



Using Long Division

Goals

- Interpret the long division method, and explain (orally) what each step in the division represents.
- Use long division to divide whole numbers that result in a whole-number quotient, and multiply the quotient by the divisor to check the answer.

Learning Targets

- I can use long division to find a quotient of two whole numbers when the quotient is a whole number.

Lesson Narrative

This lesson introduces students to **long division**. Students see that in long division the meaning of each digit is intimately tied to its place value, and that it is an efficient way to find quotients.

When using partial quotients to calculate quotients, all numbers and their meaning are fully and explicitly written out. For example, to find $657 \div 3$ we write that there are at least 3 groups of 200, record a subtraction of 600, and show a difference of 57. In long division, instead of writing out all the digits, we rely on the position of any digit—of the quotient, of the number being subtracted, or of a difference—to convey its meaning, which simplifies the calculation.

Students begin by analyzing a long division for $657 \div 3$. Having seen the same division calculated using partial quotients, students can better interpret what each digit represents and can focus on making sense of the structure of the algorithm. Next, students use the algorithm to perform division with whole-number dividends, divisors, and quotients.

An optional activity allows students to further practice using long division and to analyze a place-value error commonly made in long division.

Standards

Building On	4.NBT.B.6
Addressing	6.NS.B.2
Building Toward	6.EE.A

Instructional Routines


- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Materials to Gather

- Graph paper: Activity 2, Activity 3

Student Facing Learning Goals

 Let's use long division.



Warm-up

Activity Narrative

This *Warm-up* prompts students to make sense of a long division calculation, by familiarizing themselves with the structure and the mathematics that might be involved (MP1), before they later learn to use the algorithm to calculate quotients.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see (MP6). They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Standards

Building On 4.NBT.B.6
Building Toward 6.EE.A

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Display the images of Lin's calculations for all to see. Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing that they notice and one thing that they wonder about. Give students another minute to discuss their observations and questions.

Student Task Statement

Here are Lin's calculations for finding $657 \div 3$.

$$\begin{array}{r}
 3 \overline{) 657} \\
 \underline{- 6} \\
 05
 \end{array}
 \qquad
 \begin{array}{r}
 2 \\
 3 \overline{) 657} \\
 \underline{- 6} \downarrow \\
 05
 \end{array}
 \qquad
 \begin{array}{r}
 2 \\
 3 \overline{) 657} \\
 \underline{- 6} \\
 5 \\
 \underline{- 3} \\
 2
 \end{array}
 \qquad
 \begin{array}{r}
 2 9 \\
 3 \overline{) 657} \\
 \underline{- 6} \downarrow \\
 5 \\
 \underline{- 3} \\
 27 \\
 \underline{- 27} \\
 0
 \end{array}$$

What do you notice? What do you wonder?

Student Response

Students may notice:

- Lin arranged her calculations vertically.
- There are no partial quotients being added, but the quotient is still 219, just as in other calculations.



- The quotient is shown one digit at a time, starting with the hundreds, then the tens, then the ones.
- Lin subtracted 6, then 3, and then 27.
- There are arrows pointing down in the tens place and ones place.

Students may wonder:

- What do the arrows mean?
- Why did Lin divide one number at a time?
- Why did Lin subtract one-digit numbers at first, 6 and 3, but then subtract a two-digit number, 27, at the end?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the calculations. Next, ask students, “Is there anything on this list that you are wondering about now?” Encourage students to observe what is on display and to respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the idea of dividing by place value does not come up during the conversation, ask students to discuss this idea.

11.2 Lin Uses Long Division

🕒 25 min

Activity Narrative

This activity introduces the use of **long division** to calculate a quotient of whole numbers. Students make sense of the process of long division by studying an annotated example and relating it to the use of partial quotients and base-ten diagrams. They begin to see that long division is a variant of the method that uses partial quotients, but it is calculated and recorded differently.

When using partial quotients, the division is done in installments. The size of each installment can vary, but it is always a multiple of the divisor. Each partial quotient is written above the dividend and stacked. The sum of all partial quotients is the quotient.

In long division, the division is performed digit by digit, from the largest place to the smallest, so the resulting quotient is also recorded one digit at a time. In each step, one more digit of the quotient is calculated. Students notice that although only one digit of the quotient is written down at a time, the value that it represents is communicated through its placement.

