Scale Factors and Making Scaled Copies

Goals

- Comprehend the phrase “scale factor” and explain (orally) how it relates corresponding lengths of a figure and its scaled copy.
- Critique (orally and in writing) different strategies (expressed in words and through other representations) for creating scaled copies of a figure.
- Draw a scaled copy of a given figure using a given scale factor.
- Generalize (orally and in writing) that the relationship between the side lengths of a figure and its scaled copy is multiplicative, not additive.

Learning Targets

- I can describe what the scale factor has to do with a figure and its scaled copy.
- I can draw a scaled copy of a figure using a given scale factor.
- I know what operation to use on the side lengths of a figure to produce a scaled copy.

Lesson Narrative

This lesson develops the vocabulary for talking about scaling and scaled copies more precisely and for identifying the structures in common between two figures. Students also begin to describe the numerical relationship between the corresponding lengths in two figures using a **scale factor**. They see that when two figures are scaled copies of one another, the same scale factor relates their corresponding lengths. As students identify the scale factor that relates corresponding sides, they are making use of repeated reasoning (MP8).

Students strengthen their understanding that the relationship between scaled copies is multiplicative, not additive. Students make careful arguments about the scaling process (MP3) and have opportunities to use tools like tracing paper or index cards strategically (MP5).

Standards

Building On	**5.NBT.B.7, 5.NF.B.4**
Addressing	**7.G.A.1**
Building Toward	**7.RP.A.2**

Instructional Routines

- 5 Practices
- Math Talk
- MLR2: Collect and Display
- MLR8: Discussion Supports

Student Facing Learning Goals

Let's draw scaled copies.

2.1 Math Talk: Multiplying by a Unit Fraction

5 min

Warm-up

Activity Narrative

This is the first *Math Talk* activity in the course. See the launch for extended instructions for facilitating this activity successfully.

This *Math Talk* focuses on multiplying a whole number by a unit fraction. It encourages students to think about the relationship between multiplication and division and to rely on properties of operations to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students are identifying scale factors.

Standards

Building On **5.NBT.B.7, 5.NF.B.4**

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Launch

This is the first time students do the *Math Talk* instructional routine in this course, so it is important to explain how it works before starting.

Explain that a *Math Talk* has four problems, revealed one at a time. For each problem, students have a minute to quietly think and are to give a signal when they have an answer and a strategy. The teacher then selects students to share different strategies (likely 2–3, given limited time), and might ask questions such as "Who thought about it in a different way?" The teacher then records the responses for all to see, and might ask clarifying questions about the strategies before revealing the next problem.

Consider establishing a small, discreet hand signal that students can display when they have an answer they can support with reasoning. This signal could be a thumbs-up, a certain number of fingers that tells the number of responses they have, or another subtle signal. This is a quick way to see if the students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class.
Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support working memory, provide students with access to sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Task Statement

Find the value of each expression mentally.

- $\frac{1}{4} \cdot 20$
- $44 \cdot \frac{1}{4}$
- $\frac{1}{3} \cdot 63$
- $90 \cdot \frac{1}{6}$

Student Response

- 5. Sample reasoning: $20 \div 4 = 5$
- 11. Sample reasoning: $44 \div 4 = 11$
- 21. Sample reasoning: $60 \div 3 = 20$, $3 \div 3 = 1$, and $20 + 1 = 21$.
- 15. Sample reasoning: $90 \div 3 = 30$ and $30 \div 2 = 15$.

Activity Synthesis

Make sure the connection to division is brought up in the discussion, before moving on to the second expression.

To involve more students in the conversation, consider asking:

- "Who can restate ________'s reasoning in a different way?"
- "Did anyone use the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ________'s strategy?"
- "Do you agree or disagree? Why?"
- "What connections to previous problems do you see?"

The key takeaway is that these problems all involve multiplying by a unit fraction. One strategy that works in such cases is dividing the other factor by the denominator of the fraction.

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

2.2 Scaled Triangles

15 min

Activity Narrative

In this activity, students continue to practice identifying corresponding parts of scaled copies. By organizing corresponding lengths in a table, students see that there is a single factor that relates each length in the original triangle to its corresponding length in a copy (MP8). They learn that this number is called a **scale factor**.

As students work on the first question, listen to how they reason about which triangles are scaled copies. Identify groups who use side lengths and angles as the basis for deciding. (Students are not expected to reason formally yet, but should begin to look to lengths and angles for clues.)

As students identify corresponding sides and their measures in the second and third questions, look out for confusion about corresponding parts. Notice how students decide which sides of the right triangles correspond.

If students still have access to tracing paper, monitor for students who use this tool strategically (MP5).

