams:

Because tape diagrams are a flexible tool for illustrating and reasoning about division of fractions, they are the primary representation used in this unit. Students may, however, create other representations to support their reasoning.

A note about materials:

Base-ten blocks and paper versions of them will be useful throughout the unit. Consider preparing commercially produced base-ten blocks, if available, or printing representations of base-ten units on card stock, cutting them out, and organizing them for easy reuse.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, representing, explaining, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret and Represent

- Situations involving division (Lessons 2, 6, 9, and 12).
- Base-ten diagrams showing addition or subtraction of decimals (Lesson 14).
- Area diagrams showing products of decimals (Lesson 16).
- Calculations showing partial quotients or steps in long division (Lessons 18 and 19).
- Base-ten diagrams representing division of a whole number or a decimal by a whole number (Lessons 19).
- Situations involving measurement constraints (Lesson 22).

Explain

- How to create and make sense of division diagrams (Lesson 3).
- How to represent division situations (Lesson 6).
- How to find unknown lengths (Lesson 11).
- Processes of estimating and finding costs (Lesson 13).

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- Approaches to adding and subtracting decimals (Lesson 15).
- Methods for multiplying decimals (Lesson 17).
- A plan for optimizing costs (Lesson 22).
- Reasoning about relationships among measurements (Lesson 23).

Compare

- Verbal and numerical division representations (Lessons 4 and 5).
- Representations of division (Lesson 10).
- Base-ten diagrams with numerical calculations (Lesson 15).
- Methods for multiplying decimals (Lesson 16).
- Methods for finding quotients (Lessons 18 and 19).
- Measurements of two- and three-dimensional objects (Lesson 23).

In addition, students are expected to critique the reasoning of others about division situations and representations, generalize about multiplication and division, and justify strategies for finding sums, differences, products, and quotients.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

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lesson	new terminology	
	receptive	**productive**
Acc6.3.1	divisor dividend	quotient
Acc6.3.2	equation interpretation equal-size unknown	How many groups of ___? How many ___ in each group?
Acc6.3.3		whole equal-size
Acc6.3.4	times as ___ fraction of ___	
Acc6.3.5	container section	unknown fraction of ___
Acc6.3.7	**reciprocal** observations	times as ___ numerator denominator
Acc6.3.8		**unit rate**
Acc6.3.10		gaps
Acc6.3.11	packed	
Acc6.3.13	digits budget at least	
Acc6.3.14	base-ten diagram compose decompose vertical calculation	place value digits
Acc6.3.15	method	compose decompose
Acc6.3.16	powers of 10 partial products	product decimal point method
Acc6.3.18	**long division** partial quotients remainder	divisor
Acc6.3.19		remainder

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lesson	new terminology	
	receptive	productive
Acc6.3.20		**long division**
Acc6.3.21	precision accuracy operation	
Acc6.3.22	assumption	packed

Section A: Making Sense of Division

- Lesson 1: Size of Divisor and Size of Quotient
- Lesson 2: Meanings of Division

Section B: Dividing Fractions

- Lesson 3: How Many Groups?
- Lesson 4: What Fraction of a Group?
- Lesson 5: How Much in Each Group? (Part 1)
- Lesson 6: How Much in Each Group? (Part 2)
- Lesson 7: Finding an Algorithm for Dividing Fractions
- Lesson 8: Ratios and Rates with Fractions

Section C: Fractions in Lengths, Areas, and Volumes

- Lesson 9: Fractional Lengths
- Lesson 10: Rectangles and Triangles with Fractional Lengths
- Lesson 11: Volume of Prisms
- Lesson 12: Solving Problems Involving Fractions

Section D: Adding, Subtracting, and Multiplying Decimals

- Lesson 13: Using Decimals in a Shopping Context
- Lesson 14: Using Diagrams to Represent Addition and Subtraction
- Lesson 15: Adding and Subtracting Decimals with Many Non-Zero Digits
- Lesson 16: Methods for Multiplying Decimals
- Lesson 17: Calculating Products of Decimals

Section E: Dividing Decimals

- Lesson 18: Using Long Division
- Lesson 19: Dividing Numbers that Result in a Decimal
- Lesson 20: Dividing a Decimal by a Decimal
- Lesson 21: Solving Problems Involving Decimals

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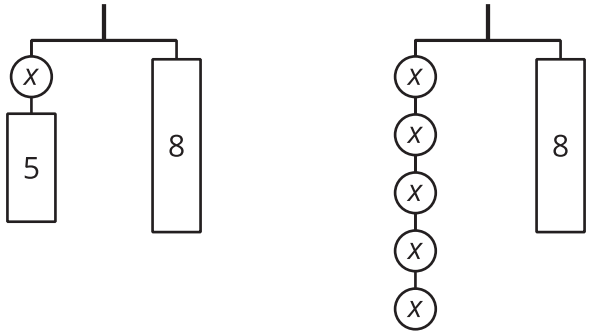
Section F: Let's Put It to Work

- Lesson 22: Fitting Boxes into Boxes
- Lesson 23: Making and Measuring Boxes

Unit 4: Equations and Expressions

In this unit, students apply their understanding of arithmetic to reason about algebraic expressions and equations.

