Drawing Triangles (Part 1)

Goals

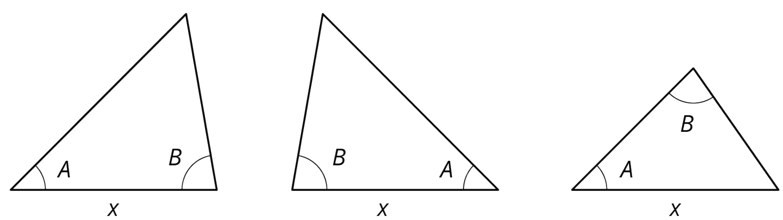
- Describe, compare, and contrast (orally and in writing) triangles that share three common measures of angles or sides.
- Draw triangles with two given angle measures and one side length, and describe (orally) how many different triangles could be drawn with the given conditions.
- Justify (orally and using other representations) whether triangles are identical copies or are "different" triangles.

Learning Targets

- Given two angle measures and one side length, I can draw different triangles with these measurements or show that these measurements determine one unique triangle or no triangle.

Lesson Narrative

In this lesson, students examine sets of triangles in which all the triangles share 3 common measures of angles or sides. For example, suppose a triangle has angles that measure A and B and a side length that measures x. Here are 3 triangles that have these measures:



This example shows 2 "different triangles" – the first two triangles are congruent, but the third is not, so it is different from the other two.

Students construct arguments and critique the reasoning of others as they decide whether triangles are congruent or not (MP3). Students do not need to memorize how many different kinds of triangles are possible given different combinations of angles and sides, and they do not need to know criteria such as angle-side-angle for determining if two triangles are identical copies.

Then students build on their observations of triangles with shared angle measures and side lengths, by drawing their own triangles with specified measures: a given angle, two given angles, and two given angles and a given side length. This helps students see the structure of certain triangles—namely that sometimes the given conditions allow only one possible triangle, sometimes more than one, and sometimes none (MP7). Students also gain experience using various tools to draw triangles with given conditions.

Teacher Notes for IM 6–8 Math Accelerated v.360
In this course, this lesson occurs after students learn what it means for two shapes to be congruent. Encourage students to use language about transformations to more precisely state why two shapes are congruent or not congruent instead of using the term "identical copies."

Standards

Addressing **7.G.A.2**

Instructional Routines

- MLR2: Collect and Display

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2, Activity 3

Required Preparation

Activity 3:

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's see how many different triangles we can draw with certain measurements.

16.1 3 Sides; 3 Angles

Warm-up

10 min

Activity Narrative

The purpose of this *Warm-up* is to begin looking at the different triangles that can be drawn when three measures are specified. The first set of triangles in this activity all share the same 3 side lengths. The second set of triangles all share the same 3 angle measures. Students consider which set of conditions mean that the created triangles must be identical.

Teacher Notes for IM 6–8 Math Accelerated v.360

In this course, this lesson occurs after students learn language about rigid transformations. It is appropriate for students to use words like "reflection" or "rotation" when determining how many different triangles there are. Encourage students to use the term "congruent" instead of "identical copies."

Students may need less time to complete this activity. Adjust the timing to 5 minutes.

Standards

Addressing **7.G.A.2**

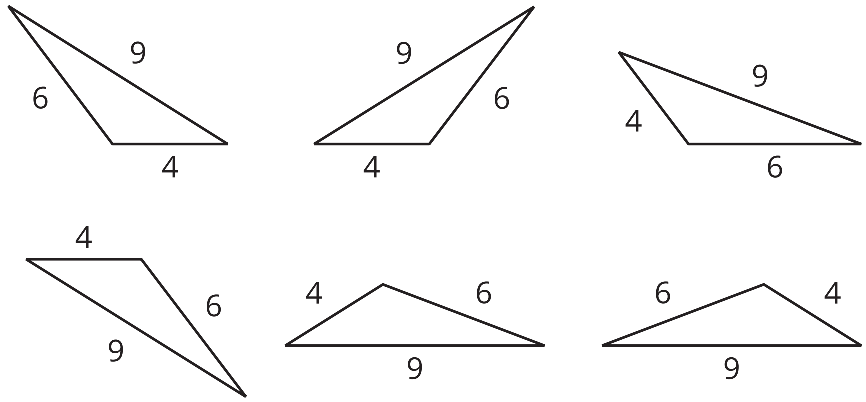
Launch

Provide access to geometry toolkits. Give students 1 minute of quiet think time, followed by a whole-class discussion.

