Unit 2, Lesson 8

Similar Triangles

Goals

- Generalize a process for identifying similar triangles and justify (orally) that finding two pairs of congruent angles is sufficient to show similarity.
- Justify (orally) that two triangles are similar by finding a sequence of transformations that takes one triangle to the other or checking that two pairs of corresponding angles are congruent.

Learning Targets

- I know how to decide if two triangles are similar just by looking at their angle measures.

Lesson Narrative

The purpose of this lesson is for students to see that if triangles have two corresponding angles with equal measure, then they are similar. Since the sum of angle measures in a triangle is always 180 degrees, having two angle measures will determine the third, making two angles all that is necessary to show similarity.

Students begin by observing features of a complex figure and crafting questions that can be asked about the figure. Then, by noticing patterns in triangles created out of dried pasta with given angle constraints, they deduce the criteria for similarity in terms of angle measures (MP7). An optional activity provides additional practice for identifying and justifying that two triangles are similar, using the same figure seen in the *Warm-up*.

Standards

Building On	**7.RP.A.2.a, 8.G.A.4**
Addressing	**8.G.A, 8.G.A.5**
Building Toward	**8.G.A.5**

Instructional Routines

- MLR2: Collect and Display
- MLR5: Co-Craft Questions
- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Blank paper: Activity 2
- Dried linguine pasta: Activity 2
- Geometry toolkits: Activity 2, Activity 3, Cool-down
- Protractors: Activity 2
- Rulers: Activity 2
- Tape: Activity 2
- Math Community Chart: Cool-down

Materials to Copy

- Making Pasta Angles and Triangles Cards (1 copy for every 4 students): Activity 2

Required Preparation

Activity 2:

Provide access to geometry toolkits, including blank paper, tape, a ruler, and a protractor. For the dried pasta that will be used to create the sides of the triangles, we recommend fettuccine or linguine so it doesn't roll off the table and is easy to break as needed.

Activity 3:

Provide access to geometry toolkits.

Student Facing Learning Goals

Let's look at similar triangles.

8.1 A Star and a Pentagon

Warm-up

5 min

Activity Narrative

This *Warm-up* prompts students to make sense of a figure by considering properties of triangles and similar polygons. This figure is also used in the optional activity.

Access for English Language Learners

This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Standards

Building On	**8.G.A.4**
Building Toward	**8.G.A.5**

Instructional Routines

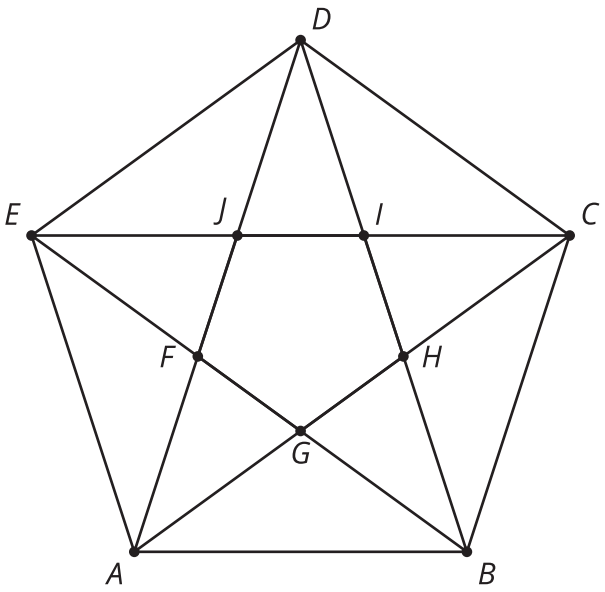
- MLR5: Co-Craft Questions

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2. Introduce the context image from the task. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Student Task Statement



Student Response

Sample responses:

- What is the measure of angle JDI?
- What is the measure of angle IHC?
- What is the measure of angle JFG?
- Which triangles are similar to triangle HIC?
- Which triangles in this picture are similar?
- Which triangles in this picture are congruent?
- How many triangles are in this picture?

Activity Synthesis

Invite several partners to share 1 question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask, “What do these questions have in common? How are they different?”