This is the first time Math Language Routine 2: *Collect and Display* is suggested in this course. In this routine, the teacher circulates and listens to student talk while jotting down words, phrases, drawings, or writing students that use. The language collected is displayed visually for the whole class to use throughout the lesson and unit. The purpose of this routine is to capture a variety of students' words and phrases—including, especially, everyday or social language and non-English—in a display that students can refer to, build on, or make connections with during future discussions, and to increase students' awareness of language used in mathematics conversations.

Access for English Language Learners

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Teacher Notes for IM 6–8 Math Accelerated v.360
The *Activity Narrative* states that this is the first time Math Language Routine 2: *Collect and Display* is suggested in this course. In IM 6–8 Math Accelerated v.360, the *Collect and Display* routine was introduced in a previous activity.

Due to students' prior work with percent increase and decrease, they may want to describe the scale factor as a percent change. For example, the side lengths of Triangle D are 50% the side lengths of Triangle O. Make sure these students understand that a 50% decrease in length corresponds to a scale factor of $\frac{1}{2}$.

Standards

Addressing	**7.G.A.1**
Building Toward	**7.RP.A.2**

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students into groups of 4. Assign each student one of the following pairs of triangles to examine for the first question.

- A and E
- B and F
- C and G
- D and H

Give students 2 minutes of quiet think time to determine if their assigned triangles are scaled copies of the original triangle. Give students another 2–3 minutes to discuss their responses and complete the first question in groups.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language students use to explain how they know whether each triangle is a scaled copy of the original. Display words and phrases such as: "corresponding," "straight," "slanted," "vertical," "right angle," "stretched," "doubled," "halved," "multiplied," "divided," "same amount," "same factor," etc.

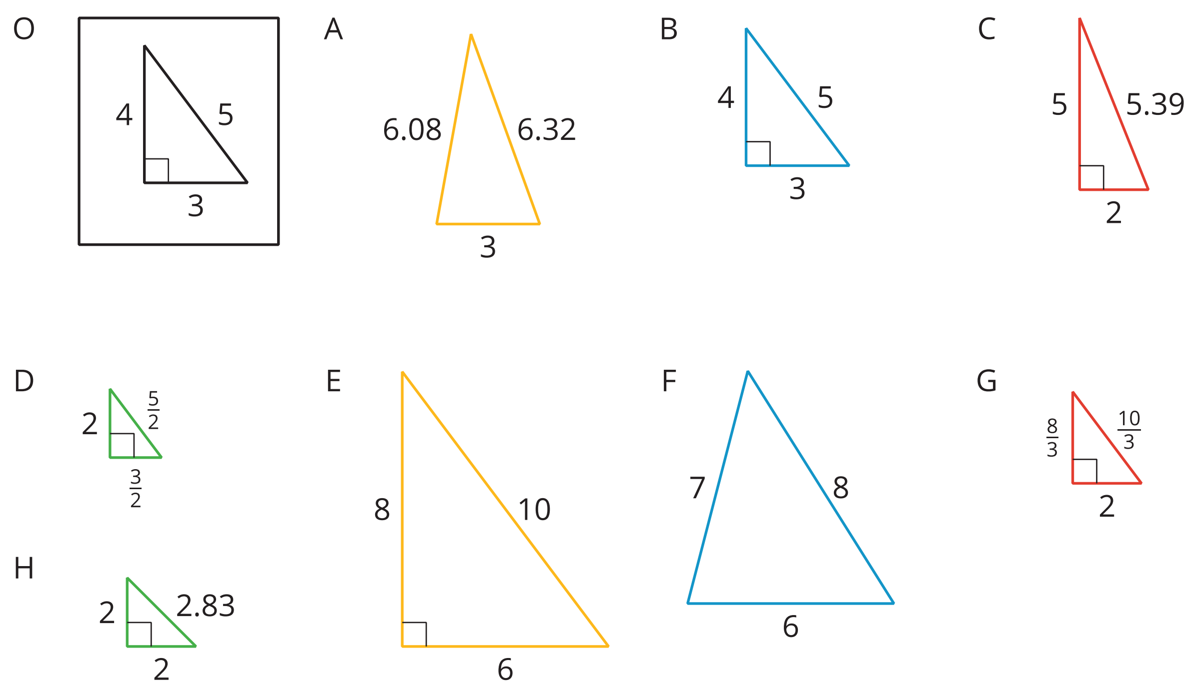
Discuss briefly as a class which triangles are scaled copies and select a couple of groups who reasoned in terms of lengths and angles to explain their reasoning. Some guiding questions:

- "What information did you use to tell scaled copies from those that are not?"
- "How were you able to tell right away that some figures are not scaled copies?"

After the class recognizes that A, C, F, and H are not scaled copies, give students quiet work time to complete the rest of the task.

Student Task Statement

Here is Triangle O, followed by a number of other triangles.



Your teacher will assign you two of the triangles to look at.