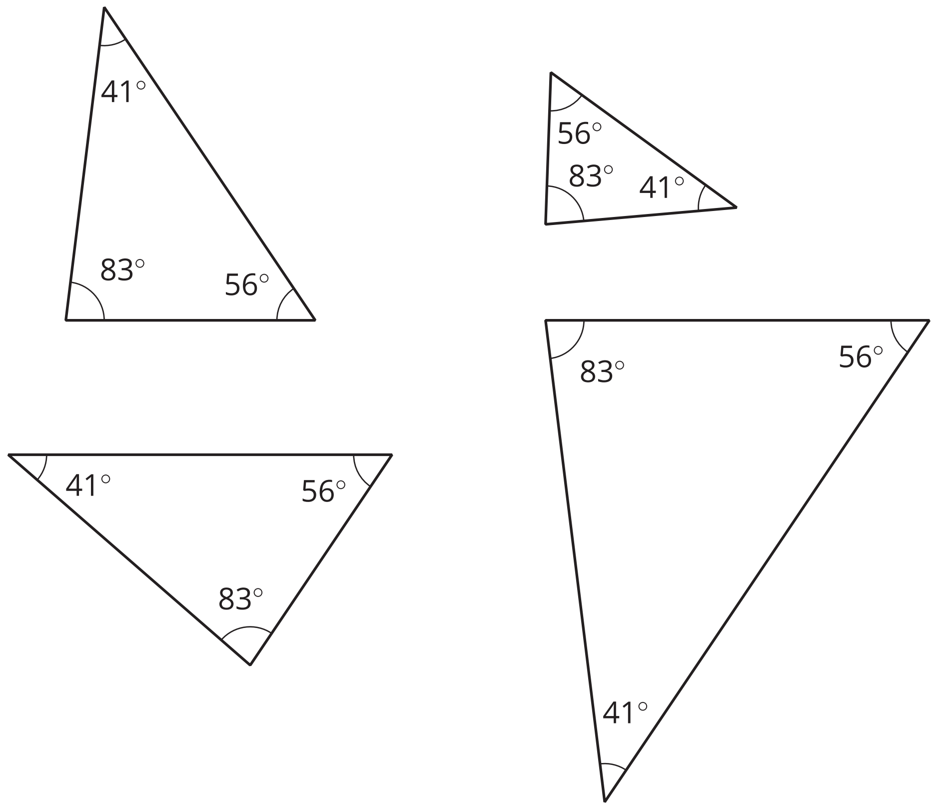
Student Task Statement

Examine each set of triangles. What do you notice? What is the same about the triangles in the set? What is different?

Set 1:



Set 2:



Student Response

1. All of the side lengths and angles are the same size. These triangles are identical copies. The triangles face different directions.
2. These triangles all have the same angles, but different side lengths. They could be scaled copies that are oriented differently.

Building on Student Thinking

Some students may say that all the triangles in the second set are “the same shape.” This statement can result from two very different misconceptions. Listen to the students’ reasoning and explain as needed:

1. Just because they are all in the same category, “triangles,” doesn’t mean they are all the same shape. If we can take two shapes and position one exactly on top of the other, so all the sides and corners line up, then they are identical

copies.

2. These triangles are scaled copies of each other, but that does not make them “the same” because their side lengths are still different. Only scaled copies made using a scale factor of 1 are identical copies.

Activity Synthesis

Invite students to share things they notice—things that are the same and things that are different about the triangles. Record and display these ideas for all to see.

If these discussion points do not come up in students’ explanations, make them explicit:

In the first set:

- All the triangles are identical copies, just in different orientations.
- They have the same 3 side lengths.
- They have the same 3 angle measures (can be checked with tracing paper or a protractor).

In the second set:

- The triangles are not identical copies.
    - Note: Students may recognize that these triangles are scaled copies of each other, since they have the same angle measures. However, this is the first time students have seen scaled copies in different orientations, and it is not essential to this lesson that students recognize that these triangles are scaled copies.
- They have the same 3 angle measures.
- They have different side lengths (can be checked with tracing paper or a ruler).

The goal is to make sure students understand that the second set has 3 different triangles (because they are different sizes) and that the first set really shows only 1 triangle in many different orientations. Tracing paper may be helpful to convince students of this.

16.2 2 Sides and 1 Angle

15 min

Activity Narrative

In this activity, students examine different orientations of triangles that all share 2 sides lengths and one angle measure. They recognize that some of these triangles are identical copies and others are different triangles (not identical copies).

In the coming lessons, students are asked to draw their own triangles. On their own, students often have trouble thinking about triangles where the three given conditions are not necessarily intended to be placed adjacent to one another. For example, when given two sides and an angle, many students will immediately think of putting the given angle between the two sides, but struggle with visualizing putting the angle anywhere else. This task is important for helping students view this as a viable option.

Teacher Notes for IM 6–8 Math Accelerated v.360
In this course, this lesson occurs after students learn language about rigid transformations and what it means for two shapes to be congruent. It is appropriate for students to use words like “reflection” or “rotation” when describing how they noticed the first set of triangles are all congruent. Similarly, students will formally study scaled copies in a later unit, so they are not likely to use the language of scaled copies when discussing the second set of triangles.