As students examine the numbers, their placements, and the operations in the algorithm and interpret them in terms of place value and the process of dividing, they engage in abstract and quantitative reasoning (MP2).

To become proficient in long division requires time, encounters with a variety of division problems, and considerable practice. Students will have opportunities to study the algorithm more closely and to use it to divide increasingly more challenging numbers over several upcoming lessons.

Standards

Addressing 6.NS.B.2

Instructional Routines

- MLR8: Discussion Supports



Launch

Tell students that they will now consider a third method—called **long division**—for solving the same division problem that Kiran and Andre had calculated using partial quotients.

Display both Andre's and Lin's calculations for all to see.

Arrange students in groups of 2. Give students 2–3 minutes of quiet time to read and make sense of the annotated example of long division, and then 3–4 minutes to discuss the first set of questions with a partner. Pause for a whole-class discussion before students use long division to answer the second set of questions.

Invite 1–2 students to explain, in their own words, what Lin had done. Then discuss students' responses to the first set of questions. Follow with questions such as:

- "After writing down the 2, Lin subtracted 6. Why was the result of the subtraction not 651 given that $657 - 6 = 651$?" (Though she wrote a subtraction of 6, she actually subtracted 600 from 657.)
- "Could Lin have written the full amounts being subtracted instead of just the non-zero digit—for example, writing 600, 50, and 7 instead of aligning 6, 5, and 7 to certain places?" (Yes, it would involve more writing, but it works just as well.)

Consider demonstrating the long-division process with another example such as $912 \div 4$ before asking students to complete the rest of the task. Provide access to graph paper. Tell students that the grid could help them line up the digits.



Access for English Language Learners

- MLR8 Discussion Supports. Invite students to repeat their explanations for Lin's steps using mathematical language: "Can you say that again, using the terms 'place value,' 'ones place,' and 'tenths place'?"
- Advances: Speaking, Representing



Access for Students with Disabilities

- Representation: Access for Perception. Read Lin's method for calculating the quotient of $657 \div 3$ aloud. Students who both listen to and read the information will benefit from extra processing time.
- Supports accessibility for: Language, Attention



Student Task Statement

- Here is how Lin found the quotient of $657 \div 3$.

Lin arranged the numbers for vertical calculations.

Her plan was to divide each digit of 657 into 3 groups, starting with the 6 hundreds.

There are 3 groups of 2 in 6, so Lin wrote 2 at the top and subtracted 6 from the 6, leaving 0.

Then, she brought down the 5 tens of 657.

There are 3 groups of 1 in 5, so she wrote 1 at the top and subtracted 3 from 5, which left a remainder of 2.

She brought down the 7 ones of 657 and wrote it next to the 2, which made 27.

There are 3 groups of 9 in 27, so she wrote 9 at the top and subtracted 27, leaving 0.

$$3 \overline{) 657}$$

$$\begin{array}{r} 2 \\ 3 \overline{) 657} \\ \underline{- 6} \\ 05 \end{array}$$

$$\begin{array}{r} 21 \\ 3 \overline{) 657} \\ \underline{- 6} \\ 5 \\ \underline{- 3} \\ 2 \end{array}$$

$$\begin{array}{r} 219 \\ 3 \overline{) 657} \\ \underline{- 6} \\ 5 \\ \underline{- 3} \\ 27 \\ \underline{- 27} \\ 0 \end{array}$$

- Study Lin's steps. Then discuss with your partner:
 - In the first step, Lin divided 6 by 3 to get 2. Why do you think she put the 2 over the 6?
 - Lin subtracted $3 \cdot 2$ then $3 \cdot 1$, and lastly $3 \cdot 9$. Earlier, Andre subtracted $3 \cdot 200$ then $3 \cdot 10$, and lastly $3 \cdot 9$. Why did they have the same quotient?
 - In the third step, why do you think Lin wrote the 7 next to the remainder of 2 rather than adding 7 and 2 to get 9?
- Lin's method is called **long division**. Use this method to find the following quotients. Check your answer by multiplying it by the divisor.
 - $846 \div 3$
 - $1,816 \div 4$
 - $768 \div 12$