1. For each of your assigned triangles, is it a scaled copy of Triangle O? Be prepared to explain your reasoning.
2. As a group, identify *all* the scaled copies of Triangle O in the collection. Discuss your thinking. If you disagree, work to reach an agreement.
3. List all the triangles that are scaled copies in the table. Record the side lengths that correspond to the side lengths of Triangle O listed in each column.

Triangle O	**3**	**4**	**5**

4. Explain or show how each copy has been scaled from the original (Triangle O).

Student Response

1. Response depends on the pair of triangles students have. Triangles B, D, E, and G are scaled copies.
2. Triangles B, D, E, and G are scaled copies. Sample reasoning: B, D, E, and G have not changed in shape (they are still right triangles). Each of their sides is the same number of times as long as the corresponding side in the original triangle. Triangles A and F do not have the same shape as Triangle O (their angles are different), so they are not scaled copies. Triangles C and H are right triangles but their sides are not the same number of times as long as the corresponding sides in the original triangle.
3.

Triangle O	3	4	5
Triangle B	3	4	5
Triangle D	$\frac{3}{2}$	2	$\frac{5}{2}$
Triangle E	6	8	10
Triangle G	2	$\frac{8}{3}$	$\frac{10}{3}$

4. Sample responses:
   - Triangle B is a same-size copy of the original. All the lengths stay the same.
   - In Triangle D, all the lengths are half of the original ones.
   - In Triangle E, all sides double in length.
   - In Triangle G, the lengths are $\frac{2}{3}$ times the corresponding lengths in the original triangle.

Building on Student Thinking

Students may think that Triangle F is a scaled copy because just like the 3-4-5 triangle, the sides are also three consecutive whole numbers. Point out that corresponding angles are not equal.

Are You Ready for More?

Choose one of the triangles that is not a scaled copy of Triangle O. Describe how you could change at least one side to make a scaled copy, while leaving at least one side unchanged.

Extension Student Response

Sample response: On Triangle F, the side of length 7 could be extended to have length 10.

Activity Synthesis

Display the image of all triangles and invite a couple of students to share how they knew which sides of the triangles correspond. Then, display a completed table in the third question for all to see.

Direct students' attention to the reference created using *Collect and Display*. Ask each group to present its observations about one triangle and how the triangle has been scaled from the original. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond. As students present, record or illustrate their reasoning on the table, such as by drawing arrows between rows and annotating with the operation that students are describing, as shown here.

Triangle O		3	4	5
Triangle D	$\cdot\frac{1}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$
Triangle E	$\cdot 2$	6	8	10
Triangle B	$\cdot 1$	3	4	5
Triangle G	$\cdot\frac{2}{3}$	2	$\frac{8}{3}$	$\frac{10}{3}$

Use the language that students use to describe the side lengths and the numerical relationships in the table to guide students toward **scale factor**. For example: "You explained that the lengths in Triangle F are all twice those in the original triangle, so we can write those as '2 times' the original numbers. Lengths in Triangle A are half of those in the original; we can write '$\frac{1}{2}$times' the original numbers. We call those multipliers—the 2 and the $\frac{1}{2}$—scale factors. We say that scaling Triangle O by a scale factor of 2 produces Triangle F, and that scaling Triangle O by a scale factor of $\frac{1}{2}$ produces Triangle A."

2.3 Which Operations?

15 min

Activity Narrative

The purpose of this activity is to contrast the effects of multiplying side lengths versus adding to or subtracting from side lengths when creating copies of a polygon. To find the corresponding side lengths on a scaled copy, the side lengths of a figure are all multiplied (or divided) by the same number. However, students often mistakenly think that adding or subtracting the same number to all the side lengths will also create a scaled copy. When students recognize that there is a multiplicative relationship between the side lengths rather than an additive one, they are looking for and making use of structure (MP7).

This activity gives students a chance to draw a scaled copy without a grid and to use paper as a measuring tool. To create a copy using a scale factor of 2, students need to mark the length of each original segment and transfer it twice onto their drawing surface, reinforcing—in a tactile way—the meaning of scale factor. The angles in the polygon are right angles (and a 270 degree angle in one case) and can be made using the corner of an index card.

Monitor for students who use these strategies to create Andre's drawing in the last question:

- Add 4 units to all the sides and draw a shape that is not a scaled copy (either by changing the angles or creating a figure that isn't closed).
- Sketch a figure that looks like a scaled copy and label it with side lengths that are 4 units longer, without attending to whether the numbers match the actual lengths.
- Show that adding 4 units would not result in a consistent scale factor between pairs of corresponding side lengths.
- Use a ruler to measure the side lengths of the original shape and multiply these measurements by 2.

- Use an index card to measure the side lengths of the original shape and draw lengths that are twice as long.
- Use an index card to measure and recreate the right angles in the figure.

Plan to have students present in this order to support moving them from additive to multiplicative reasoning about the side lengths.