Standards

Addressing **7.G.A.2**

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 2. Give students 2–3 minutes of quiet work time followed by time to discuss their explanations with a partner. Follow with a whole-class discussion. Provide access to geometry toolkits.

Access for English Language Learners

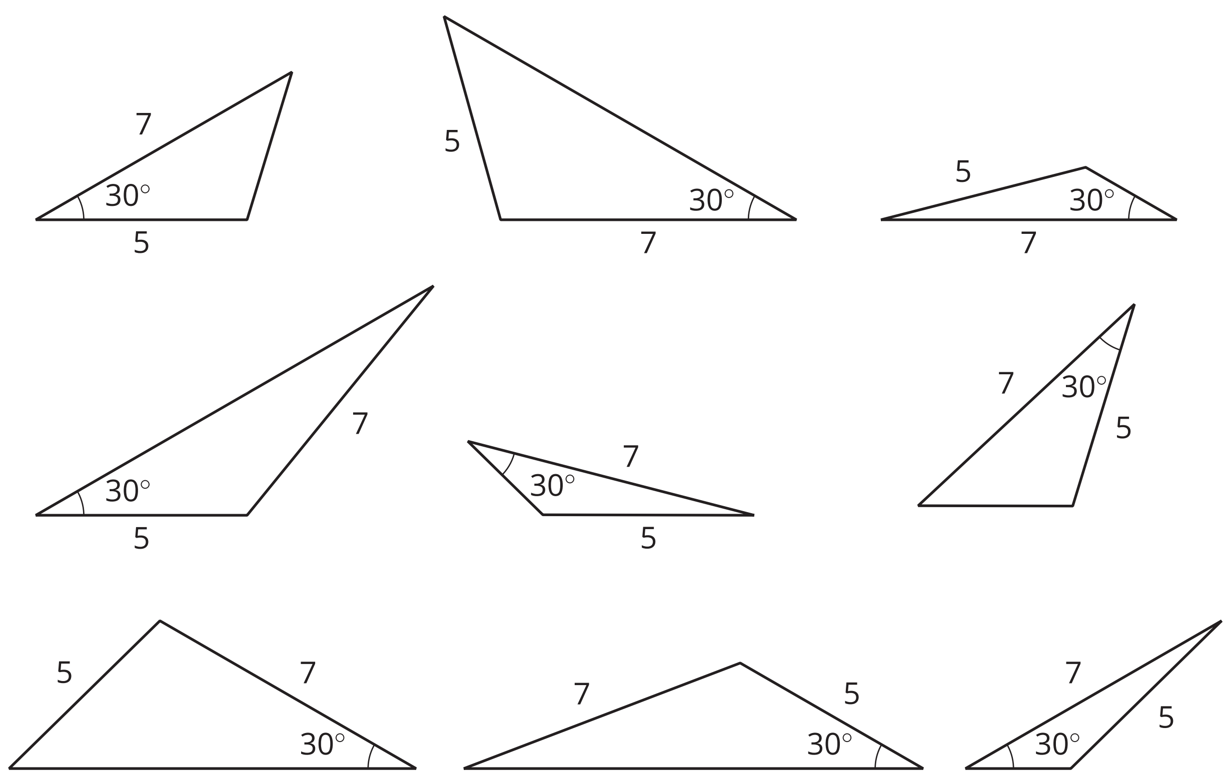
MLR2 Collect and Display. Collect the language that students use to compare triangles and to describe the relative position of sides and angles. Display words and phrases, such as “side length,” “angle measure,” “between,” “next to,” and “turned.” During the *Activity Synthesis*, invite students to suggest ways to update the display: “What are some other words or phrases we should include?” Invite students to borrow language from the display as needed.
Advances: Conversing, Reading

Access for Students with Disabilities

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide students with a printed copy of the triangles for them to cut out and rearrange to determine the number of different triangles.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Student Task Statement