In the first section, students work with equations of the form $x + p = q$ and $px = q$ where p and q are positive rational numbers. They use tape diagrams and hanger diagrams to reason about the meaning of equations, and to develop an understanding that to solve an equation is to find a value that would make the equation true. Students end the section by identifying, interpreting, and writing equations to represent and solve real-world problems.



$x + 5 = 8$ $\qquad$ $5x = 8$

In the second section, students write algebraic expressions and evaluate them for given values. They identify and write equivalent expressions, reasoning using diagrams, the distributive property, and other properties of operations.

The third section is all about exponents. First, students learn to use exponents ^{2} and ^{3} to express areas of squares and volumes of cubes and their units. Next, they write expressions with a whole-number exponent and a base that may be a whole number, a fraction, or a variable. They analyze such expressions for equivalence, as well as use the conventional order of operations to evaluate them. Students also identify solutions to simple exponential equations.

In the last two sections, students analyze real-world relationships between two quantities where one quantity depends on the other. They use tables, graphs, and equations to represent and reason about such relationships.

The work here prepares students to work with proportional relationships in a later unit, as well as to solve equations that are more complex in grade 7.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, describing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Tape diagrams involving letters that stand for numbers (Lesson 1).
- The parts of an equation (Lesson 5).
- Numerical expressions involving exponents (Lesson 12).
- Different representations of the same relationship between quantities (Lesson 17).

Describe

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- Solutions to equations (Lesson 2).
- Stories represented by given equations (Lesson 5).
- Patterns of growth that can be represented using exponents (Lesson 12).
- Relationships between independent and dependent variables using tables, graphs, and equations (Lesson 16).

Explain

- The meaning of a solution using hanger diagrams (Lesson 3).
- How to solve an equation (Lesson 4).
- How to determine whether two expressions are equivalent, including with reference to diagrams (Lesson 7).
- Strategies for determining whether expressions are equivalent (Lesson 14).

In addition, students are expected to compare descriptions of situations, expressions, equations, diagrams, tables, and graphs. They generalize about properties of operations and strategies for solving equations. Students also justify claims about equivalent expressions and justify reasoning when evaluating expressions.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

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lesson	new terminology	
	receptive	productive
Acc6.4.1	value (of a variable)	operation
Acc6.4.2	**variable** **coefficient** **solution to an equation** true equation false equation	value (of a variable)
Acc6.4.3	each side (of an equal sign) balanced hanger diagram	
Acc6.4.4	solve (an equation)	each side (of an equal sign)
Acc6.4.7	**equivalent expressions** commutative property	
Acc6.4.8	distributive property area as a product area as a sum	
Acc6.4.9	**term**	**equivalent expressions**
Acc6.4.11	**squared** **cubed** **exponent** edge length	
Acc6.4.12	to the power	
Acc6.4.13		to the power **exponent**
Acc6.4.15		**solution to an equation**
Acc6.4.16	**independent variable** **dependent variable** horizontal axis vertical axis	**variable** relationship
Acc6.4.17	coordinates	

Section A: Equations in One Variable

- Lesson 1: Tape Diagrams and Equations
- Lesson 2: Truth and Equations
- Lesson 3: Staying in Balance
- Lesson 4: Practice Solving Equations
- Lesson 5: Represent Situations with Equations

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Section B: Equal and Equivalent

- Lesson 6: Write Expressions with Variables
- Lesson 7: Equal and Equivalent
- Lesson 8: The Distributive Property, Part 1
- Lesson 9: The Distributive Property, Part 2
- Lesson 10: The Distributive Property, Part 3

Section C: Expressions with Exponents

- Lesson 11: Squares and Cubes
- Lesson 12: Meaning of Exponents
- Lesson 13: Expressions with Exponents
- Lesson 14: Evaluating Expressions with Exponents
- Lesson 15: Equivalent Exponential Expressions

Section D: Relationships Between Quantities

- Lesson 16: Two Related Quantities, Part 1
- Lesson 17: Two Related Quantities, Part 2

Section E: Let's Put It to Work

- Lesson 18: Tables, Equations, and Graphs, Oh My!

Unit 5: Proportional Relationships

In this unit, students develop the idea of a proportional relationship. They work with proportional relationships that are represented in tables, as equations, and on graphs. This builds on previous work with equivalent ratios and helps prepare students for the study of linear functions in later courses.

In a table of equivalent ratios, a multiplicative relationship between a pair of rows is given by a scale factor, while the multiplicative relationship between the columns is given by a unit rate. Students learn that the relationship between pairs of values in the two columns is called a "proportional relationship," and the unit rate that describes this relationship is called a "constant of proportionality." Students use equations of the form $y = kx$ to represent proportional relationships and solve problems. They determine whether given tables and equations could represent a proportional relationship.

Then students investigate graphs of proportional relationships. They recognize that the graph of a proportional relationship is a straight line through $(0, 0)$. They interpret points on the graph, including the point $(1, k)$. Here is an example of a graph, an equation, and a table that all represent the same proportional relationship.