Standards

Addressing	**7.G.A.1**
Building Toward	**7.RP.A.2**

Instructional Routines

- 5 Practices
- MLR8: Discussion Supports

Launch

Give students 2–3 minutes of quiet think time for the first three questions, then pause for a whole-class discussion. Consider asking questions like these:

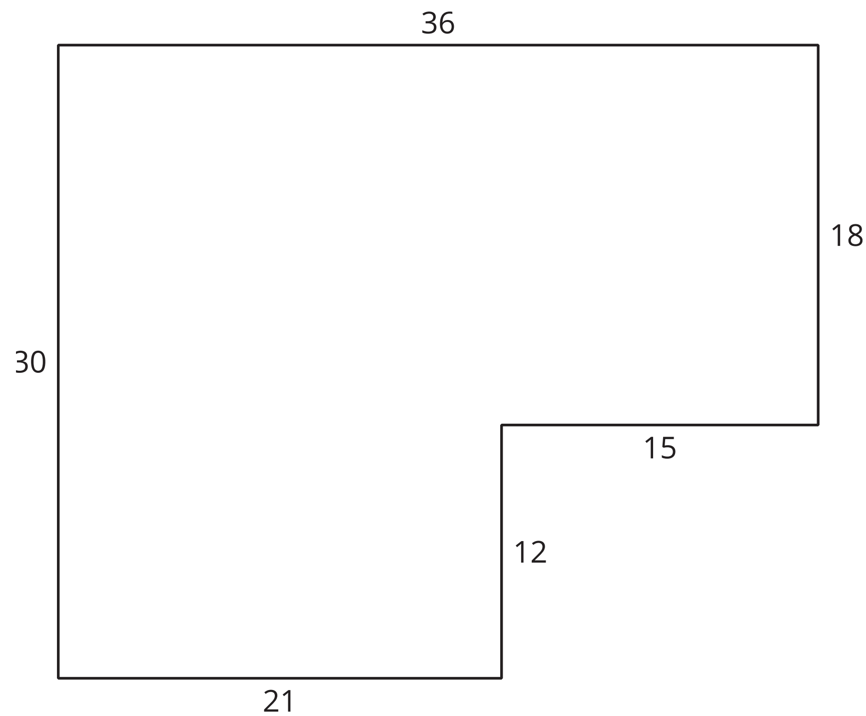
- “What is the scale factor used to create Jada’s drawing? What about for Diego’s drawing?” ($\frac{1}{3}$ for Jada’s. There isn't one for Diego's, because it is not a scaled copy.)
- “What can you say about the corresponding angles in Jada and Diego’s drawings?” (They are all equal, even though one is a scaled copy and one is not.)

Then ask students to read the last question and check that they understand which side of the polygon Andre would like to be 8 units long on his drawing. Provide access to index cards, so that students can use it as a measuring tool. Consider not explicitly directing students as to its use to give them a chance to use tools strategically. If needed, show how to mark the 4-unit length along the edge of a card and to use the mark to determine the needed lengths for the copy.

Give students 4–5 minutes of quiet work time, and then 2 minutes to share their work with a partner. Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

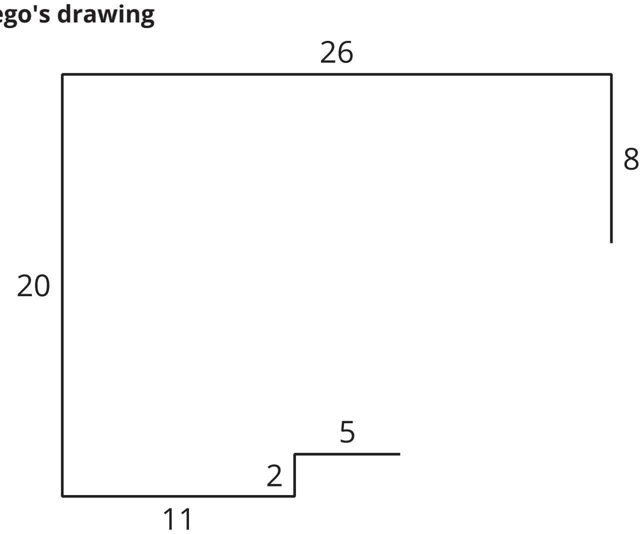
Student Task Statement

Diego and Jada want to scale this polygon so the side that corresponds to 15 units in the original is 5 units in the scaled copy.

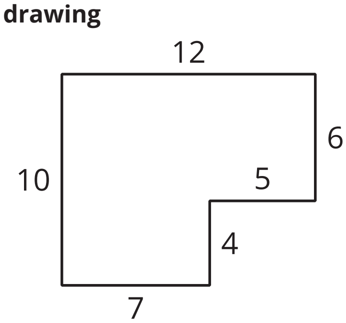


Diego and Jada each use a different operation to find the new side lengths. Here are their finished drawings.