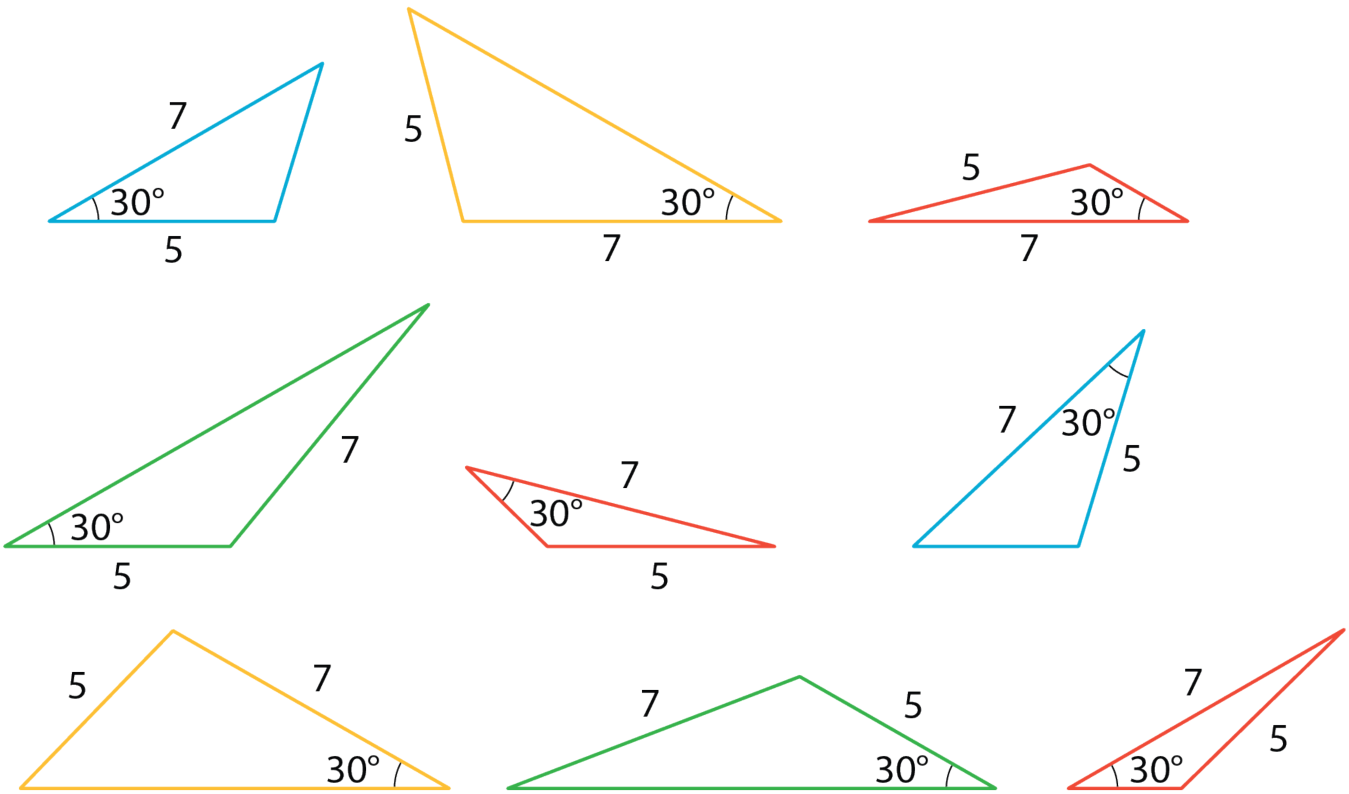
Examine this set of triangles.



1. What is the same about the triangles in the set? What is different?
2. How many different triangles are there? Explain or show your reasoning.

Student Response

1. All of the triangles have one side of length 5 and one side of length 7. And all of them have a 30-degree angle.The triangles are oriented differently, and the two sides and one angle are in a different order.
2. There are 4 different triangles.



Sample reasoning:

- The triangles marked in blue have the common measurements in this order: 7 units, 30°, 5 units.
- The triangles marked in green have the common measurements in this order: 30°, 5 units, 7 units.
- The triangles marked in both yellow and red have the common measurements in this order: 30°, 7 units, 5 units, but the yellow triangles are larger and the red triangles are smaller.

Building on Student Thinking

Some students may say that there are 9 different triangles, because they do not recognize that some of them are identical copies oriented differently. Prompt them to use tracing paper to compare the triangles.

Activity Synthesis

Select students to share the similarities and differences between the triangles in the set.

Trace a few of the triangles from the set and show how you can turn, flip, or move some of them to line up while others cannot be lined up. Ask students what this means about all the triangles in the set (they are not all identical to each other). Explain that, “While there are certainly times when the position of a triangle is important (‘I wouldn’t want my roof upside down!’), for this unit in geometry, we will consider shapes to be the same if they are identical copies.”

To highlight the differences among the triangles, ask students:

- “Is there only one possible triangle that could be created from the given conditions?” (No, there were 4.)
- “How would you explain what is different about these four triangles?” (Some have the 30° angle between the two sides of known length and others have the 30° angle next to the side of unknown length.)

Explain to students that it seems that the order in which the conditions are included in the triangle (for example, is the angle between the two sides or not?) matters in creating different triangles. Emphasize that the three required pieces (2 sides and 1 angle) do not have to all be put next to one another. When they are asked to draw triangles with three or more conditions, they should consider the way in which the conditions are arranged in their drawing. For example, think about whether the given angle must go between the two sides or not.

