

# Rational and Irrational Numbers

Let's learn about irrational numbers.

## 4.1 Math Talk: Positive Solutions

Solve each equation mentally.

- $x^2 = 36$

- $x^2 = \frac{9}{4}$

- $x^2 = \frac{1}{4}$

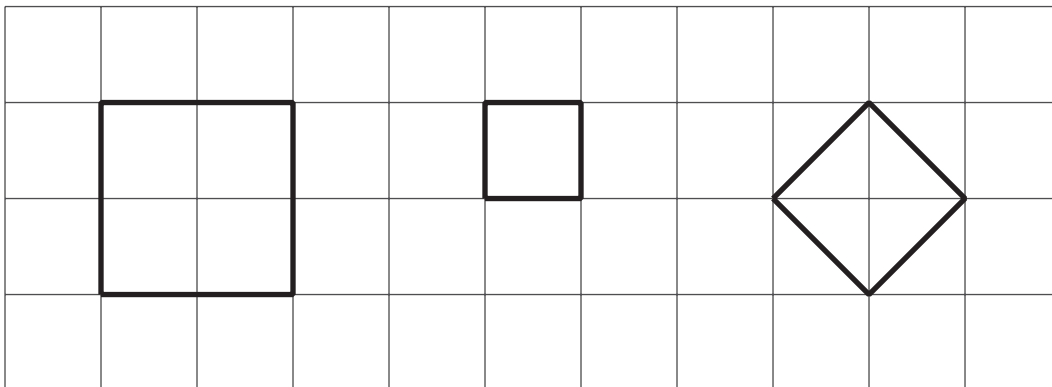
- $x^2 = \frac{49}{25}$



## 4.2 Three Squares

For each square:

1. Label the area.
2. Label the side length.
3. Write an equation that shows the relationship between the side length and the area.



## 4.3 Looking for a Solution

Are any of these numbers a solution to the equation  $x^2 = 2$ ? Explain your reasoning.

- 1
- $\frac{1}{2}$
- $\frac{3}{2}$
- $\frac{7}{5}$

## 4.4

Looking for  $\sqrt{2}$ 

A **rational number** is a number that can be expressed as a positive or negative fraction.

1. Find some more rational numbers that are close to  $\sqrt{2}$ .

2. Can you find a rational number that is exactly  $\sqrt{2}$ ?



## Are you ready for more?

If you have an older calculator and evaluate the expression  $\left(\frac{577}{408}\right)^2$ , it will tell you that the answer is 2, which might lead you to think that  $\sqrt{2} = \frac{577}{408}$ .

1. Explain why you might be suspicious of the calculator's result.

2. Find an explanation for why  $408^2 \cdot 2$  could not possibly equal  $577^2$ . How does this show that  $\left(\frac{577}{408}\right)^2$  could not equal 2?

3. Repeat these questions for  $\left(\frac{1414213562375}{10000000000000}\right)^2 \neq 2$ , an equation that even many modern calculators and computers will get wrong.



## Lesson 4 Summary

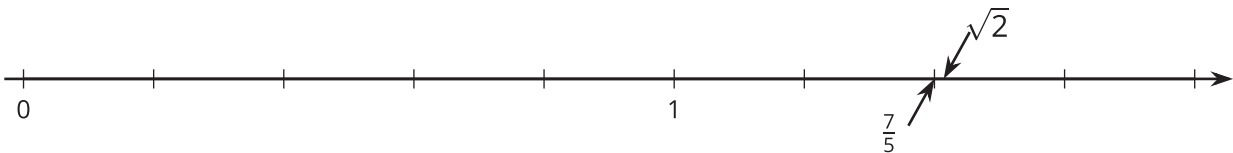
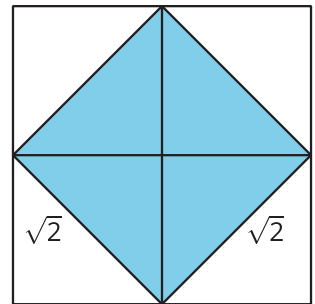
A square whose area is 25 square units has a side length of  $\sqrt{25}$  units, which means that  $\sqrt{25} \cdot \sqrt{25} = 25$ . Since  $5 \cdot 5 = 25$ , we know that  $\sqrt{25} = 5$ .

$\sqrt{25}$  is an example of a rational number. A **rational number** is a fraction or its opposite. In an earlier grade we learned that  $\frac{a}{b}$  is a point on the number line found by dividing the interval from 0 to 1 into  $b$  equal parts and finding the point that is  $a$  of them to the right of 0. We can always write a fraction in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are integers (and  $b$  is not 0), but there are other ways to write them. For example, we can write  $\sqrt{25} = \frac{5}{1} = 5$  or  $-\frac{1}{\sqrt{4}} = -\frac{1}{2}$ . Because fractions and *ratios* are closely related ideas, fractions and their opposites are called *rational* numbers.

Here are some examples of rational numbers:  $\frac{7}{4}$ , 0,  $\frac{6}{3}$ , 0.2,  $-\frac{1}{3}$ , -5,  $\sqrt{9}$ ,  $-\frac{\sqrt{16}}{\sqrt{100}}$

Now consider a square whose area is 2 square units with a side length of  $\sqrt{2}$  units. This means that  $\sqrt{2} \cdot \sqrt{2} = 2$ .

An **irrational number** is a number that is not rational, meaning it cannot be expressed as a positive or negative fraction. For example,  $\sqrt{2}$  has a location on the number line (it's a tiny bit to the right of  $\frac{7}{5}$ ), but its location can not be found by dividing the segment from 0 to 1 into  $b$  equal parts and going  $a$  of those parts away from 0.



$\frac{17}{12}$  is close to  $\sqrt{2}$  because  $(\frac{17}{12})^2 = \frac{289}{144}$ , which is very close to 2 since  $\frac{288}{144} = 2$ . We could keep looking forever for rational numbers that are solutions to  $x^2 = 2$ , and we would not find any since  $\sqrt{2}$  is an irrational number.

The square root of any whole number is either a whole number, like  $\sqrt{36} = 6$  or  $\sqrt{64} = 8$ , or an irrational number. Here are some examples of irrational numbers:  $\sqrt{10}$ ,  $-\sqrt{3}$ ,  $\frac{\sqrt{5}}{2}$ ,  $\pi$ .