Diego's drawing



Jada's drawing



1. What operation do you think Diego used to calculate the lengths for his drawing?
2. What operation do you think Jada used to calculate the lengths for her drawing?
3. Did each method produce a scaled copy of the polygon? Explain your reasoning.

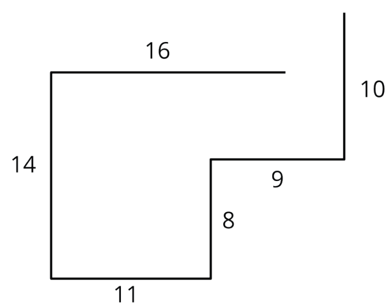
   Pause here for a whole-class discussion.

4. Andre wants to make a scaled copy of Jada's drawing so that the side that corresponds to 4 units in Jada's polygon is 8 units in his scaled copy. Create the scaled copy that Andre wants. If you get stuck, consider using the edge of an index card or paper to measure the lengths needed to draw the copy.

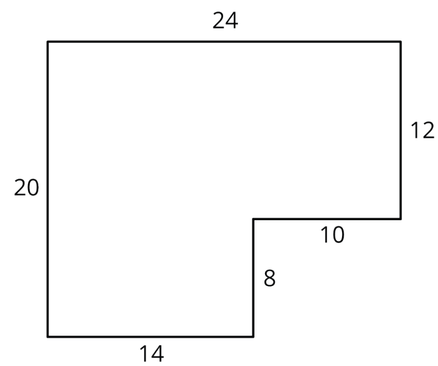
Student Response

1. Since we can get from 15 to 5 by subtracting 10, Diego may have subtracted 10 units from the length of every side. Subtracting 10 from each side length in the original gives Diego's picture.
2. Jada went from 15 to 5 by multiplying by $\frac{1}{3}$ or dividing by 3. Multiplying each side by $\frac{1}{3}$ in the original gives Jada's picture.
3. No, only Jada's method produces a scaled copy. Sample reasoning: Subtracting 10 from each length did not work because now the figure is no longer a polygon. There is a big gap between the two sides that should meet. To create a scaled copy, every length needs to be multiplied (or divided) by the same number.
4. See the figure on the right.

Adding 4 units to each side



A correctly drawn figure



Are You Ready for More?

The side lengths of Triangle B are all 5 more than the side lengths of Triangle A. Can Triangle B be a scaled copy of Triangle A? Explain your reasoning.

Extension Student Response

Yes, if Triangle A is equilateral, then its side lengths are all the same. Adding 5 to each side, the lengths will still be the same and so Triangle B will also be equilateral.

If Triangle A is not equilateral, then Triangle B will not be a scaled copy of Triangle A. To see why, notice that adding 5 to a side length of 5 doubles the side length. Adding 5 to a side length that is greater than 5 changes the side by a scale factor less than 2. Adding 5 to a side length less than 5 changes the side length by a scale factor greater than 2. So if one side length of Triangle A is 5, all side lengths have to be 5 or else Triangle B will not be a scaled copy of Triangle A. This reasoning works for side lengths other than 5.

Activity Synthesis

The purpose of the activity is to explicitly call out a potential misunderstanding about how scale factors work, emphasizing that scale factors work by multiplying existing side lengths by a common factor, rather than by adding or subtracting a common length to each.

Invite previously selected students to share their explanations or illustrations that adding 4 units to the length of each segment would not work (for example, the copy is no longer a polygon, or the copy has angles that are different from those in the original figure). Sequence the discussion of the approaches in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

As students share their approaches, consider asking:

- "What scale factor did you use to create your copy? Why?"
- "How did you measure the side lengths for the copy?"
- "How did you measure the angles for the copy?"

After students have finished sharing with the whole class, connect the different responses to the learning goals by asking questions such as:

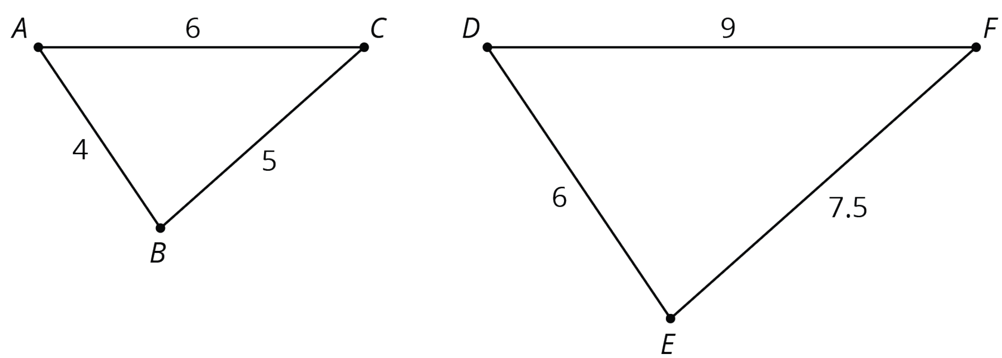
- "How did the scale factor show up in each method?"
- "What worked well in ________'s approach? What did not work well?"
- "What role does multiplication play in each approach?"

