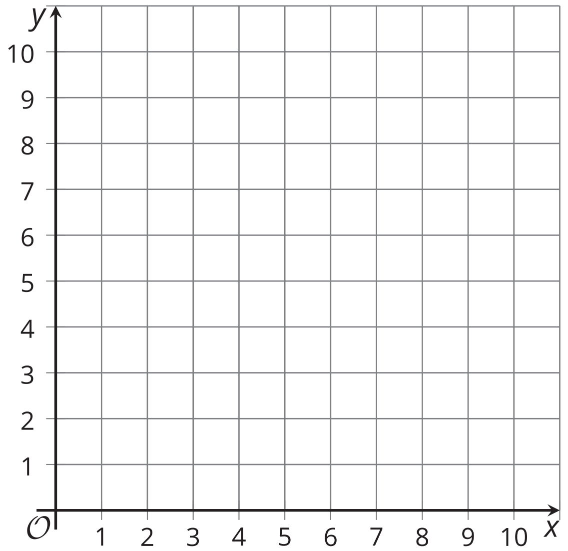
Lesson 15: Weighted Averages

- Let’s split segments using averages and ratios.

15.1: Part Way: Points

For the questions in this activity, use the coordinate grid if it is helpful to you.



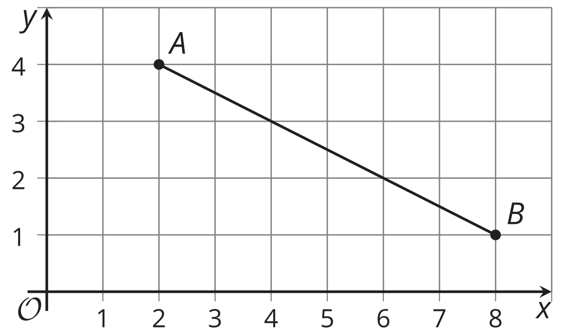
1. What is the midpoint of the segment connecting $(1, 2)$ and $(5, 2)$?

2. What is the midpoint of the segment connecting $(5, 2)$ and $(5, 10)$?

3. What is the midpoint of the segment connecting $(1, 2)$ and $(5, 10)$?

15.2: Part Way: Segment

Point A has coordinates $(2, 4)$. Point B has coordinates $(8, 1)$.



1. Find the point that partitions segment AB in a $2 : 1$ ratio.

2. Calculate $C = \frac{1}{3}A + \frac{2}{3}B$.

3. What do you notice about your answers to the first 2 questions?

4. For 2 new points K and L, write an expression for the point that partitions segment KL in a $3 : 1$ ratio.

Are you ready for more?

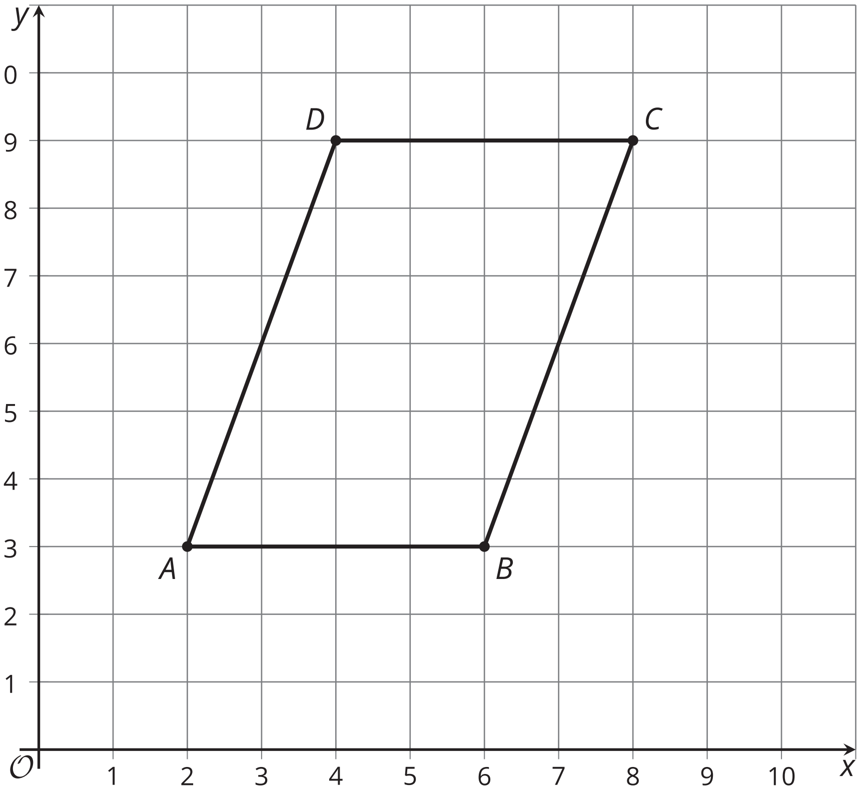
Consider the general quadrilateral $QRST$ with $Q = (0, 0)$, $R = (a, b)$, $S = (c, d)$, and $T = (e, f)$.