16.3 How Many Can You Draw?

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students are asked to draw as many different triangles as they can with the given conditions. The purpose of this activity is to provide an opportunity for students to see the three main results for this unit: a situation in which only a unique triangle can be made, a situation in which it is impossible to create a triangle from the given conditions, and a situation in which multiple triangles can be created from the conditions. Students make use of the structure of triangle side lengths and angles as they explore these conditions (MP7).

Students are not expected to remember which conditions lead to which results, but should become more familiar with some methods for attempting to create different triangles. They will practice including various conditions into the triangles, including the conditions in different combinations, and practice recognizing when the resulting triangles are identical copies or not.

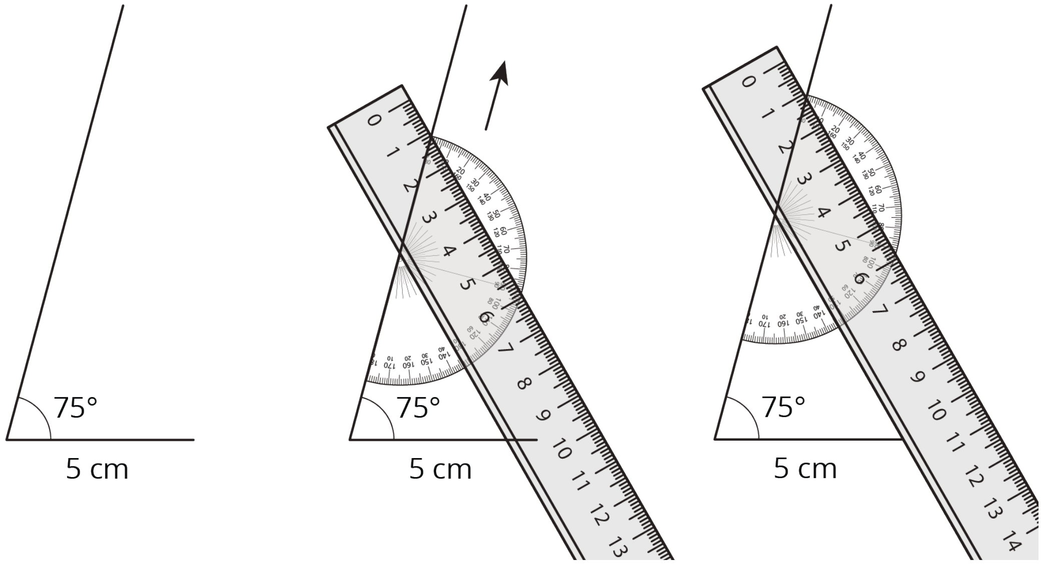
In the digital version of the activity, students use an applet to create triangles with given conditions. The applet allows students to change the conditions dynamically.

Teacher Notes for IM 6–8 Math Accelerated v.360
In this course, this lesson occurs after students learn that the sum of the interior angles of a triangle is 180 degrees. It is appropriate for students to use this fact when reasoning about unique triangles.

Also, the *Activity Synthesis* refers to a previous activity that is not included in this course. If time allows, demonstrate for students that one way to draw a triangle (Lin's method) with one angle measuring 75°, one angle measuring 45°, and one side measuring 5 cm is to:

1. Draw the 5 cm segment.
2. Draw the 75° angle on one end of the segment, with a very long ray.
3. Place a protractor along the ray.
4. Line up a ruler at the 45° mark on the protractor.
5. Keeping the ruler and protractor together, slide them along the ray until the edge of the ruler intersects with the other end of the 5 cm segment.

6. Keeping the ruler in place on the paper, remove the protractor from underneath it.
7. Draw a line along the ruler from the ray to the segment.



Consider asking students to try to draw a triangle with one angle measuring 60°, one angle measuring 90°, and one side measuring 4 cm the way Lin would.

Standards

Addressing **7.G.A.2**

Launch

Keep students in the same groups. Tell students they must try at least two different times to draw a triangle with the measurements given in each problem. Give students 5 minutes of quiet work time followed by time to discuss their different triangles with a partner. Follow with a whole-class discussion. Provide access to geometry toolkits.