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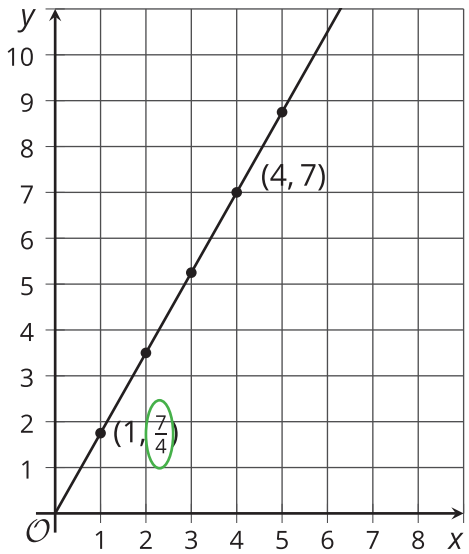
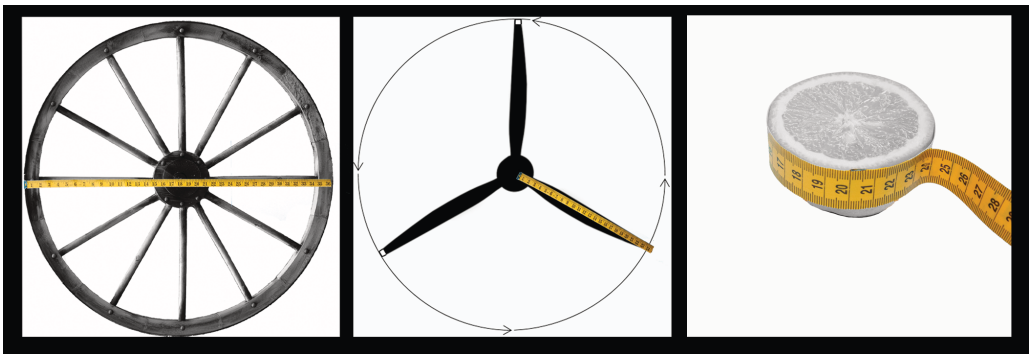


$y = \frac{7}{4}x$

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

Next, students apply their knowledge of proportional relationships to the context of measuring circles. This builds on students' work from previous grades with perimeter and area of polygons. Students will build on this work in later courses when they study the volume of spheres, cylinders, and cones.

The terms "center," "radius," "diameter," and "circumference" are introduced. Then students investigate the relationship between circumference and diameter and see that it is a proportional relationship. They apply this relationship to solve problems. Next, students explore the area of circular regions. They see an informal derivation that shows where the formula $A = \pi r^2$ comes from and then use this formula to solve problems.



A note on using the terms "ratio," "proportional relationship," and "unit rate":

In these materials, the term "ratio" is used to mean a type of association between two or more quantities. A quantity is a measurement that can be specified by a number and a unit, for example 4 oranges, 4 centimeters, or "my height in feet." A proportional relationship is a collection of equivalent ratios.

A unit rate is the numerical part of a rate per 1 unit, for example, the 6 in 6 miles per hour. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are never called ratios. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are identified as "unit rates" for the ratio $a : b$. In high school—after the study of ratios, rates, and proportional relationships—students discard the term "unit rate" and start referring to a to b, $a : b$, and $\frac{a}{b}$ as "ratios."

In grades 6–8, students write rates without abbreviated units, for example as "3 miles per hour" or "3 miles in every 1 hour." Use of notation for derived units such as $\frac{\text{mi}}{\text{hr}}$ waits for high school—except for the special cases of area and volume.

A note on using the term "circle":

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Strictly speaking, a circle is one-dimensional. It is the boundary of a two-dimensional region, rather than the region itself. The circular region is called a "disk." Because students are not yet expected to make this distinction, these materials refer to both disks and the boundaries of disks as "circles," using illustrations to eliminate ambiguity.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, justifying, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- Approaches to solving problems involving proportional relationships (Lesson 3).
- Proportional relationships with nonproportional relationships (Lesson 5).
- Tables, descriptions, and graphs representing the same situations (Lesson 7).
- Graphs of proportional relationships (Lesson 8).
- The relationships of square diagonals and perimeters to square diagonals and areas (Lesson 10).
- The relationships of diameters and circumferences to diameters and areas (Lesson 15).

Justify

- Reasoning about circumference and perimeter (Lesson 13).
- Estimates for the areas of circles (Lesson 15).
- Reasoning about areas of curved figures (Lesson 16).
- Whether or not a relationship is proportional (Lesson 17).
- Reasoning about the cost of stained-glass windows (Lesson 20).

Generalize

- About proportional relationships (Lesson 1).
- About equations that represent proportional relationships (Lesson 2).
- About categories for sorting circles (Lesson 11).
- About the relationships between circumference and diameter (Lesson 12).