Access for English Language Learners

MLR8 Discussion Supports. Provide students with the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking

Lesson Synthesis

Display this image for all to see.



Ask students:

- "Triangle DEF is a scaled copy of triangle ABC. What is the scale factor?" (The scale factor is 1.5 or $\frac{3}{2}$.)
- "How do we draw a scaled copy of a figure?" (To draw a scaled copy of a figure, we need to multiply all of the lengths by the scale factor.)
- "Can we create scaled copies by adding or subtracting the same value from all lengths? Why or why not?" (No, adding or subtracting the same value to all lengths will usually not create a scaled copy.)

2.4 More Scaled Copies

Cool-down

5 min

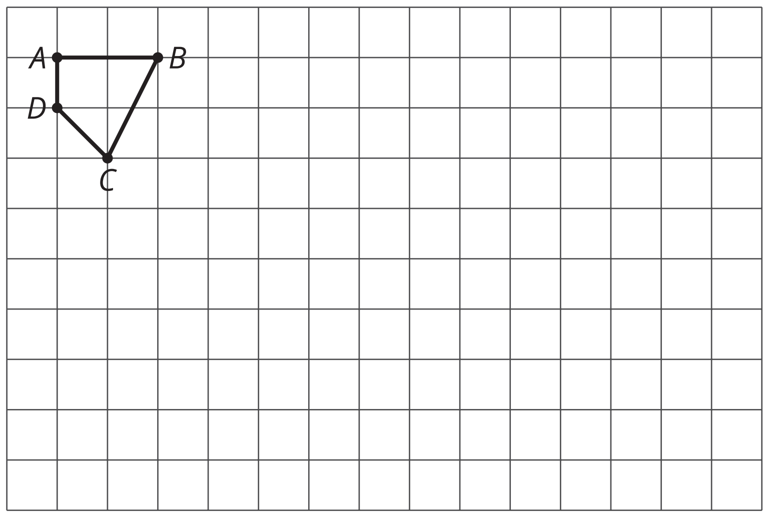
Standards

Addressing 7.G.A.1

Building Toward 7.RP.A.2

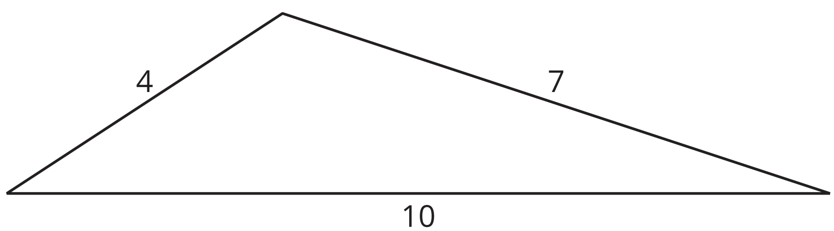
Student Task Statement

1. Create a scaled copy of $ABCD$ using a scale factor of 4.



2. Triangle Z is a scaled copy of Triangle M.

M



Select all the sets of values that could be the side lengths of Triangle Z.

A. 8, 11, and 14.

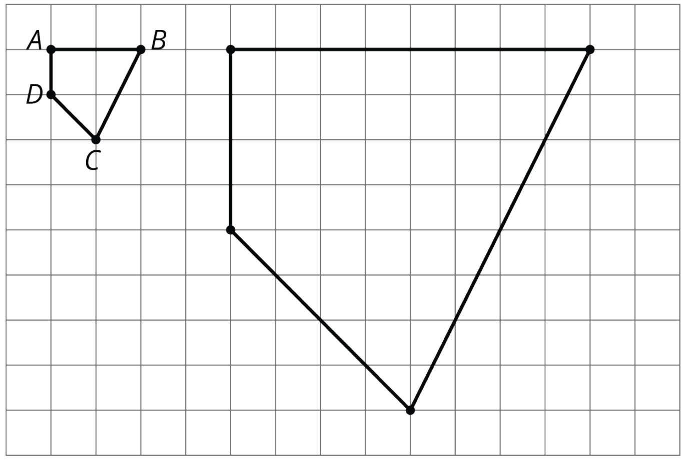
B. 10, 17.5, and 25.

