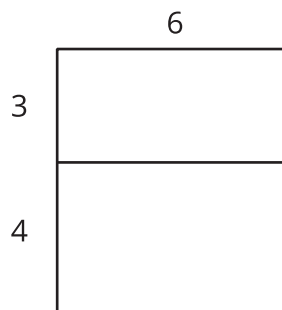




# Equivalent Quadratic Expressions

Let's use diagrams to help us rewrite quadratic expressions.

## 8.1 Diagrams of Products



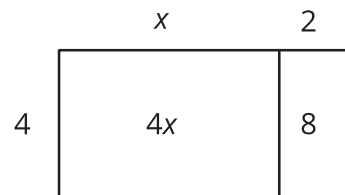
1. Explain why the diagram shows that  $6(3 + 4) = 6 \cdot 3 + 6 \cdot 4$ .

2. Draw a diagram to show that  $5(x + 2) = 5x + 10$ .

## 8.2

## Drawing Diagrams to Represent More Products

Applying the distributive property to multiply out the factors of, or expand,  $4(x + 2)$  gives us  $4x + 8$ , so we know the two expressions are equivalent. We can use a rectangle with side lengths of  $(x + 2)$  and 4 to illustrate the multiplication.

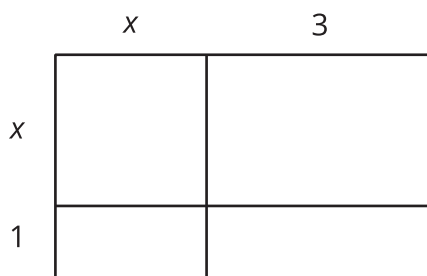


1. Draw a diagram to show that  $n(2n + 5)$  and  $2n^2 + 5n$  are equivalent expressions.
2. For each expression, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.
  - a.  $6\left(\frac{1}{3}n + 2\right)$
  - b.  $p(4p + 9)$
  - c.  $5r\left(r + \frac{3}{5}\right)$
  - d.  $(0.5w + 7)w$

## 8.3

## Using Diagrams to Find Equivalent Quadratic Expressions

1. Here is a diagram of a rectangle with side lengths of  $x + 1$  and  $x + 3$ . Use this diagram to show that  $(x + 1)(x + 3)$  and  $x^2 + 4x + 3$  are equivalent expressions.

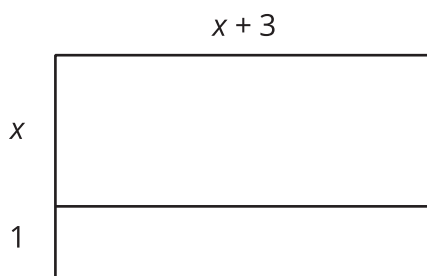


2. Draw diagrams to help you write an equivalent expression for each of the following:

a.  $(2x + 1)(x + 3)$

b.  $(x + 5)^2$

3. Here is a diagram of a rectangle with the same area as in the first question. Use this diagram to show that  $(x + 1)(x + 3)$  and  $x(x + 3) + 1(x + 3)$  are equivalent expressions. Then explain how you could rewrite that expression as  $x^2 + 4x + 3$ , without a diagram.



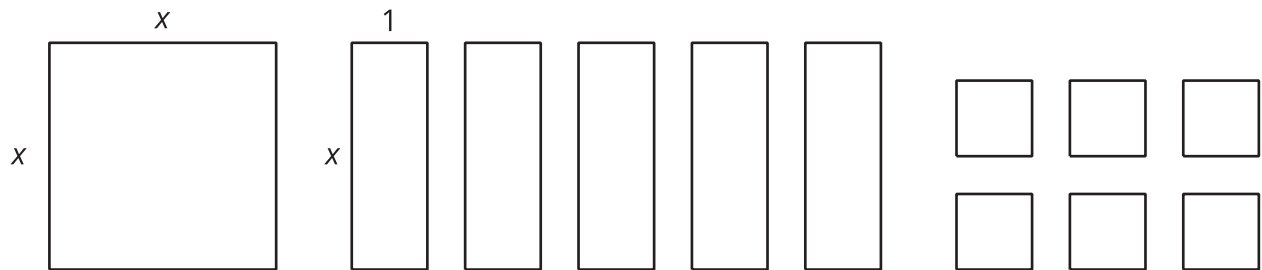
4. Write an equivalent expression for each expression:

a.  $(x + 2)(x + 6)$

b.  $(x + m)(x + n)$



### Are you ready for more?

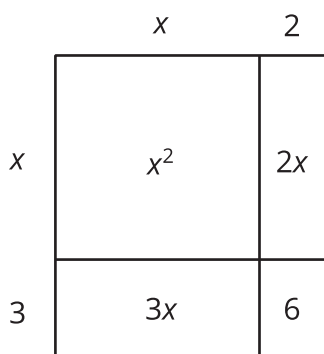


1. Is it possible to arrange an  $x$ -by- $x$  square, five  $x$ -by-1 rectangles and six 1-by-1 squares into a single large rectangle? Explain or show your reasoning.
2. What does this tell you about an equivalent expression for  $x^2 + 5x + 6$ ?
3. Keeping the  $x$ -by- $x$  square and the five  $x$ -by-1 rectangles, can you form a different rectangle by using a different number of 1-by-1 squares than what is shown?

## Lesson 8 Summary

A quadratic function can often be defined by many different but equivalent expressions. For example, we saw earlier that the predicted revenue, in thousands of dollars, from selling a downloadable movie at  $x$  dollars can be expressed with  $x(18 - x)$ , which can also be written as  $18x - x^2$ .

Sometimes a quadratic expression is a product of two factors that are each a linear expression, for example  $(x + 2)(x + 3)$ . We can write an equivalent expression by thinking about each factor, the  $(x + 2)$  and  $(x + 3)$ , as the side lengths of a rectangle, with each side length being decomposed into a variable expression and a number.



Multiplying  $(x + 2)$  and  $(x + 3)$  gives the area of the rectangle. Adding the areas of the four sub-rectangles also gives the area of the rectangle. This means that  $(x + 2)(x + 3)$  is equivalent to  $x^2 + 2x + 3x + 6$ , or to  $x^2 + 5x + 6$ .

Notice that the diagram illustrates the distributive property being applied. Each term of one factor (say, the  $x$  and the  $2$  in  $x + 2$ ) is multiplied by every term in the other factor (the  $x$  and the  $3$  in  $x + 3$ ).

$$\begin{aligned} & (x + 2)(x + 3) \\ &= x(x + 3) + 2(x + 3) \\ &= x^2 + 3x + 2x + (2)(3) \\ &= x^2 + (3 + 2)x + (2)(3) \end{aligned}$$

In general, when a quadratic expression is written in the form of  $(x + p)(x + q)$ , we can apply the distributive property to rewrite it as  $x^2 + px + qx + pq$ , or as  $x^2 + (p + q)x + pq$ .