If questions about similarity or scale factor are not mentioned by students, ask students "What is a mathematical question about similar figures that you could ask using this diagram?"

8.2 Making Pasta Angles and Triangles

30 min

Activity Narrative

In this activity, students create triangles with given angles using dried pasta. When comparing triangles constructed with just one angle measure in common, they notice that the triangles are all different, usually only sharing the one given angle. When comparing triangles constructed with two angle measures in common, they notice that the triangles are all similar (MP7).

This is true because a triangle is determined by the length of one side and the two angles made with that side. The length of that side can then be scaled with a dilation to give any particular similar triangle. This means that triangles with two pairs of congruent corresponding angles must be similar, commonly known as the angle-angle (AA) triangle similarity theorem. Students do not need to know the AA triangle similarity theorem by name at this time, but they will see this idea again in a later activity.

Standards

Building On **7.RP.A.2.a, 8.G.A.4**

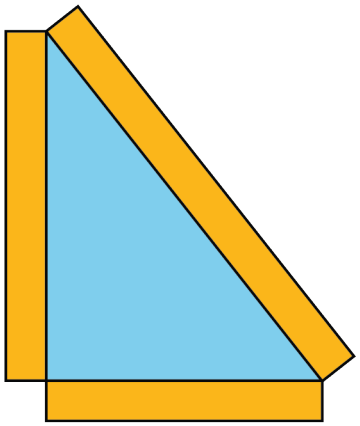
Addressing **8.G.A.5**

Instructional Routines

- MLR2: Collect and Display

Launch

Tell students that they are going to build triangles using dried pasta and a specific set of given angles. Then they will find classmates who used the same angle(s) and compare triangles. Display the image for all to see.



In the image the approximate angle measures are 90°, 50°, and 40°. Demonstrate how students can trace their angle(s) on a sheet of paper before they build their pasta triangle.

Provide students with 1 strip of 3 angles (A, B, and C) pre-cut from the blackline master. Each student should also have access to tape, extra paper to tape their triangles, and a ruler from their geometry toolkits.

Pause students after they have made and compared triangles for the first problem, making sure they find some non-similar triangles. Pause students again after the second problem, making sure they find similar triangles. Consider doing a gallery walk to see how the first sets of triangles differ and how the second sets of triangles are alike. Then have students work on the third problem.

Access for English Language Learners

MLR2 Collect and Display. Direct attention to words collected and displayed from a previous lesson or activity. Invite students to borrow language from the display as needed, and update it throughout the lesson. Circulate, listen for, and collect the language students use as they find others who use the same angles and decide whether their triangles are similar or not. Record words and phrases such as "The triangles are not similar because the angles do not match."
Advances: Conversing, Reading

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Use visible timers or audible alerts to help learners anticipate and prepare to transition between different parts of this activity.
Supports accessibility for: Organization, Attention

Student Task Statement

Your teacher will give you dried pasta, a set of 3 angles labeled A, B, and C, blank paper, and tape.

1. Create a triangle using 3 pieces of pasta and angle A. Your triangle *must* include the angle you were given, but you are otherwise free to make any triangle you like. Tape your pasta triangle to a sheet of paper so it won't move.
   a. After you have created your triangle, measure each side length with a ruler and record the length on the paper next to the side. Then measure the angles to the nearest 5° using a protractor and record these measurements on your paper.
   b. Find 2 others in the room who have the same angle A and compare your triangles. What is the same? What is different?
   c. Are the triangles congruent? Are the triangles similar? Explain your reasoning.
2. Now use more pasta and all 3 angles A, B, and C to create 1 new triangle. Tape this pasta triangle on a separate blank sheet of paper.
   a. After you have created your triangle, measure each side length with a ruler and record the length on the paper next to the side. Then measure the angles to the nearest 5° using a protractor and record these measurements on your paper.
   b. Find 2 others in the room who used your same 3 angles and compare your triangles. What is the same? What is different?
   c. Are the triangles congruent? Are the triangles similar? Explain your reasoning.