C. 6, 9, and 11.

D. 6, 10.5, and 15.

E. 8, 14, and 20.

Student Response



1.
2. B, D, E

Responding to Student Thinking

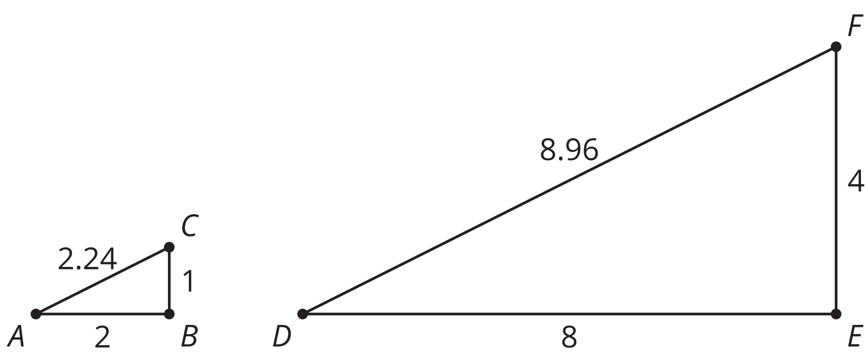
Points to Emphasize
If students struggle with finding all the possible measurements of a similar triangle, focus on scaled lengths when opportunities arise over the next several lessons. For example, in this activity, have students highlight how all distances in a scaled copy (not just the side lengths of the figure) are related by the same scale factor:

Accelerated 7, Unit 2, Lesson 3, Activity 2 Measuring the Three Quadrilaterals

Lesson 2 Summary

Creating a scaled copy involves *multiplying* the lengths in the original figure by a **scale factor**.

For example, to make a scaled copy of triangle ABC where the base is 8 units, we would use a scale factor of 4. This means multiplying all the side lengths by 4, so in triangle DEF, each side is 4 times as long as the corresponding side in triangle ABC.



Glossary

- scale factor

Lesson 2 Practice Problems

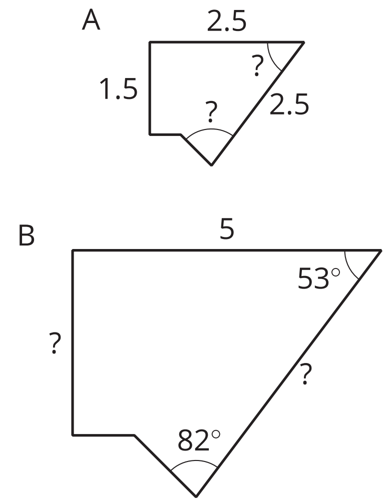
1 Student Task Statement

Polygon B is a scaled copy of Polygon A.

a. What is the scale factor from Polygon A to Polygon B? Explain your reasoning.

b. Find the missing length of each side marked with a "?" in Polygon B.

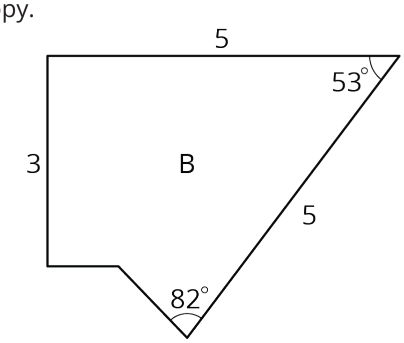
c. Determine the measure of each angle marked with a "?" in Polygon A.



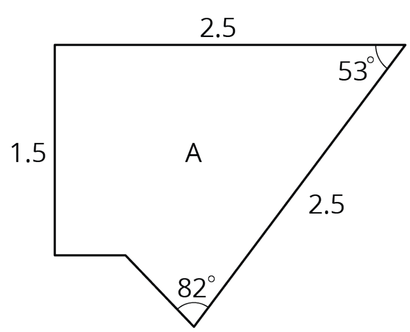
Solution

a. 2 because the top horizontal side has length 2.5 units in Polygon A and 5 units in Polygon B

b. All sides scale by the same factor of 2, so the side that is 2.5 units in Polygon A is 5 units in the copy, and the 1.5-unit-long one is 3 units in the copy.

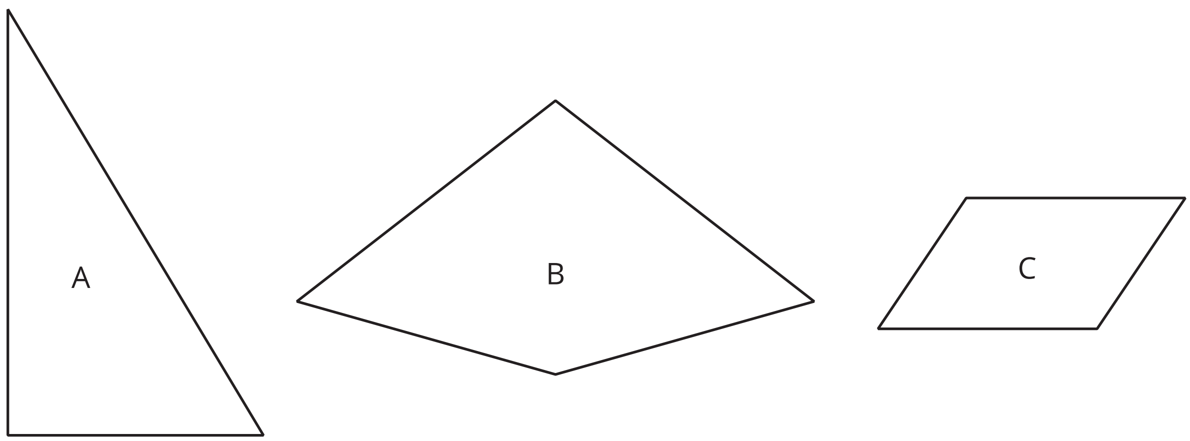


c. 53° and 82° because scaled copies have the same corresponding angles



2 Student Task Statement

Here are 3 polygons.