Access for Students with Disabilities

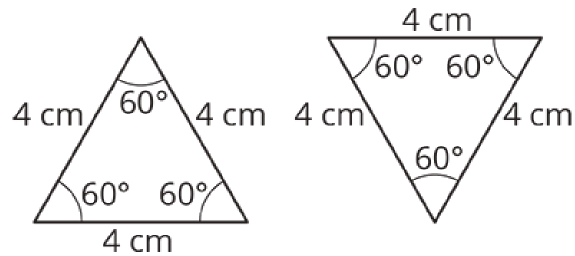
Action and Expression: Internalize Executive Functions. To support organization, provide students with a printed graphic organizer to categorize the different triangles by condition.
Supports accessibility for: Language, Organization

Student Task Statement

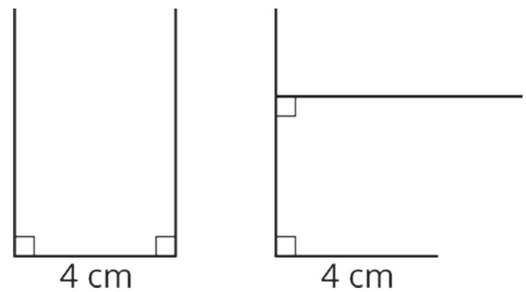
1. Draw as many different triangles as you can with each of these sets of measurements:
   a. Two angles measure 60°, and one side measures 4 cm.
   b. Two angles measure 90°, and one side measures 4 cm.
   c. One angle measures 60°, one angle measures 90°, and one side measures 4 cm.
2. Which of these sets of measurements determine one unique triangle? Explain or show your reasoning.

Student Response

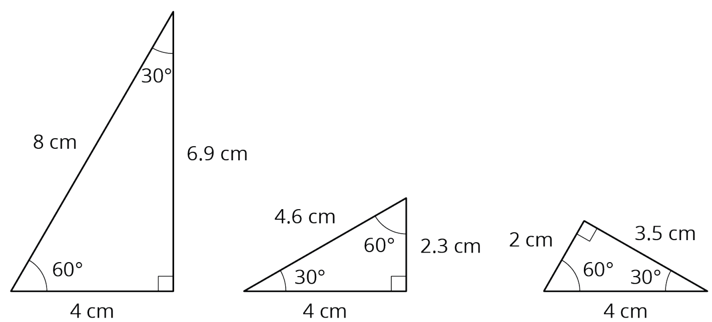
1. Sample responses:
    a. Two orientations of the same triangle.



    b. Two attempts to draw a triangle with two 90° angles and a 4-cm side. There is no possible triangle with these conditions.



    c. Three different triangles can be made with the conditions.



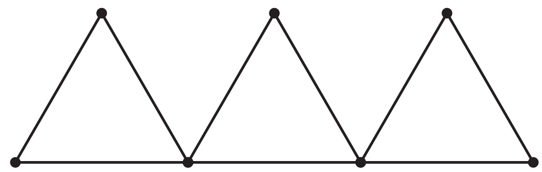
2. Only the first set of measurements determine a unique triangle.

Building on Student Thinking

Some students may draw two different orientations of the same triangle for the third set of conditions, with the 4-cm side in between the 60° and 90° angles. Prompt them to use tracing paper to check whether their two triangles are really different (not identical copies).

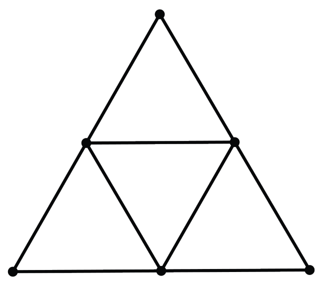
Some students may say the third set of measurements determines one unique triangle, because they assume the side length must go between the two given angle measures. Remind them of the discussion about Lin's triangle in the previous activity.

Are You Ready for More?



In the diagram, 9 toothpicks are used to make three equilateral triangles. Figure out a way to move only 3 of the toothpicks so that the diagram has exactly 5 equilateral triangles.

Extension Student Response



There are four small equilateral triangles and one large one.

Activity Synthesis

Ask students to indicate how many *different* triangles (triangles that are not identical copies) they could draw for each set of conditions. Select students to share their drawings and reasoning about the uniqueness of each problem. Discuss the methods that students used as they thought about other triangles that might fit the conditions.

Consider asking some of the following questions:

- "Which conditions produced a unique triangle?" (the first set of conditions)
- "Were there conditions that produced more than one triangle?" (the third set of conditions)
- "Were there conditions you could not draw a triangle for?" (the second set of conditions)
- "Why could you not draw a triangle for the second set of conditions?" (because two sides are parallel and will never intersect)

If not mentioned by students, explain to students that for the third set of conditions it is possible that all students drew identical copies using the 4-cm length as the side between the 60° and 90° angles. Consider asking them to think of the previous activity and to try to draw the triangle the way Lin would.

In grade 7, students do not need to know that the angle measures within triangle have a sum of 180°. Tell them that next year they will learn more about why these different conditions determine different numbers of triangles.

Lesson Synthesis

Here are some questions for discussion:

- If you have a drawing of two triangles, how can you tell if they are congruent? (If I trace one triangle and can move the tracing to perfectly line up with the other, then they are congruent.)