Student Response

- Sample response:
 - The 6 represents 600. Lin was really dividing 600 by 3, which is 200. The 2 needs to be written in the hundreds place to tell us its actual value.
 - The $3 \cdot 2$ Lin subtracted from 6 represents $3 \cdot 200$ subtracted from 600, since the 6 and the 2 are both in the hundreds place. Similarly, the $3 \cdot 1$ Lin subtracted from 5 represents $3 \cdot 10$ subtracted from 5 tens (or 50). Lin's work shows the same steps Andre took without writing out as many digits in the calculations.
 - The 2 represents two tens (or 20). The 7 represents 7 ones. So Lin is adding 2 tens to 7 ones, and the result is 27.



a.

$$\begin{array}{r} 282 \\ 3 \overline{) 846} \\ \underline{-6} \\ 24 \\ \underline{-24} \\ 6 \\ \underline{-6} \\ 0 \end{array}$$

b.

$$\begin{array}{r} 4 \overline{) 1816} \\ \underline{- 16} \\ 21 \\ \underline{- 20} \\ 16 \\ \underline{- 16} \\ 0 \end{array}$$

C.

$$\begin{array}{r} 64 \\ 12 \overline{) 768} \\ \underline{- 72} \\ 48 \\ \underline{- 48} \\ 0 \end{array}$$

Building on Student Thinking

Students may recognize that they need to subtract multiples of the divisor in each place but not always subtract the greatest multiple. For example, when calculating $846 \div 3$ they might subtract 3 instead of 6 from the 8 in the hundreds place, and end up with 54 for the next round of division, which would lead to a two-digit quotient for that round. Urge students to check if they have found the greatest whole-number multiple of the divisor in each place before moving on to the next place. One way to check is to see if the result of the subtraction (in this case, 5, the result of $8 - 3$), is greater than the divisor, and if so, to adjust the calculation accordingly.

Activity Synthesis

Display the worked-out long divisions for all to see. Select a student to explain the steps for at least one of the division problems.

Draw students' attention to the division $1,816 \div 4$, in which the first digit of the dividend is smaller than the divisor. Select 1–2 students to share how they approached this situation. If not brought up in students' explanation, discuss how we could reason about these.

- “If we were using base-ten diagrams to represent $1,816 \div 4$, we would have 1 piece representing a thousand. How would we divide that piece into 4 groups?” (Decompose it into 10 hundreds, add them to the 8 pieces representing 8 hundreds, and then distribute the 18 hundreds into 4 groups.)
- “How can we apply the same idea to long division when there are not enough thousands to divide into 4 groups?” (We can think of the 1 thousand and 8 hundreds as 18 hundreds and divide that value instead.)
- “How many hundreds would go into each group if we divide 18 hundreds into 4 groups?” (4 hundreds, with a remainder of 2 hundreds.)
- “Where should we write the 4? Why?” (In the hundreds place, because it represents 4 hundreds.)
- “How do we deal with the remaining 2 hundreds?” (Decompose them into 20 tens, combine those with the 1 ten, and divide 21 tens by 4.)

Activity Narrative

In this activity, students continue to practice using long division to find quotients. Here, the presence of 0's in the dividend and the quotient presents an added layer of complexity, prompting students to reason abstractly and quantitatively (MP2) as they make sense of the meaning of each digit in the numbers they are dealing with.

Standards

Addressing 6.NS.B.2

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in groups of 2. Give students 6–7 minutes to work independently or with their partner. Follow with a whole-class discussion. Provide access to graph paper.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation, such as “First, I _____ because . . .,” “Why did you . . .?,” “How did you get . . .?”

Supports accessibility for: Language, Social-Emotional Functioning

Student Task Statement

1. Use long division to calculate each quotient.

- a. $1001 \div 7$
- b. $2996 \div 14$

2. Here is Priya's calculation of $906 \div 3$.

$$\begin{array}{r}
 \overline{3 2 0} \\
 3 \overline{) 9 0 } \\
 \underline{- 9} \\
 0 \\
 \underline{-} \\
 6 \\
 \underline{-} \\
 0
 \end{array}$$

- a. Priya wrote 320 for the value of $906 \div 3$. Check her answer by multiplying it by 3. What product do you get?
- b. What does the product tell you about Priya's answer? Explain your reasoning. If you think her answer is incorrect, describe the error and show the correct calculation and answer.