1. Find the midpoints of each side of this quadrilateral.

2. Show that if these midpoints are connected consecutively, the new quadrilateral formed is a parallelogram.

15.3: Part Way: Quadrilateral

Here is quadrilateral $ABCD$.



1. Find the point that partitions segment AB in a $1:4$ ratio. Label it B'.

2. Find the point that partitions segment AD in a $1:4$ ratio. Label it D'.

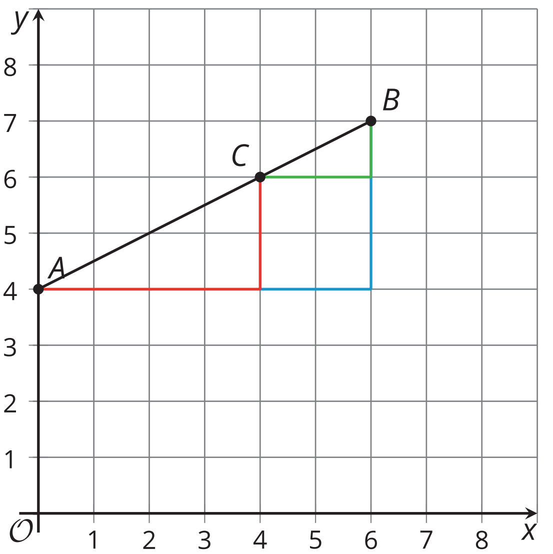
3. Find the point that partitions segment AC in a $1:4$ ratio. Label it C'.

4. Is $AB'C'D'$ a dilation of $ABCD$? Justify your answer.

Lesson 15 Summary

To find the midpoint of a line segment, we can average the coordinates of the endpoints. For example, to find the midpoint of the segment from $A = (0, 4)$ to $B = (6, 7)$, average the coordinates of A and B: $\left(\frac{0+6}{2}, \frac{4+7}{2}\right) = (3, 5.5)$. Another way to write what we just did is $\frac{1}{2}(A + B)$ or $\frac{1}{2}A + \frac{1}{2}B$.

Now, let's find the point that is $\frac{2}{3}$ of the way from A to B. In other words, we'll find point C so that segments AC and CB are in a $2 : 1$ ratio.



In the horizontal direction, segment AB stretches from $x = 0$ to $x = 6$. The distance from 0 to 6 is 6 units, so we calculate $\frac{2}{3}$ of 6 to get 4. Point C will be 4 horizontal units away from A, which means an x-coordinate of 4.

In the vertical direction, segment AB stretches from $y = 4$ to $y = 7$. The distance from 4 to 7 is 3 units, so we can calculate $\frac{2}{3}$ of 3 to get 2. Point C must be 2 vertical units away from A, which means a y-coordinate of 6.

It is possible to do this all at once by saying $C = \frac{1}{3}A + \frac{2}{3}B$. This is called a weighted average. Instead of finding the point in the middle, we want to find a point closer to B than to A. So we give point B more weight—it has a coefficient of $\frac{2}{3}$ rather than $\frac{1}{2}$ as in the midpoint calculation. To calculate $C = \frac{1}{3}A + \frac{2}{3}B$, substitute and evaluate.

$\frac{1}{3}A + \frac{2}{3}B$

$\frac{1}{3}(0, 4) + \frac{2}{3}(6, 7)$

$\left(0, \frac{4}{3}\right) + \left(4, \frac{14}{3}\right)$

$(4, 6)$

Either way, we found that the coordinates of C are $(4, 6)$.