Student Response

1. a. No written response required.
   b. Sample response: The triangles only have the 1 angle in common. The side lengths and the other 2 angles are different.
   c. Most triangles will not be congruent or similar. Sample response: The triangles are not congruent or similar because they have different sets of angle measures.
2. a. No written response required.
   b. Sample response: All 3 angles are the same but the side lengths are different.

c. The three-angle triangles are similar. Sample response: Corresponding side lengths are all multiplied by the same scale factor, but the measured side lengths may not be exactly proportional because of possible measurement error.

Building on Student Thinking

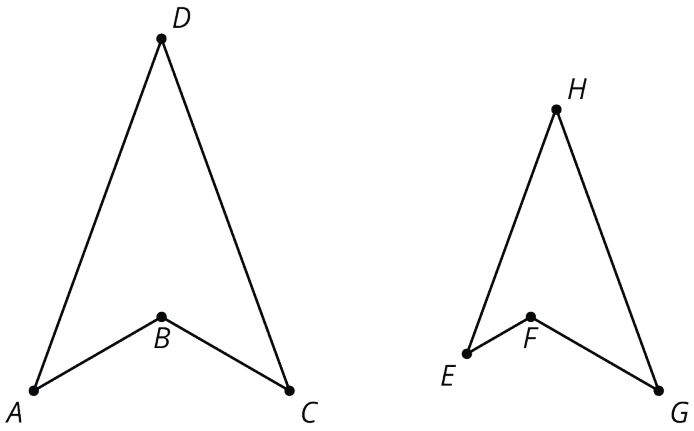
Students may need a reminder that the sum of angles in any triangle is 180 degrees.

Some students may think that their triangle side lengths are not proportional or that their angle measures do not add up to 180 degrees. Remind them that their measurements are only approximate.

Are You Ready for More?

Quadrilaterals $ABCD$ and $EFGH$ have 4 angles with measures of 240°, 40°, 40°, and 40°. Do $ABCD$ and $EFGH$ have to be similar?

Extension Student Response



No. Start with a 240° angle ABC and place D so that BAD and BCD are 40° angles. Angle ADC will also be 40°. This quadrilateral can be made no matter where A and C are placed. The figure on the right shows another example of this construction. By placing E closer to 240° angle EFG, the angles do not change but the result is a non-similar quadrilateral.

Activity Synthesis

The goal of this discussion is to make sure students understand that if triangles share 2 pairs of congruent angles then they are similar. Discuss with students:

- "Did the angles in your triangles always add up to 180 degrees?" (Yes, but because the angles were rounded to the nearest 5 degrees sometimes they weren't exactly 180 degrees.)
- "What did you notice in the first problem when you compared your triangle with other classmates who used the same angle?" (The triangles were all different.)
- "What did you notice in the problem where you had to use 2 angles to make a triangle?" (The triangles all looked the same but were different sizes. They were similar.)
- "Did you need all 3 angle measurements to build a triangle in the second problem?" (No, once we knew 2 angles,

that was enough to build the pasta triangle, the third angle automatically being the right measure.)

- "How did you decide whether or not the sides of your triangle were proportional to other triangles?" (By dividing 2 side lengths of one triangle and comparing that to the quotient of the 2 corresponding sides in the other triangle. Measurement error may mean that quotients computed may not be exactly equal even if the triangles are similar.)
- "How did you check whether or not your triangle was similar to another?" (Lining up 1 angle and then dilating would tell me if the triangles were similar, and observing that the angles were different or the sides were not proportional would tell me they were not similar.)

8.3 Similar Figures in a Regular Pentagon

10 min

Optional

Activity Narrative

This optional activity presents students with a complex figure and asks them to find triangles similar to a given triangle. This activity provides an additional opportunity for students to practice explaining why two triangles are similar (MP3).

Monitor for students who use these methods to show that their chosen triangles are similar to triangle DJI:

- Finding a sequence of translations, rotations, reflections, and dilations
- Checking that two (or three) corresponding angles are congruent

Standards

Addressing **8.G.A**

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Provide access to geometry toolkits.