Student Response

1. a

$$\begin{array}{r} 143 \\ 7 \overline{) 1001} \\ \underline{- 7} \\ 30 \\ \underline{- 28} \\ 21 \\ \underline{- 21} \\ 0 \end{array}$$

b

$$\begin{array}{r} 214 \\ 14 \overline{) 2996} \\ \underline{- 28} \\ 19 \\ \underline{- 14} \\ 56 \\ \underline{- 56} \\ 0 \end{array}$$

2. a. $320 \cdot 3 = 960$.

b. Priya's answer is incorrect. Sample reasoning: Multiplying the answer by 3 should give 906, but it gives 960.

The 2 of the quotient should be in the ones place. Since there are only 6 ones remaining, she should have taken away 3 times 2, not 3 times 20. The correct answer is 302.

$$\begin{array}{r} 302 \\ 3 \overline{) 906} \\ \underline{- 9} \\ 0 \\ \underline{- 0} \\ 6 \\ \underline{- 6} \\ 0 \end{array}$$

Activity Synthesis

Display the solutions to the first two division problems and give students time to check their work. Then, focus the discussion on ways to tell if a long division is done correctly. Discuss questions such as:

- "How can we check if 143 is the quotient of $1,001 \div 7$?" (We can multiply 143 by 7. If the division was done correctly, then the product would be 1,001, which is the case.)
- "Why does multiplying Priya's answer, 320, by the divisor 3, give 960 instead of 906? Where did the error happen?" (Priya placed the result of dividing the 6 in 906 by 3 in the tens place, even though the 6 represents ones. It should've been 2 ones.)

Make sure students notice that although checking an answer can tell us that we have made a mistake, it will not necessarily show where the mistake is. This way of checking also works only if we perform the multiplication correctly.



Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students in explaining their analysis of Priya's answer. Examples:

- "The product shows that Priya's answer is correct (or incorrect) because . . ."
- "She did _____ correctly and (but) she . . ."



- “The correct answer should be _____ because . . .”

Advances: Writing, Conversing

Lesson Synthesis

Invite students to compare and contrast long division with other methods of division. Ask questions such as:

- “How is long division like other methods of dividing that we have used before?” (It involves putting base-ten units into equal-size groups, decomposing larger units into smaller ones, and paying attention to place value.)
- “How is long division similar to the vertical calculations that use partial quotients? (We divide in “installments” and subtract parts of the dividend below it until there is nothing left. The quotient is above the dividend. The steps are recorded in a vertical format.)
- “How is it different?” (There is a very specific order to follow. We divide digit by digit—from left to right, and subtract as large a group as possible at any step. Not all the digits of the amounts being divided, multiplied, or subtracted are written out. The place of the digit tells us its value.)

Then use an example from the lesson to highlight some ideas about long division:

- We start by dividing the digit with the highest place value and work downward.
- Any remainder in one place is worth 10 times as much in the place to its right and is combined with the value that is in that place before being divided. This keeps going until there is nothing left of the dividend.
- The placement of each digit of the quotient matters because it conveys the value of the digit

11.4

Dividing by 5

Cool-down

5 min



Standards

Addressing 6.NS.B.2



Student Task Statement



Use long division to find the value of $1,875 \div 5$. Then check your answer by multiplying it by 5.

Student Response

375.

$$\begin{array}{r}
 \overline{) 1875} \\
 \underline{- 15} \\
 37 \\
 \underline{- 35} \\
 25 \\
 \underline{- 25} \\
 0
 \end{array}$$



Responding to Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 11 Summary

Long division is another method for calculating quotients. It relies on place value to perform and record the division.

When we use long division, we work from left to right and with one digit at a time, starting with the leftmost digit of the dividend. We remove the largest group possible each time, using the placement of the digit to indicate the size of each group.

Here is an example of how to find $948 \div 3$ using long division.

$$\begin{array}{r} 316 \\ 3 \overline{) 948} \\ \underline{- 9} \longleftarrow 3 \text{ groups of 3 (hundreds)} \\ 4 \\ \underline{- 3} \longleftarrow 3 \text{ groups of 1 (ten)} \\ 1 8 \\ \underline{- 1} 8 \longleftarrow 3 \text{ groups of 6 (ones)} \\ 0 \end{array}$$

- We start by dividing 9 hundreds into 3 groups, which means 3 hundreds in each group. Instead of writing 300, we simply write 3 in the hundreds place, knowing that it means 3 hundreds.
- There are no remaining hundreds, so we work with the tens. We can make 3 groups of 1 ten out of 4 tens, so we write 1 in the tens place above the 4 of 948. Subtracting 3 tens from 4 tens, we have a remainder of 1 ten.
- We know that 1 ten is 10 ones. Combining these with the 8 ones from 948, we have 18 ones. We can make 3 groups of 6 ones, so we write 6 in the ones place.