In addition, students are expected to explain how to determine whether or not a relationship is proportional, how to use different approximations of π, how to find the area of composite shapes, and how to compare and represent situations with different constants of proportionality. Students are also asked to interpret situations involving proportional relationships, floor plans and maps, situations involving circles, and situations involving circumference and area.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

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lesson	new terminology	
Acc6.5.1	**constant of proportionality** **proportional relationship**	___ is proportional to ___
Acc6.5.2	steady situation	**reciprocal**
Acc6.5.4		constant of proportionality proportional relationship
Acc6.5.5		constant
Acc6.5.7	**origin** **coordinate plane** quantity axes	coordinates
Acc6.5.9	x-coordinate y-coordinate	**origin**
Acc6.5.10	perimeter	
Acc6.5.11	**radius** **diameter** **circumference** center (of a circle)	**circle**
Acc6.5.12	**pi (π)**	
Acc6.5.13	half-circle rotation approximation	**diameter** **circumference** **pi (π)**
Acc6.5.14	floor plan	approximate estimate
Acc6.5.15	**area of a circle** formula	**radius**
Acc6.5.16	in terms of π	**area of a circle**
Acc6.5.17		axes
Acc6.5.18	reasonable	
Acc6.5.19		**squared** center (of a circle) formula
Acc6.5.20	design	

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Section A: Representing Proportional Relationships with Equations

- Lesson 1: Proportional Relationships and Equations
- Lesson 2: Two Equations for Each Relationship
- Lesson 3: Writing Equations to Represent Relationships

Section B: Comparing Proportional and Nonproportional Relationships

- Lesson 4: Comparing Relationships with Tables
- Lesson 5: Comparing Relationships with Equations
- Lesson 6: Solving Problems about Proportional Relationships

Section C: Representing Proportional Relationships with Graphs

- Lesson 7: Graphs of Proportional Relationships
- Lesson 8: Using Graphs to Compare Relationships
- Lesson 9: Two Graphs for Each Relationship

Section D: Circumference of a Circle

- Lesson 10: How Well Can You Measure?
- Lesson 11: Exploring Circles
- Lesson 12: Exploring Circumference
- Lesson 13: Applying Circumference

Section E: Area of a Circle

- Lesson 14: Estimating Areas
- Lesson 15: Area of a Circle
- Lesson 16: Applying Area of Circles

Section F: Let's Put It to Work

- Lesson 17: Four Representations
- Lesson 18: Using Water Efficiently
- Lesson 19: Distinguishing Circumference and Area
- Lesson 20: Stained-Glass Windows

Unit 6: Percent Increase and Decrease

In this unit, students deepen their understanding of proportional relationships and percentages. They solve multi-step problems and work with situations that involve fractional amounts. This builds on the work students did in previous units with ratios, rates, percentages, and proportional relationships. Students will build on this work in high school with exponential functions representing compounded percent increase and decrease.

Students begin the unit by revisiting proportional relationships, but this time the given values are fractional amounts. To determine the constant of proportionality, students must compute the quotient of two fractions. Students also make sense of situations where an increase or decrease is expressed as a fraction of the initial amount. They create diagrams and apply the distributive property to generate expressions that represent these situations. They also use long division

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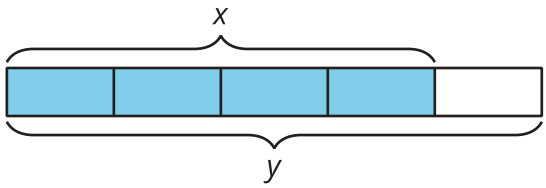
to write fractions as decimals, including their first introduction to repeating decimals.

Next, students make sense of situations where an increase or decrease is expressed as a percentage of the initial amount. They continue creating diagrams and writing equations to represent the situations. They solve for any one of the three quantities—the initial amount, the final amount, or the percentage of the change—given the other two quantities. They also reason about fractional percentages.

Then students apply percent increase and decrease to solve problems in a variety of real-world situations, such as tax, tip, interest, markup, discount, depreciation, and commission. Lastly, students make sense of situations where the difference between a correct measurement and an incorrect measurement is expressed as a percentage of the correct amount.



$y = x + \frac{1}{4}x$

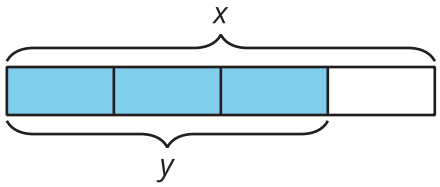
$y = (1 + \frac{1}{4})x$

$y = \frac{5}{4}x$

$y = (1 + 0.25)x$

$y = 1.25x$

"a 25% increase"



$y = x - \frac{1}{4}x$

$y = (1 - \frac{1}{4})x$

$y = \frac{3}{4}x$

$y = (1 - 0.25)x$

$y = 0.75x$

"a 25% decrease"

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, explaining, and representing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Concrete problems involving percent increase and decrease (Lesson 5).
- Problems involving sales tax and tip (Lesson 8).
- Concrete situations involving percent error (Lesson 11).

Explain

- How to solve concrete and abstract problems involving an amount plus (or minus) a fraction of that amount (Lesson 2).
- How to solve percent change problems (Lesson 4).
- Strategies for solving percent problems with fractional percentages (Lesson 7).
- How to measure lengths and interpret measurement error (Lesson 11).
- Strategies for solving percent error problems (Lesson 11).

Represent

- Situations involving percent increase and decrease (Lessons 6 and 12).
- Situations from the news involving percent change (Lesson 13).

