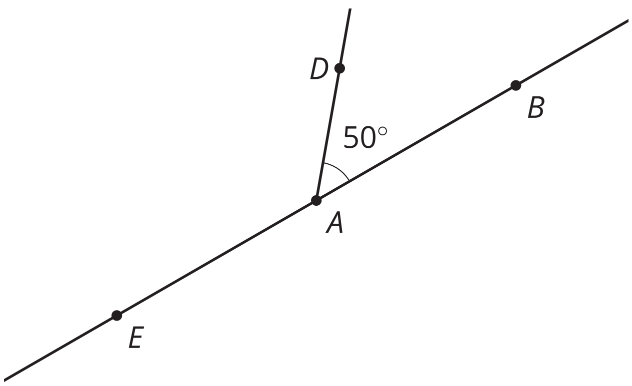
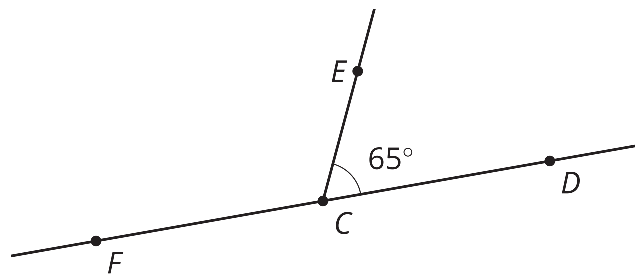
Evidence, Angles, and Proof

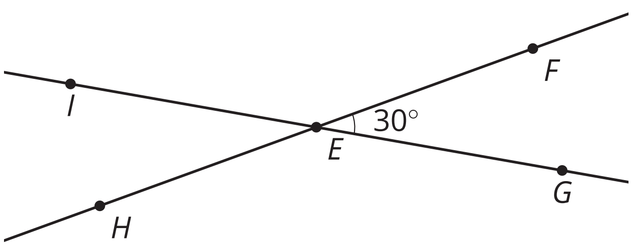
Let’s make convincing explanations.

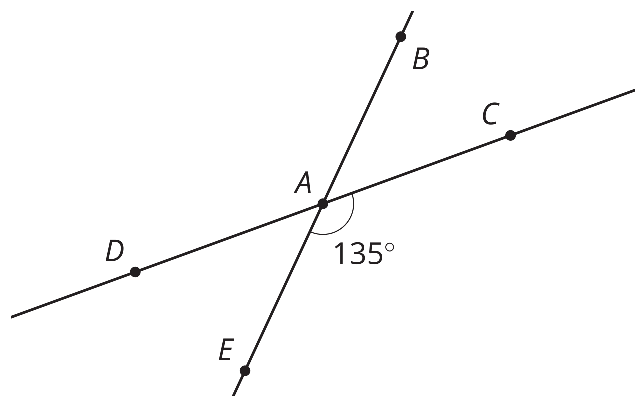
19.1 Math Talk: Supplementary Angles

Mentally evaluate all of the missing angle measures in each figure.





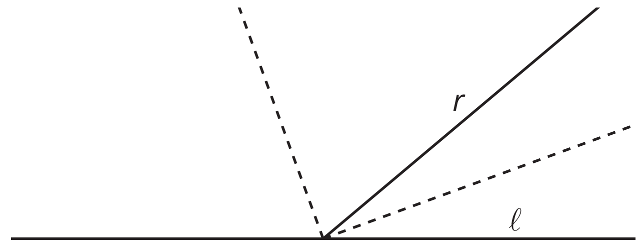




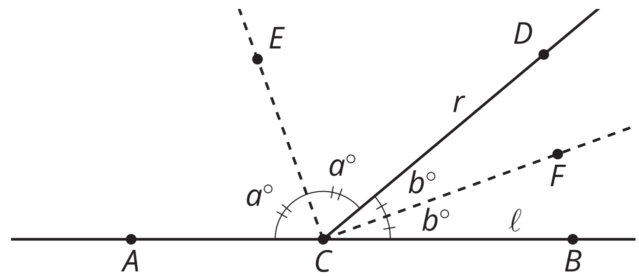
19.2 That Can't Be Right, Can It?

Elena and Diego are working together on this problem: Here is a figure where ray r meets line ℓ. The dashed rays are angle bisectors.

Diego makes this conjecture: "The angle formed between the angle bisectors is always a right angle, no matter what the angle between r and ℓ is."



Elena says, "It's difficult to tell specifically which angles you're talking about." She labels the diagram and restates the conjecture as the following: "Ray CE bisects angle ACD into 2 congruent angles. Ray CF bisects angle BCD into 2 congruent angles. We conjecture angle ECF is a right angle."



Diego adds more information to the diagram as he tells Elena, "We can put letters here to represent the angle measures. So these 2 angles are each a°, and these are b°. That means our conjecture is $a+b=90$."