In total, there are 3 groups of 3 hundreds, 1 ten, and 6 ones in 948, so $948 \div 3 = 316$.

Glossary

-  • long division

Lesson 11 Practice Problems

1 Student Task Statement

Kiran is using long division to find $696 \div 12$.

$$12 \overline{) 696}$$

He starts by dividing 69 by 12. In which decimal place should Kiran place the first digit of the quotient (5)?

- A. Hundreds
- B. Tens
- C. Ones
- D. Tenths

Solution

B

2 Student Task Statement

Here is a long-division calculation of $917 \div 7$.

$$\begin{array}{r} 131 \\ 7 \overline{) 917} \\ \underline{- 7} \\ 21 \\ \underline{- 21} \\ 7 \\ \underline{- 7} \\ 0 \end{array}$$

- a. There is a 7 under the 9 of 917. What does this 7 represent?
- b. What does the subtraction of 7 from 9 mean?
- c. Why is a 1 written next to the 2 from $9 - 7$?

Solution

- a. Sample response: The 7 under the 9 represents 700 (because it is written directly under the hundreds place of 917).
- b. Sample response: It means a subtraction of 7 groups of 1 hundred from 9 hundreds.
- c. Sample response: To represent the 10 in 917. There are 2 hundreds left after 7 hundreds are subtracted from 9 hundreds. The 2 hundreds are combined with the 1 ten from 917, which makes 21 tens.



3 Student Task Statement

Han's calculation of $972 \div 9$ is shown here.

$$\begin{array}{r}
 180 \\
 9 \overline{) 972} \\
 \underline{- 9} \\
 72 \\
 \underline{- 72} \\
 0 \\
 \underline{- 0} \\
 0
 \end{array}$$

- Find $180 \cdot 9$.
- Use your calculation of $180 \cdot 9$ to explain how you know Han has made a mistake.
- Identify and correct Han's mistake.

Solution

- $180 \cdot 9 = 1620$
- If Han were correct, the product of 180 and 9 would be 972.
- Sample response: Han's mistake is that when he brought down the 7 from 972 and saw that 7 tens could not be divided into 9 groups (or 7 is not a multiple of 9), he did not write 0 above the 7 before bringing down the 2 ones. Here is the correct long division calculation:

$$\begin{array}{r}
 108 \\
 9 \overline{) 972} \\
 \underline{- 9} \\
 07 \\
 \underline{- 0} \\
 72 \\
 \underline{- 72} \\
 0
 \end{array}$$

4 Student Task Statement

Find each quotient.

a.

$$5 \overline{) 465}$$

b.

$$12 \overline{) 924}$$

c.

$$3 \overline{) 1107}$$

Solution

- a. 93
- b. 77
- c. 369

a.

$$\begin{array}{r} 93 \\ 5 \overline{) 465} \\ \underline{-45} \\ 15 \\ \underline{-15} \\ 0 \end{array}$$

b.

$$\begin{array}{r} 77 \\ 12 \overline{) 924} \\ \underline{-84} \\ 84 \\ \underline{-84} \\ 0 \end{array}$$

c.

$$\begin{array}{r} 369 \\ 3 \overline{) 1107} \\ \underline{-9} \\ 20 \\ \underline{-18} \\ 27 \\ \underline{-27} \\ 0 \end{array}$$

5

from Unit 5, Lesson 7



Student Task Statement

Suppose one ounce of craft paint will cover 1.2 square feet. How many square feet would 14.25 ounces of craft paint cover?

- A. 0.171 square foot
- B. 1.71 square feet
- C. 17.1 square feet
- D. 171 square feet

Solution

C

6

from Unit 5, Lesson 4



Student Task Statement

The mass of one coin is 16.718 grams. The mass of a second coin is 27.22 grams. How much greater is the mass of the second coin than the first? Show your reasoning.

Solution

10.502 grams. Sample reasoning: $27.22 - 16.718 = 10.502$