In addition, students are expected to compare decimal and fraction representations, compare representations of an increase (or decrease) of an amount by a fraction or decimal, generalize about using constants of proportionality to

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solve problems efficiently and about relationships with percent increase and decrease, and justify why specific information is needed to solve percent change problems.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
Acc6.6.2	(a fraction) more than (a fraction) less than initial/original amount final/new amount	**tape diagram** distributive property
Acc6.6.3	**repeating decimal** decimal representation	
Acc6.6.4	**percent increase** **percent decrease**	(a fraction) more than (a fraction) less than
Acc6.6.5	discount	initial/original amount final/new amount
Acc6.6.8	sales tax tax rate tip	**percent increase**
Acc6.6.9	interest commission markup markdown	**percent decrease**
Acc6.6.10		discount
Acc6.6.11	**measurement error** **percent error**	

Section A: Proportional Relationships with Fractions

- Lesson 1: Revisiting Proportional Relationships
- Lesson 2: More than That, Less than That
- Lesson 3: Say It with Decimals

Section B: Percent Increase and Decrease

- Lesson 4: Increasing and Decreasing
- Lesson 5: One Hundred Percent
- Lesson 6: Percent Increase and Decrease with Equations
- Lesson 7: Part of a Percent

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Section C: Applying Percentages

- Lesson 8: Tax and Tip
- Lesson 9: Percentage Contexts
- Lesson 10: Solving Multi-step Percentage Problems
- Lesson 11: Expressing Error as a Percentage

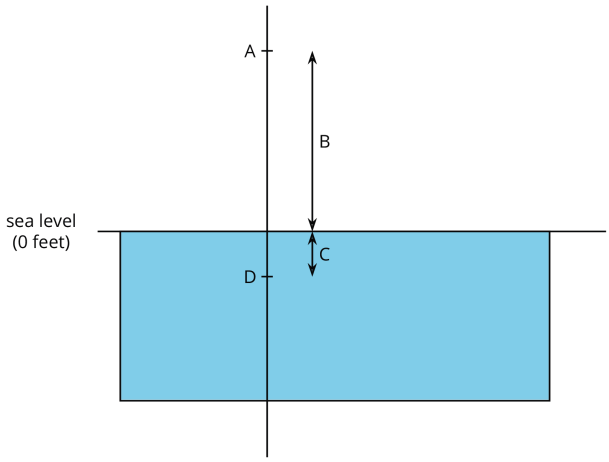
Section D: Let's Put It to Work

- Lesson 12: Changes on the Earth
- Lesson 13: Posing Percentage Problems

Unit 7: Rational Numbers

In this unit, students learn about negative numbers and ways to represent them on a number line and the coordinate plane. They perform operations on rational numbers, which are all numbers that can be written as a positive or negative fraction or zero.

Students begin by considering situations involving temperature or elevation and interpreting what negative numbers mean in those contexts. Previously, when students worked only with nonnegative numbers, magnitude and order were indistinguishable. In this unit, when comparing two signed numbers, students learn to distinguish between the absolute value of a number (magnitude) and a number's relative position on the number line (order).



- A represents 15 feet.
- B represents $|15|$ feet.
- C represents $|-4|$ feet.
- D represents -4 feet.

Then students use tables and number line diagrams to represent changes in temperature or elevation. They extend addition and subtraction from fractions to all rational numbers. And they see that $a - b$ is equivalent to $a + (-b)$.

Then students use ordered pairs to describe pairs of numbers that include negative numbers. In grade 5, they plotted pairs of positive numbers on the coordinate grid. Here, they plot pairs of rational numbers in all four quadrants of the coordinate plane. They interpret the meanings of plotted points in given contexts and use coordinates to calculate horizontal or vertical distances between two points.

Next, students examine multiplication and division. They work with constant velocity, which is a signed number that indicates direction and speed. This allows products of signed numbers to be interpreted in terms of position, direction of movement, and time before or after a specific point. Students use the relationship between multiplication and division to understand how division extends to rational numbers.

Then students work with expressions that use the four operations on rational numbers. They also solve problems that involve interpreting negative numbers in context. They solve linear equations of the form $p + x = q$ or $px = q$, where p and q are rational numbers.

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A note on using the terms "expression," "equation," and "signed number":

In these materials, an *expression* is built from numbers, variables, operation symbols ($+$, $-$, $\cdot$, $\div$), parentheses, and exponents. (Exponents—in particular, negative exponents—are not a focus of this unit. Students work with integer exponents in a future course and noninteger exponents in high school.) An *equation* is a statement that two expressions are equal, thus it always has an equal sign. *Signed numbers* include all rational numbers, written as decimals or in the form $\frac{a}{b}$.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, representing, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Situations involving negative numbers (Lessons 1 and 5).
- Graphs involving positive and negative numbers (Lesson 12).
- Tables and situations involving signed numbers (throughout unit).

Represent

- Addition of signed numbers on a number line (Lesson 6).
- Situations involving signed numbers (Lessons 7, 10, and 16).
- Changes in elevation (Lesson 10).
- Position, speed, and direction (Lesson 14).

Generalize

- About subtracting and adding signed numbers (Lesson 9).
- About differences and magnitude (Lesson 10).
- About multiplying negative numbers (Lessons 14 and 15).
- About additive and multiplicative inverses (Lesson 20).