- When trying to draw different triangles with the same set of conditions, what are some things to try? (Change the order of the conditions in the triangle.)
- Sometimes a set of conditions results in a unique triangle. What other results can come from a set of conditions? (It could be impossible, or they could be used to make multiple triangles.)
- If you are given a side length and two angles, what would you do to try to get started making different triangles? (Draw a line segment with the given length and put the two angles on each end. Then I would try leaving one angle on one end, but using Lin's method of using a protractor and sliding it along for the other angle to create a triangle. Finally, I would do something similar, but switch which angle is next to the given length.)

16.4 Comparing Andre's and Noah's Triangles

5 min

Cool-down

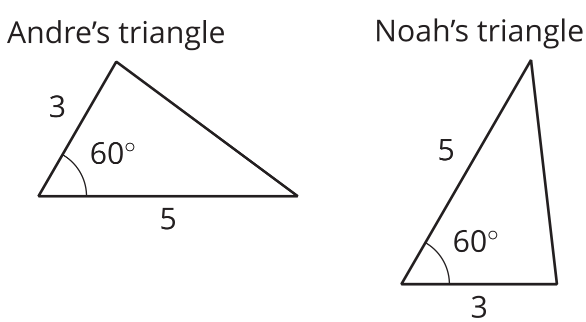
Teacher Notes for IM 6–8 Math Accelerated v.360
It is appropriate for students to use words like "reflection" or "rotation" when describing whether Andre and Noah drew different triangles.

Standards

Addressing **7.G.A.2**

Student Task Statement

Andre and Noah each drew a triangle with side lengths of 5 cm and 3 cm and an angle that measures 60°, and then they showed each other their drawings.



1. Did Andre and Noah draw different triangles? Explain your reasoning.
2. Explain what Andre and Noah would have to do to draw another triangle that is different from what either of them has already drawn.

Student Response

1. These are both the same triangle. In both cases, the 60° angle is between the 3-cm and 5-cm sides. If you trace one triangle, flip it and turn it, it can line up exactly with the other triangle.
2. To draw a different triangle, they should try putting the 60° angle next to the side of unknown length, instead of between the two known sides.

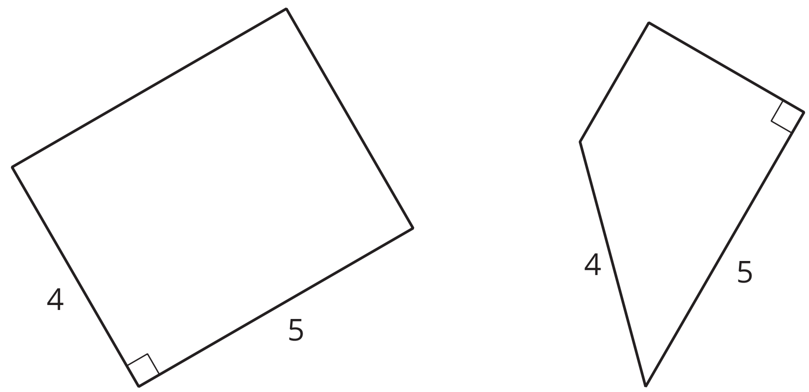
Responding to Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 16 Summary

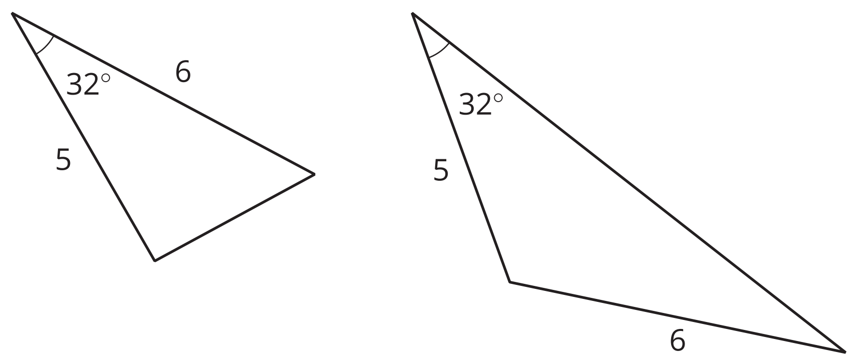
Both of these quadrilaterals have a right angle and side lengths 4 and 5:

However, in one case, the right angle is *between* the two given side lengths, and in the other, it is not.

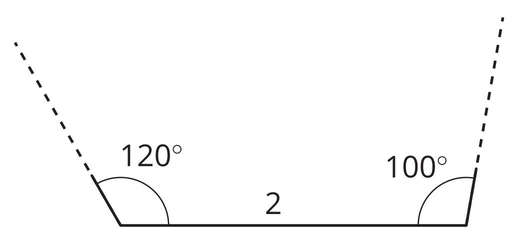


If we create two triangles with three equal measures, but these measures are not next to each other in the same order, that usually means that the triangles are different.