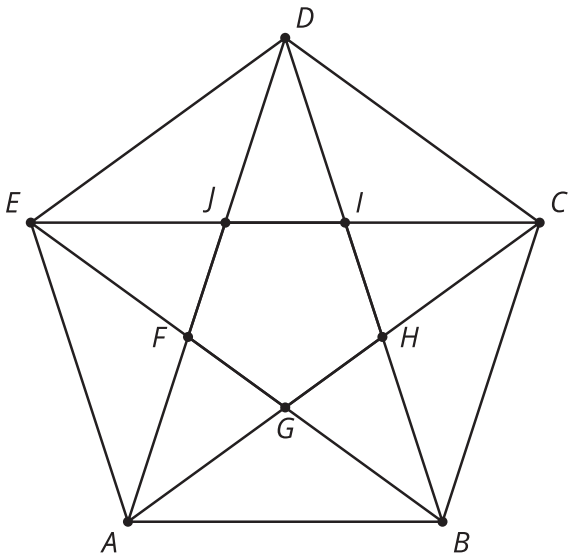
If students created a mathematical question about similarity or scale factors during the *Warm-up*, highlight those questions now. Give students 5 minutes of quiet work time followed by a partner then whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to illustrate which triangles are similar to DJI. Annotate side and angle measurements in the appropriate places on the image.
Supports accessibility for: Visual-Spatial Processing

Student Task Statement

1. This diagram has several triangles that are similar to triangle DJI.



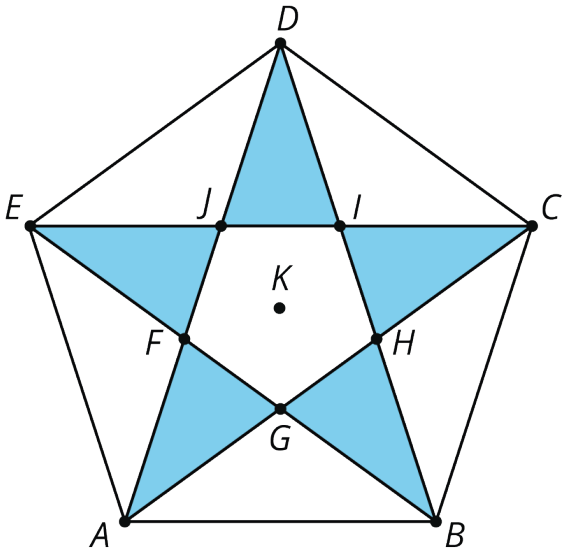
a. Three different scale factors were used to make triangles similar to DJI. In the diagram, find at least 1 triangle of each size that is similar to DJI.

b. Explain how you know each of these 3 triangles is similar to DJI.

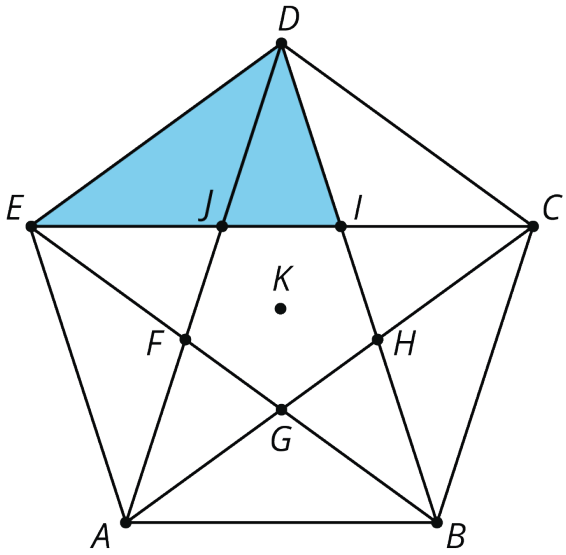
2. Find a triangle in the diagram that is not similar to DJI.