In addition, students are expected to use language to compare magnitudes of positive and negative numbers, compare features of ordered pairs, and compare appropriate axes for different sets of coordinates, Students are also expected to explain how to order rational numbers, how to determine distances on the coordinate plane, how to determine changes in temperature, how to find information using inverses, and how to model situations involving signed numbers.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

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lesson	new terminology	
	receptive	productive
Acc6.7.1	**positive number** **negative number** temperature degrees Celsius elevation sea level closer to 0 farther from 0	number line below zero
Acc6.7.2	**rational number** **sign** **inequality**	greater than less than
Acc6.7.3	**opposite (numbers)** from least to greatest	
Acc6.7.4	**absolute value**	**positive number** **negative number** distance (away) from 0
Acc6.7.5		closer to 0 farther from 0
Acc6.7.6	signed numbers	temperature
Acc6.7.7	sum expression	
Acc6.7.8	**deposit** **withdrawal** account balance debt	
Acc6.7.10	difference	**absolute value** distance
Acc6.7.11	**quadrant** **coordinate plane** x-coordinate y-coordinate (line) segment	axis
Acc6.7.12	degrees Fahrenheit	degrees Celsius
Acc6.7.13		**absolute value** x-coordinate y-coordinate
Acc6.7.14	velocity	

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lesson	new terminology	
	receptive	productive
Acc6.7.16	**solution (to an equation)** factor	
Acc6.7.18	additive inverse multiplicative inverse rational number variable	sum difference
Acc6.7.20		**opposite** **solution (to an equation)**
Acc6.7.23		increase decrease

Section A: Negative Numbers and Absolute Value

- Lesson 1: Positive and Negative Numbers
- Lesson 2: Comparing Positive and Negative Numbers
- Lesson 3: Ordering Rational Numbers
- Lesson 4: Absolute Value of Numbers
- Lesson 5: Comparing Numbers and Distance from Zero

Section B: Adding and Subtracting Rational Numbers

- Lesson 6: Changing Temperatures
- Lesson 7: Changing Elevation
- Lesson 8: Money and Debts
- Lesson 9: Representing Subtraction
- Lesson 10: Finding Differences

Section C: The Coordinate Plane

- Lesson 11: Constructing the Coordinate Plane
- Lesson 12: Interpreting Points in a Coordinate Plane
- Lesson 13: Distances in the Coordinate Plane

Section D: Multiplying and Dividing Rational Numbers

- Lesson 14: Multiplying Rational Numbers
- Lesson 15: Multiply!
- Lesson 16: Dividing Rational Numbers
- Lesson 17: Negative Rates

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Section E: Four Operations with Rational Numbers

- Lesson 18: Expressions with Rational Numbers
- Lesson 19: Solving Problems with Rational Numbers
- Lesson 20: Solving Equations with Rational Numbers
- Lesson 21: Representing Contexts with Equations

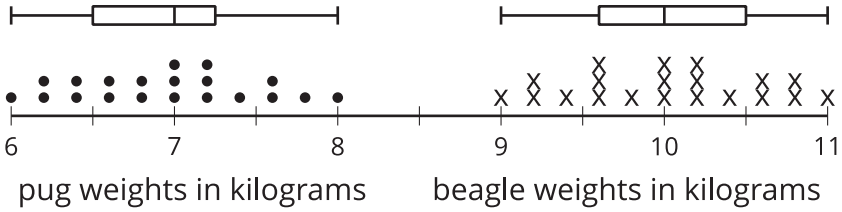
Section F: Let's Put It to Work

- Lesson 22: Drawing in the Coordinate Plane
- Lesson 23: The Stock Market

Unit 8: Data Sets, Distributions, and Sampling

This unit is a brief overview of some key statistical concepts. First, students learn about populations and study variables associated with a population. They begin by classifying questions as either statistical or non-statistical—based on whether variable data is necessary to answer the question. This leads to further investigation into variability and data displays, such as dot plots and histograms. As students visualize data, they begin to describe the distribution of data more precisely as they work with mean and mean absolute deviation (MAD).

After working with those statistics, students begin to recognize that some distributions are not well-suited to description by mean and MAD. Students are introduced to median, range, and interquartile range as additional measures of center and variability that can be used to describe distributions in some situations. That also leads to the box plot as an additional way to visualize data.



Next, students examine different ways to collect data from samples within a population to understand why random selection is useful. Then students generate samples and estimate information about the population from sample data.

The unit concludes with an optional section exploring probability. Students are introduced to probability as a way to quantify how likely an event is to happen. They explore the connection between probability and results of repeated experiments, ways to examine the sample space for more complex experiments, and simulating experiments.

Note that the introduction of mean absolute deviation is used as an introductory model for understanding variability. Although standard deviation is more mathematically useful, its calculation and meaning may be difficult for students at this level without an understanding of normal distributions. In later courses, when student understanding of variability and their exposure to additional distributions is expanded, students will learn about standard deviation and evolve their understanding away from mean absolute deviation.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, interpreting, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- Questions that produce numerical and categorical data (Lesson 1).

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- Dot plots and histograms (Lesson 3).
- Features and distributions of data sets (Lessons 4 and 5).
- Measures of center with samples (Lesson 9).
- Sampling methods (Lesson 10).
- Methods for writing sample spaces (Lesson 15).

Interpret

- Dot plots (Lessons 2 and 5).
- Histograms (Lesson 3).
- Mean of a data set (Lesson 4).
- Five-number summaries and box plots (Lesson 7).
- Situations involving populations and samples (Lesson 8).
- Situations involving sample spaces and probability (Lesson 16).

