Unit 2, Lesson 13

# Polynomial Division (Part 2)

Let's learn a different way to divide polynomials.

## 13.1 Notice and Wonder: Different Divisions

What do you notice? What do you wonder?

$$\begin{array}{r|l} & 2 \\ 11 & 2\ 7\ 7\ 2 \\ & \underline{2\ 2} \\ & 5 \end{array}$$

$$\begin{array}{r|l} & 2\ 5 \\ 11 & 2\ 7\ 7\ 2 \\ & \underline{2\ 2} \\ & 5\ 7 \\ & \underline{5\ 5} \\ & 2 \end{array}$$

$$\begin{array}{r|l} & 2\ 5\ 2 \\ 11 & 2\ 7\ 7\ 2 \\ & \underline{2\ 2} \\ & 5\ 7 \\ & \underline{5\ 5} \\ & 2\ 2 \\ & \underline{2\ 2} \\ & 0 \end{array}$$

$$\begin{array}{r} 2x^2 \\ x+1\overline{)2x^3+7x^2+7x+2} \\ \underline{-2x^3-2x^2} \\ 5x^2+7x \end{array}$$

## 13.2 Polynomial Long Division

1. Diego used the long division shown here to figure out that $6x^2 - 7x - 5 = (2x+1)(3x-5)$. Show what it would look like if he had used a diagram.

$$\begin{array}{r} 3x-5 \\ 2x+1\overline{)6x^2-7x-5} \\ \underline{-6x^2-3x} \\ -10x-5 \\ \underline{10x+5} \\ 0 \end{array}$$

| | | |
|---|---|---|
| $2x$ | $6x^2$ | |
| 1 | | |

Pause here for a whole-class discussion.

2. $(x-2)$ is a factor of $2x^3-7x^2+x+10$, which means there is some other factor $A$ where $2x^3-7x^2+x+10=(x-2)(A)$. Finish the division started here to find the value of $A$.

$$x-2\overline{)2x^3-7x^2+x+10}$$
$$\underline{-2x^3+4x^2}$$
$$2x^2$$

3. Jada used the diagram shown here to figure out that $2x^3+13x^2+16x+5=(2x+1)(x^2+6x+5)$. Show what it would look like if she had used long division.

| | $x^2$ | $6x$ | 5 |
|---|---|---|---|
| $2x$ | $2x^3$ | $12x^2$ | $10x$ |
| 1 | $x^2$ | $6x$ | 5 |

$$2x+1\overline{)2x^3+13x^2+16x+5}$$

## Are you ready for more?

1. What is $(x^4-1)\div(x-1)$?

2. Use your response to predict what $(x^7 - 1) \div (x - 1)$ is, and then use division to check your prediction.

## 13.3 More Long Division

Here are some polynomial functions with known factors. Rewrite each polynomial as a product of linear factors using long division.

1. $A(x) = x^3 - 7x^2 - 16x + 112, (x - 7)$

$$
\begin{array}{r}
x^2 \\
x - 7 \overline{)\, x^3 - 7x^2 - 16x + 112} \\
\underline{-x^3 + 7x^2} \\
\end{array}
$$

2. $C(x) = x^3 - 3x^2 - 13x + 15, (x + 3)$

# 13.4 Missing Numbers

Here are pairs of equivalent expressions, one in standard form and the other in factored form. Find the missing numbers.

1. $x^2 + 9x + 14$ and $(x + 2)(x + \square)$

2. $x^2 - 9x + 20$ and $(x - \square)(x - \square)$

3. $2x^2 + 2x - 24$ and $2(x + \square)(x - 3)$

4. $\square x^3 + 11x^2 - 17x + 6$ and $(-x + 3)(2x - 1)(x - 2)$

5. $6x^3 + 2x^2 - 16x + 8$ and $(x - 1)(2x + 4)(\square x - 2)$

6. $2x^3 + 7x^2 - 7x - 12$ and $(2x - 3)(x + \square)(x + \square)$

7. $x^3 + 6x^2 + \square x - 10$ and $(x + 2)(x - 1)(x + \square)$

## Lesson 13 Summary

In earlier grades, we learned that one way to divide numbers, like 1,573 divided by 11, is by using long division.

$$\begin{array}{r} 1 \\ 11\overline{)\,1573} \\ \underline{11} \\ 4 \end{array} \qquad \begin{array}{r} 14 \\ 11\overline{)\,1573} \\ \underline{11} \\ 47 \\ \underline{44} \\ 3 \end{array} \qquad \begin{array}{r} 143 \\ 11\overline{)\,1573} \\ \underline{11} \\ 47 \\ \underline{44} \\ 33 \\ \underline{33} \\ 0 \end{array}$$

Here the division has been completed in stages, focusing on the highest power of 10 (1,000) in the dividend 1,573, and working down. This long division shows that $1{,}573 = (11)(143)$.

Similar to integers, we can also use long division on polynomials. Instead of focusing on powers of 10, in polynomial long division we focus on powers of $x$. Just as we started with the highest power or 10, we start with the highest power of $x$, the leading term, and work down to the constant term. For example, here is $x^3 + 5x^2 + 7x + 3$ divided by $x + 1$ completed in three stages. Notice how terms of the same degree are in the same columns.

$$\begin{array}{r} x^2 \\ x+1\overline{)\,x^3+5x^2+7x+3} \\ \underline{-x^3-x^2} \\ 4x^2+7x \end{array} \qquad \begin{array}{r} x^2+4x \\ x+1\overline{)\,x^3+5x^2+7x+3} \\ \underline{-x^3-x^2} \\ 4x^2+7x \\ \underline{-4x^2-4x} \end{array} \qquad \begin{array}{r} x^2+4x+3 \\ x+1\overline{)\,x^3+5x^2+7x+3} \\ \underline{-x^3-x^2} \\ 4x^2+7x \\ \underline{-4x^2-4x} \\ 3x+3 \end{array}$$

At each stage, the focus is only on the term with the largest exponent that's left. At the conclusion, we can see that $x^3 + 5x^2 + 7x + 3 = (x + 1)(x^2 + 4x + 3)$.