Student Response

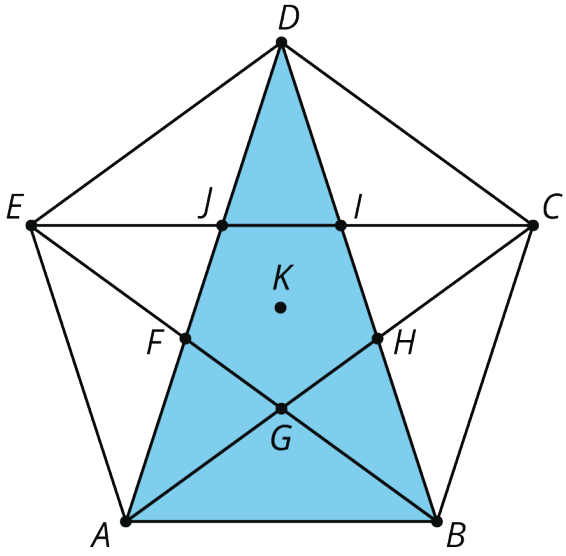
1. Sample response:
   a. Options are (one from each)
      - Congruent to DJI (scale factor 1): DJI, EJF, AFG, BGH, CHI



      - Middle sized (scale factor about 1.5 from DJI): $EID, DJC, EGA, AEJ, BAF, ABH, CGB, BIC, DHC, DEF$



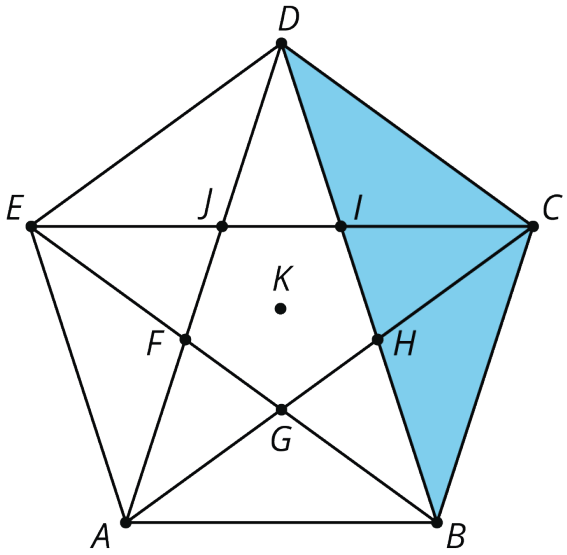
- Large sized (scale factor about 2.5 from DJI): DAB, BCE, ACD, BDE, CEA



b. Sample responses:

- Check two of the angles to be the same as DJI.
- Find transformations that take DJI to the other triangles.
- Measure the angles to be congruent and the side lengths to be proportional.

2. Options are: DEJ, EFA, ABG, BCH, CDI, BDC, CDE, DEA, ABE, ABC, DFB, EGC, DAH, BIE, ACJ.



Building on Student Thinking

If students predict similar triangles just by looking at the triangles, make sure they justify their decisions by including specific measurements.

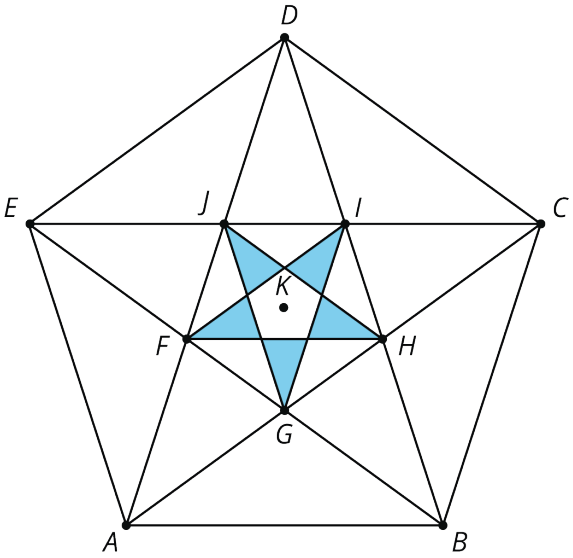
Are You Ready for More?

How can you draw more lines to create additional triangles similar to triangle DJI?