Justify

- Reasoning for matching data sets to questions (Lesson 1).
- Reasoning about mean and median (Lesson 6).
- Which samples are or are not representative of a larger population (Lesson 9).
- Which samples correspond with different populations (Lesson 11).
- Whether situations are surprising and possible (Lesson 14).

In addition, students are expected to represent data using dot plots, histograms, five-number summaries, and box plots, and to represent probabilities using sample spaces. Students also have opportunities to use language to describe features of a data set, describe patterns observed in repeated experiments, and explain how to use a simulation to answer questions about the situation.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

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lesson	new terminology	
	receptive	productive
Acc6.8.1	**numerical data** **categorical data** **dot plot** **statistical question** **variability** **distribution** **frequency**	
Acc6.8.2	**center** **spread** typical	**variability**
Acc6.8.3	**histogram** bins	**distribution** **center** **spread**
Acc6.8.4	**average** **mean** **measure of center** fair share balance point	
Acc6.8.5	**mean absolute deviation (MAD)** measure of spread symmetrical	**mean** typical
Acc6.8.6	**median** peak cluster unusual value	**measure of center**
Acc6.8.7	**range** **quartile** **interquartile range (IQR)** **box plot** whisker five-number summary	**median** measure of spread minimum maximum
Acc6.8.8	**population** **sample** survey	**mean absolute deviation (MAD)**
Acc6.8.9	**representative**	
Acc6.8.10	random sample	

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lesson	new terminology	
	receptive	productive
Acc6.8.11	measure of variability	**population** **sample** random sample symmetrical
Acc6.8.12		**representative** measure of variability
Acc6.8.13	**event** **chance experiment** **outcome** **probability** **random** **sample space**	likely unlikely impossible certain
Acc6.8.14		**outcome** **probability**
Acc6.8.15	tree (diagram)	**sample space**
Acc6.8.16		tree (diagram)
Acc6.8.17	**simulation**	**random**

Section A: Dot Plots and Histograms

- Lesson 1: Representing Data
- Lesson 2: Using Dot Plots to Answer Statistical Questions
- Lesson 3: Interpreting Histograms

Section B: Measures of Center and Variability

- Lesson 4: The Mean
- Lesson 5: Variability and MAD
- Lesson 6: The Median
- Lesson 7: Box Plots and Interquartile Range

Section C: Sampling

- Lesson 8: Larger Populations
- Lesson 9: What Makes a Good Sample?
- Lesson 10: Sampling in a Fair Way
- Lesson 11: Estimating Population Measures of Center
- Lesson 12: More about Sampling Variability

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Section D: Probability

- Lesson 13: What Are Probabilities?
- Lesson 14: Estimating Probabilities through Repeated Experiments
- Lesson 15: Keeping Track of All Possible Outcomes
- Lesson 16: Multi-step Experiments
- Lesson 17: Designing Simulations

Unit 9: Putting It All Together

In this optional unit, students use concepts and skills from previous units to solve problems. The first section explores a variety of different contexts, such as population density, Fermi problems, measurement error, and energy usage. The second section has five lessons about different systems of voting. In the last section, students build a trundle wheel and design a five-kilometer race course.

All related standards in this unit have been addressed in prior units. These sections provide an optional opportunity for students to go more deeply and make connections between domains.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as critiquing, comparing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique

- Reasoning about Fermi problems (Lesson 1).
- Peer reasoning about percent error in length, area, and volume measurement (Lesson 7).
- Claims about percentages (Lesson 10).
- Reasoning about the fairness of voting systems (Lesson 13).
- Peer methods of measuring distance (Lesson 15).

Compare

- Sources of energy (Lessons 2 and 3).
- Rectangles and fractions (Lesson 5).
- Regions with different population densities (Lesson 8).
- Voting systems (Lesson 11).
- Advantages and disadvantages of different methods (Lesson 16).

Justify

- Reasoning about Fermi problems (Lesson 1).
- Reasoning about the fairness of voting systems (Lessons 11 and 13).

In addition, students are also expected to describe distributions of voters and methods for measuring distance, including how to build and use a trundle wheel. Students also have opportunities to interpret and represent characteristics of the world population and generalize about decomposition of area and numbers

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students'

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use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	**productive**
Acc6.9.2	kilowatt-hour(kWh)	
Acc6.9.5	mixed number	
Acc6.9.7		**percent error**
Acc6.9.8	population density	
Acc6.9.9	in favor majority	
Acc6.9.10	plurality run off	majority
Acc6.9.12	in all fair	
Acc6.9.15	trundle wheel	
Acc6.9.17		trundle wheel

Section A: Making Connections

- Lesson 1: Fermi Problems
- Lesson 2: Energy Flow
- Lesson 3: Making Paper
- Lesson 4: If Our Class Were the World
- Lesson 5: Rectangle Madness
- Lesson 6: Rectangle Fractions
- Lesson 7: Measurement Error
- Lesson 8: How Crowded Is This Neighborhood?

Section B: Voting

- Lesson 9: How Do We Choose?
- Lesson 10: More than Two Choices
- Lesson 11: Comparing Voting Systems
- Lesson 12: Picking Representatives
- Lesson 13: Designing Districts

Section C: Designing a Course

- Lesson 14: Measuring Long Distances over Uneven Terrain

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- Lesson 15: Building a Trundle Wheel
- Lesson 16: Using a Trundle Wheel to Measure Distances
- Lesson 17: Designing a 5K Course

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Pacing Guide

Number of days includes assessments. Upper bound of range includes optional lessons.

