Multiplying Complex Numbers

Let's multiply complex numbers.

13.1 i Squared

Write each expression in the form $a + bi$, where a and b are real numbers.

1. $4i \cdot 3i$

2. $4i \cdot -3i$

3. $-2i \cdot -5i$

4. $-5i \cdot 5i$

5. $(-5i)^2$

13.2 Multiplying Imaginary Numbers

Take turns with your partner matching an expression in column A with an equivalent expression in column B.

- For each match that you find, explain to your partner how you know it's a match.
- For each match that your partner finds, listen carefully to the explanation. If you disagree, discuss your thinking and work to reach an agreement.

A	B
$5 \cdot 7i$	-9
$5i \cdot 7i$	$35i$
$3i^2$	-35
$(3i)^2$	1
$8i^3$	9
i^4	-3
$-i^2$	-1
$(-i)^2$	$-8i$

13.3 Multiplying Complex Numbers

Write each product in the form $a + bi$, where a and b are real numbers.

1. $(8 + i)(-5 + 3i)$

2. $(-3 + 9i)(5i)$

3. $(3 + 2i)^2$

4. $(3 + 2i)(3 - 2i)$

Are you ready for more?

On October 16, 1843, while walking across the Broom Bridge in Dublin, Ireland, Sir William Rowan Hamilton came up with an idea for numbers that would work sort of like complex numbers. Instead of just the number i (and its opposite $-i$) squaring to give -1, he imagined three numbers i, j, and k (each with an opposite) that squared to give -1.

The way these numbers multiplied with each other was very interesting. i times j would give k, j times k would give i, and k times i would give j. But the multiplication he imagined did not have a commutative property. When those numbers were multiplied in the opposite order, they'd give the opposite number. So j times i would give $-k$, k times j would give $-i$, and i times k would give $-j$. A *quaternion* is a number that can be written in the form $a + bi + cj + dk$ where a, b, c, and d are real numbers.

Let $w = 2 + 3i - j$ and $z = 2i + 3k$. Write each given expression in the form $a + bi + cj + dk$.

1. $w + z$

2. wz

3. zw

Lesson 13 Summary

One way to multiply two complex numbers is to use the distributive property.

$$(2 + 3i)(4 + 5i) = 8 + 10i + 12i + 15i^2$$

Remember that $i^2 = -1$, so:

$$(2 + 3i)(4 + 5i) = 8 + 10i + 12i - 15$$

When we add the real parts together and the imaginary parts together, we get:

$$(2 + 3i)(4 + 5i) = -7 + 22i$$