Elena exclaims, "Oh! I see it now. Angle ACB is 180 degrees, so $a+a+b+b=180$. Then the middle part has to be a right angle."

Diego writes down a summary of their conversation: "For any straight line ℓ and ray r, the angle bisectors of the 2 angles form a right angle. That's because there are 2 pairs of congruent angles, a° and b°, that sum to 180. So $a+b$ has to equal 90 degrees, a right angle."

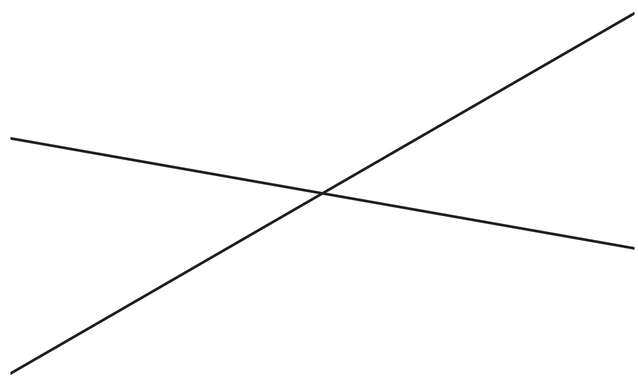
1. Take turns with your partner to explain what is happening in this conversation.
    a. For each paragraph that you read, explain to your partner what Elena or Diego is saying and doing.
    b. For each paragraph that your partner reads, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

2. Diego says, “These 2 angles are each $a°$.” Which 2 angles is he referring to? How does he know they are the same size?

3. Why does $a + b$ have to equal 90 degrees?

19.3 Convince Me

Here are 2 intersecting lines that create 2 pairs of vertical angles:



1. What is the relationship between vertical angles? Write down a conjecture. Label the diagram to make it easier to write your conjecture precisely.

2. How do you know your conjecture is true for all possible pairs of vertical angles? Explain your reasoning.

Are you ready for more?

One reason mathematicians like to have rigorous proofs even when conjectures seem to be true is that sometimes conjectures turn out to not be true. Here is one famous example: If we draw n points on a circle and connect each pair of points, how many regions does that divide the circle into? If we draw only 1 point, there are no line segments to connect, so there is just 1 region in the circle. If we draw 2 points, they are connected by a line segment which divides the circle into 2 regions.

1. If we draw 3 points on a circle and connect each pair of points with a line segment, how many regions do we get in our circle?
2. If we draw 4 points on a circle and connect each pair of points with a line segment, how many regions do we get in our circle?
3. If we draw 5 points on a circle and connect each pair of points with a line segment, how many regions do we get in our circle?
4. Make a conjecture about how many regions we get if we draw n points on a circle and connect each pair of points with a line segment.

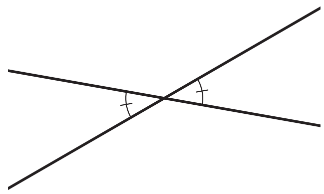
5. Test your conjecture with 6 points on a circle. How many regions do we get if we use the conjecture? How many regions are there actually?

Lesson 19 Summary

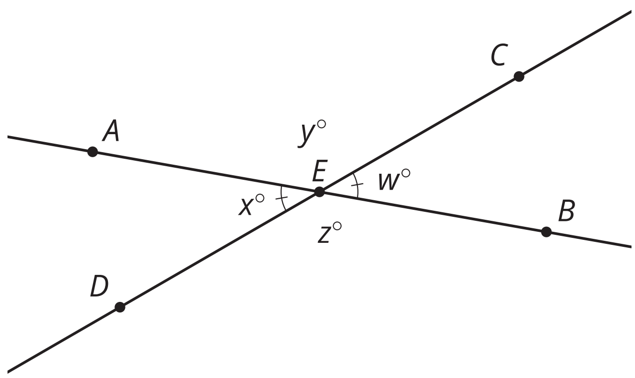
In many situations, it is important to understand the reasons why an idea is true. Here are some questions to ask when trying to convince ourselves or others that a statement is true:

- How do we know this is true?
- Would these reasons convince someone who didn't think it was true?
- Is this true always, or only in certain cases?
- Can we find any situations where this is false?

In this lesson, we reasoned that pairs of vertical angles are always congruent to each other:



We saw this by labeling the diagram and making precise arguments having to do with transformations or angle relationships. For example, label the diagram with points:



Rotate the figure 180 degrees around point E. Then ray EA goes to ray EB, and ray ED goes to ray EC. That means the rotation takes angle AED onto angle BEC, so angle AED is congruent to angle BEC.

Many true statements have multiple explanations. Another line of reasoning uses angle relationships. Notice that angles AED and AEC together form line CD. That means that $x + y = 180$. Similarly, $y + w = 180$. That means that both x and w are equal to $180 - y$, so they are equal to each other. Since angle AED and angle CEB have the same degree measure, they must be congruent.