	Accelerated 6	Accelerated 7
week 1 week 2 week 3	Unit 1 Areas (14–15 days) Optional Lesson: 13	Unit 1 Rigid Transformations and Congruence (20–21 days) (MA) Optional Lesson: 18
week 4 week 5 week 6 week 7 week 8 week 9	Unit 2 Ratios, Rates, and Percentages (26–31 days) (MA) Optional Lessons: 14, 15, 26, 27, 28	Unit 2 Scale Drawings, Similarity, and Slope (20–22 days) (MA) Optional Lessons: 18, 19
week 10 week 11 week 12 week 13 week 14	Unit 3 Fractions and Decimals (22–26 days) (MA) Optional Lessons: 6, 12, 22, 23	Unit 3 Equations and Inequalities (18–20 days) Optional Lessons: 11, 18
week 15 week 16 week 17 week 18	Unit 4 Equations and Expressions (18–20 days) Optional Lessons: 10, 18	Unit 4 Expressions and More Equations (13–14 days) Optional Lesson: 12
week 19 week 20 week 21 week 22	Unit 5 Proportional Relationships (18–22 days) Optional Lessons: 17, 18, 19, 20	Unit 5 Linear Relationships (25–31 days) (MA) Optional Lessons: 10, 24, 25, 26, 27, 28
week 23 week 24 week 25	Unit 6 Percent Increase and Decrease (12–15 days) Optional Lessons: 7, 12, 13	Unit 6 Functions and Volume (21–26 days) (MA) Optional Lessons: 14, 17, 18, 22, 23
week 26 week 27 week 28 week 29 week 30	Unit 7 Rational Numbers (23–26 days) (MA) Optional Lessons: 17, 22, 23	Unit 7 Exponents and Scientific Notation (15–16 days) Optional Lesson: 14
week 31 week 32 week 33	Unit 8 Data Sets, Distributions, and Sampling (13–19 days) Optional Lessons: 12–17	Unit 8 Pythagorean Theorem and Irrational Numbers (15–17 days) Optional Lessons: 10, 15
week 34 week 35 week 36	Unit 9 Putting It All Together (0–17 days) Optional Lessons: all	Unit 9 Putting It All Together (0–12 days) Optional Lessons: all

(MA) = Unit has Mid-Unit Assessment

Total number of days for each course = Lessons + Assessments – Optional Lessons

Accelerated 6 = 146 Days Accelerated 7 = 147 Days

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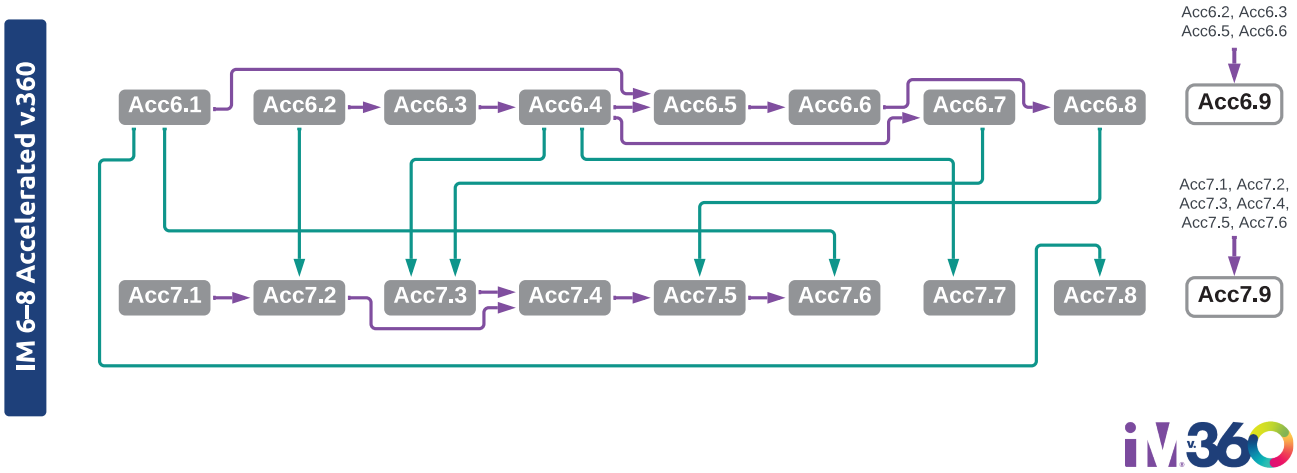
Dependency Chart



In the unit dependency chart, an arrow indicates that a particular unit is designed for students who already know the material in a previous unit. Reversing the order of the units would have a negative effect on mathematical or pedagogical coherence. Examples:

- There is an arrow from Acc6.6 to Acc6.8, because students are expected to use their skills in representing percentages (from Acc6.6) when solving problems about probability (in Acc6.8).
- There is an arrow from Acc7.2 to Acc7.4, because students are expected to use their knowledge of similar triangles and slope (from Acc7.2) when they represent linear relationships in a variety of ways (in Acc7.4).

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