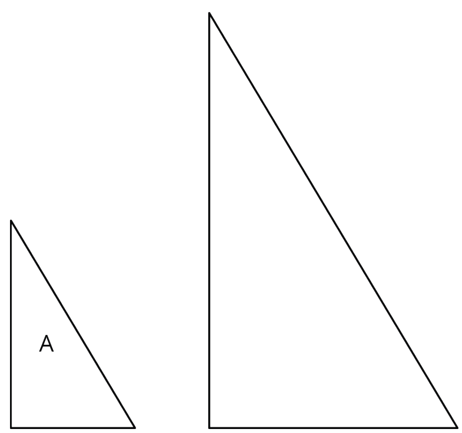
Draw a scaled copy of Polygon A using a scale factor of 2.

Draw a scaled copy of Polygon B using a scale factor of $\frac{1}{2}$.

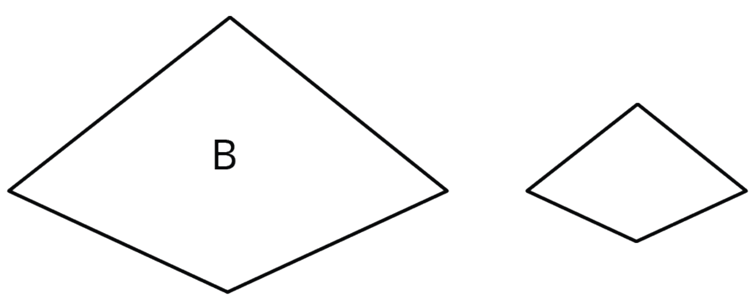
Draw a scaled copy of Polygon C using a scale factor of $\frac{3}{2}$.

Solution

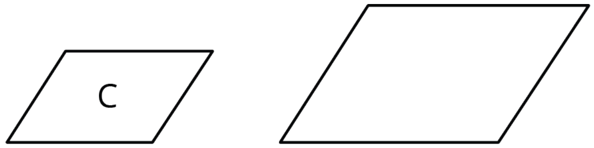
1.



2.



3.



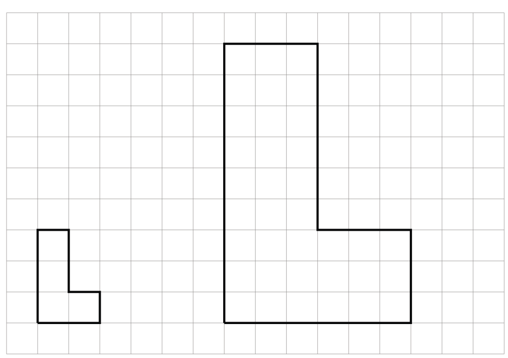
3 Student Task Statement

a. Draw a scaled copy of this polygon so that the scaled copy has a perimeter of 30 units.

b. What is the scale factor? Explain how you know.

Solution

a.



b. The perimeter of the original polygon is 10 units. Since the perimeter of a scaled copy is multiplied by the scale factor, a scale factor of 3 needs to be applied to get a copy with a perimeter of 30.

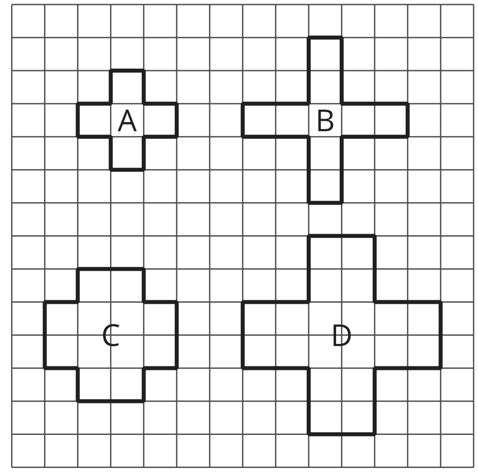
4 from Unit 2, Lesson 1

Student Task Statement

Priya and Tyler are discussing the figures.

- Priya says B, C, and D are scaled copies of A.
- Tyler says B and D are scaled copies of A.

Do you agree with either of them? Explain your reasoning.



Solution

Sample response: I agree with neither one. Only D is a scaled copy of A. In D, the length of each segment of the plus sign is twice the length of its corresponding segment in A. In B and C, some segments are double their corresponding lengths in A but some are not.