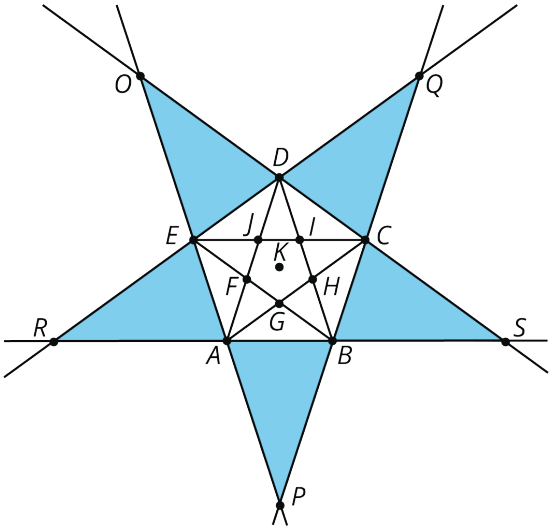
Extension Student Response

Sample responses:

- Draw in the star inscribed in the inner pentagon $FGHIJ$, or just 1 of these segments.



- Extend the sides of pentagon $ABCDE$ until the lines intersect.



Activity Synthesis

The goal of this discussion is to compare different strategies for showing that 2 triangles are similar. Invite students to share some of the triangles they found and explain how they determined the triangles to be similar.

Access for English Language Learners

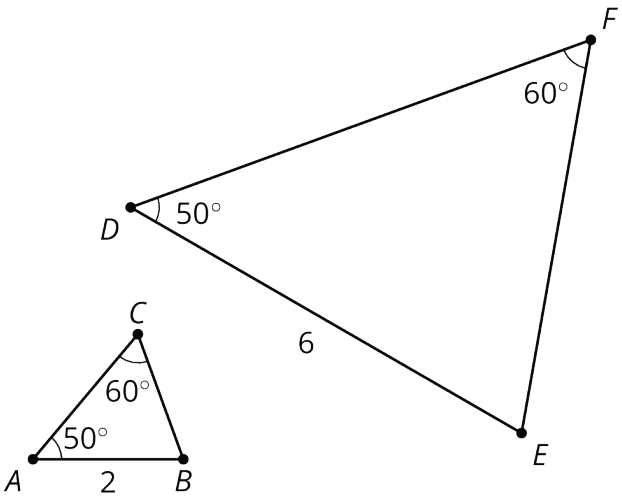
MLR7 Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different representations. Ask, "How are these strategies the same?" "How are they different?" and "Are there any benefits or drawbacks to one strategy compared to another?"
Advances: Representing, Conversing

Lesson Synthesis

The goal of this discussion is for students to understand that when 2 triangles share 2 pairs of congruent angles, they are similar. Begin by asking students what they know about a triangle that has a 50° angle. Discuss with students:

- "Could this triangle be isosceles?" (Yes, it could have two 50° angles and one 80° angle.)
- "Could this triangle *not* be isosceles?" (Yes, it could have one 50° angle, one 20° angle, and one 110° angle.)
- "Are all triangles with a 50° angle similar?" (No, because the other two pairs of angles could be different and the sides might not be proportional.)

Next, ask students what they know about a triangle that has 2 angles that measure 50° and 60°. Then show this image.



Discuss with students:

- "Are these two triangles similar?" (Yes, we can dilate triangle ABC with center A and scale factor 3, and then apply rigid transformations so that angle A matches up with angle D and angle C matches up with angle F. The vertices also match up and so the 2 triangles are similar.)
- "Did we need to know the measure of the third angle in each triangle to determine they were similar?" (No, since the sum of the interior angles of a triangle is always 180°, if two corresponding angles are congruent, then the third angle will also be congruent.)
- "Consider a third triangle that is also similar to triangle ABC. What do we know about this third triangle?" (It has 2 angles that measure 50° and 60°. It is also similar to triangle DEF.)

Add to or create a new class display listing different ways to show that 2 triangles are similar, such as:

- Describe a series of transformations that take one figure to the other.
- Find 2 congruent corresponding angles in triangles.
- Find 3 congruent corresponding angles in triangles.
- If 2 triangles are both similar to the same triangle, then they are similar to each other.

This display should be posted in the classroom for the remaining lessons within this unit.