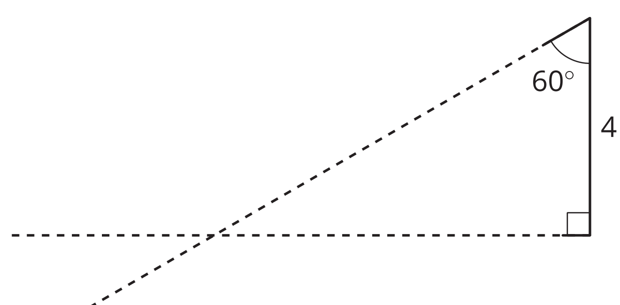
Here is an example:



Sometimes, we are given two different angle measures and a side length, and it is impossible to draw a triangle. For example, there is no triangle with side length 2 and angle measures 120° and 100°:



Sometimes, we are given two different angle measures and a side length between them, and we *can* draw a unique triangle. For example, if we draw a triangle with a side length of 4 between angles 90° and 60°, there is only one way in which they can meet up and make a triangle:

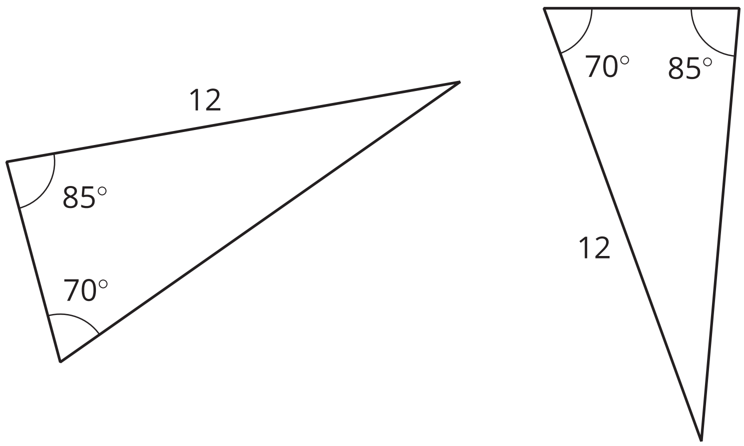


Any triangle drawn with these three conditions will be identical to the one above, with the same side lengths and the same angle measures.

Lesson 16 Practice Problems

1 Student Task Statement

Are these two triangles identical? Explain how you know.

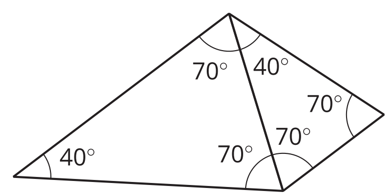


Solution

No, these two triangles are not identical. They have two of the same angle measures and one side length is the same, but the sides and angles are arranged differently in the triangles. In the triangle on the left, the side marked 12 is adjacent to the 85° angle. In the triangle on the right, the side marked 12 is adjacent to the 70° angle.

2 Student Task Statement

Are these triangles identical? Explain your reasoning.



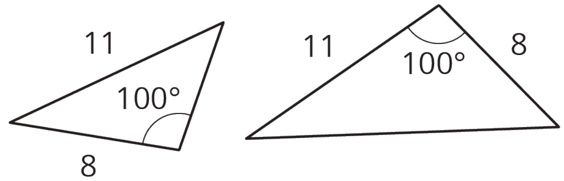
Solution

No, they are not identical. Although they have the same angle measurements, two of the side lengths are different.

3 Student Task Statement

Tyler claims that if two triangles each have a side length of 11 units and a side length of 8 units, and also an angle measuring 100°, they must be identical to each other. Do you agree? Explain your reasoning.

Solution



No, it is possible to build two different triangles with these measurements.

4 Student Task Statement

Use a protractor to try to draw each triangle. Which of these three triangles is impossible to draw?

a. A triangle where one angle measures 20° and another angle measures 45°

b. A triangle where one angle measures 120° and another angle measures 50°

c. A triangle where one angle measures 90° and another angle measures 100°

Solution

It is impossible to draw a triangle where one angle measures 90° and another angle measures 100°.

5 Student Task Statement

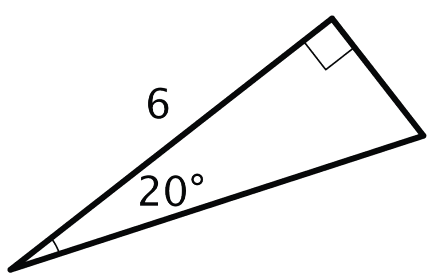
A triangle has an angle measuring 90°, an angle measuring 20°, and a side that is 6 units long. The 6-unit side is in between the 90° and 20° angles.

a. Sketch this triangle and label your sketch with the given measures.

b. How many unique triangles can you draw like this?

Solution

a.



b. There is only one triangle that fits this description, as long as the 6-unit side is between the two given angles.