8.4 Finding Similar Triangles

5 min

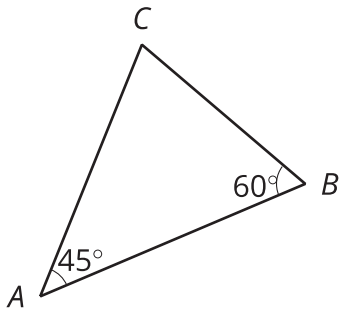
Cool-down

Standards

Addressing **8.G.A**

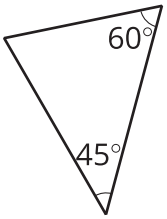
Student Task Statement

Here is triangle ABC.

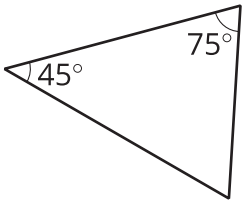


Select **all** triangles that are similar to triangle ABC.

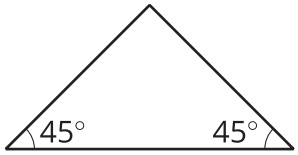
A



B



C



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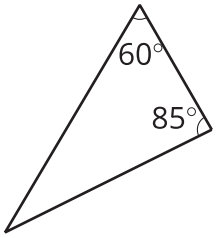
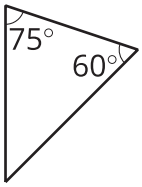


E



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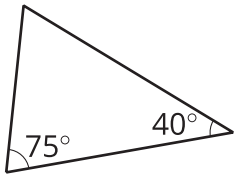


Student Response

A, B, E

Responding to Student Thinking

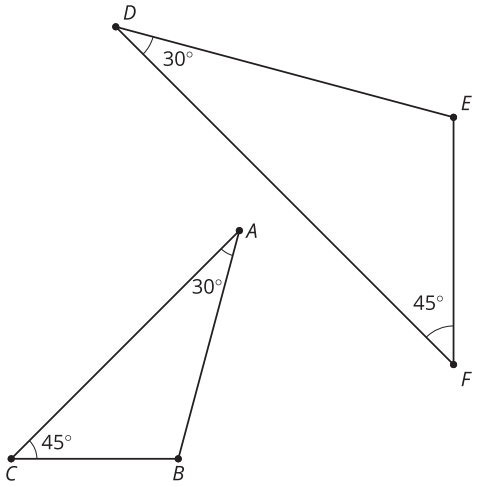
Points to Emphasize
If students struggle with identifying similar triangles, as opportunities arise over the next several lessons, focus on showing two triangles are similar using their angles. For example, in the activity referred to here, emphasize that in addition to finding a series of transformations to show triangle similarity, two (or three) pairs of congruent angles can also show that two triangles are similar.

Grade 8, Unit 2, Lesson 10, Activity 2 Similar Triangles on the Same Line

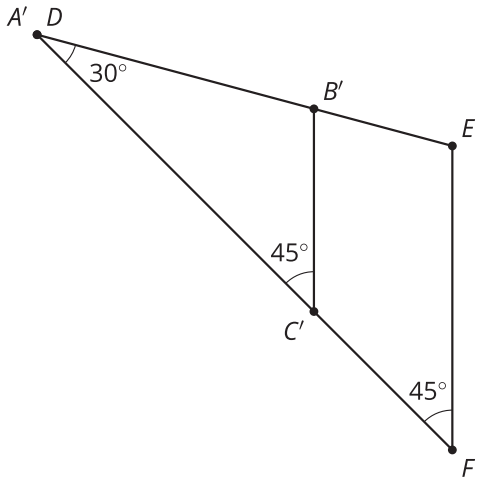
Lesson 8 Summary

Two polygons are similar when there is a sequence of translations, rotations, reflections, and dilations taking one polygon to the other. When the polygons are triangles, we only need to check that both triangles have two corresponding angles to show they are similar.

For example, triangle ABC and triangle DEF both have a 30-degree angle and a 45-degree angle.



We can translate A to D and then rotate around point D so that the two 30-degree angles are aligned, giving this picture:



Then a dilation with center D and appropriate scale factor will move C' to F. This dilation also moves B' to E, showing that triangles ABC and DEF are similar.

Lesson 8 Practice Problems

1 Student Task Statement

For each pair of triangles, some of the angle measures are given in degrees. Use the information to decide if the triangles are similar or not. Explain how you know.

- Triangle A: 53, 71, _____; Triangle B: 53, 71, _____
- Triangle C: 90, 37, _____; Triangle D: 90, 53, _____
- Triangle E: 63, 45, _____; Triangle F: 14, 71, _____
- Triangle G: 121, _____, _____; Triangle H: 70, _____, _____

Solution

- Similar. Sample reasoning: They have 2 pairs of angles with equal measurement.
- Similar. Sample reasoning: Since the angles in a triangle add up to 180°, the missing angle in Triangle C must be 53°. The 2 triangles therefore have 2 pairs of angles with equal measurement.
- Not similar. Sample reasoning: Similar triangles have equal angle measurements, and these 2 triangles do not have any angle measures in common.
- Not similar. Sample reasoning: Similar triangles have equal angle measurements, but no triangle can have angles which measure 121 and 70 degrees, as these add up to more than 180°.

2 Student Task Statement

a. Draw 2 equilateral triangles that are not congruent.

b. Measure the side lengths and angles of your triangles. Are the 2 triangles similar?

c. Do you think 2 equilateral triangles will be similar *always*, *sometimes*, or *never*? Explain your reasoning.

Solution

a. Answers vary

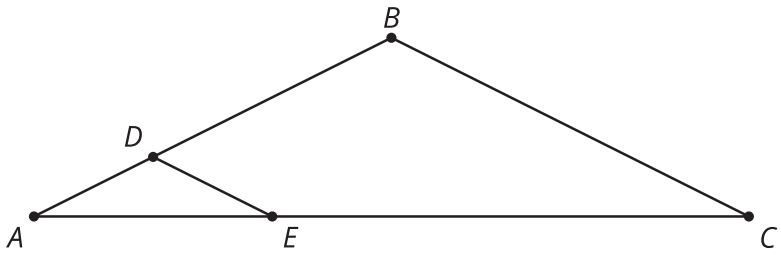
b. Yes

c. Always. Sample reasoning: All equilateral triangles have the same angle measures, so they are all similar.

3 Student Task Statement

In the figure, line segment BC is parallel to line segment DE.

Explain why triangle ABC is similar to triangle ADE.



Solution

Sample responses:

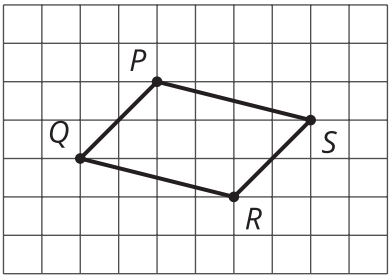
- Triangles ABC and ADE share angle A. Line segment AC is a transversal for parallel lines segments BC and DE. Therefore, angles ADE and ABC are congruent. Since they share 2 congruent angles, triangles ABC and ADE are similar.
- A dilation with center A and a scale factor of about $\frac{1}{3}$ will take triangle ABC to triangle ADE.

4 from Unit 2, Lesson 4

Student Task Statement

The quadrilateral $PQRS$ in the diagram is a parallelogram.

Let $P'Q'R'S'$ be the image of $PQRS$ after applying a dilation centered at a point O (not shown) with scale factor 3.



Which of the following is true?

A. $P'Q' = PQ$

B. $P'Q' = 3PQ$

C. $PQ = 3P'Q'$

D. $P'Q' = \frac{1}{3}PQ$

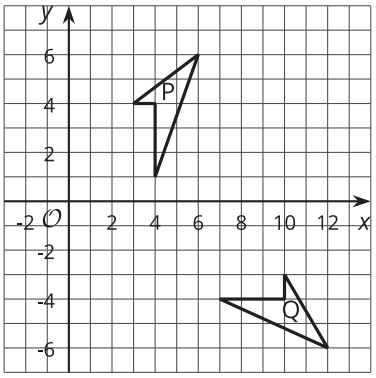
Solution

B

5 from Unit 1, Lesson 6

Student Task Statement

Describe a sequence of transformations for which Quadrilateral P is the image of Quadrilateral Q.



Solution

Sample response: Translate Quadrilateral Q 3 units left and 5 units up. Now they share a point. Rotate using this point as the center, 90 degrees counterclockwise.