Solving Inequalities with Absolute Values

Goals

- Draw solutions to absolute value inequalities on a number line.
- Interpret absolute value inequalities using distance on a number line.

Learning Targets

- I can solve absolute value inequalities by reasoning about distance on a number line.

Lesson Narrative

In this lesson, students solve absolute value inequalities by reasoning about distances on a number line. They begin by examining a context in which there is a manufacturing tolerance. Students write values that fall within that tolerance and draw the range of solutions on a number line. Then, they write an inequality to represent the distance from the target value. In the following activity, students solve absolute value inequalities and draw the solution on a number line using reasoning about distance from a number.

In these materials, there is no emphasis on using "and" and "or" in a precise way. Instead, students are asked to draw their solutions on a number line. This helps students focus on understanding the meaning of their solutions.

Standards

Addressing **HSA-CED.A.1, HSA-REI.B.3**

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR6: Three Reads

Required Materials

Materials to Gather

- Nut and bolt: Activity 2

Student Facing Learning Goals

Let's solve absolute value inequalities.

16.1 Interpreting Absolute Value Inequalities

5 min

Warm-up

Activity Narrative

In this *Warm-up*, students use their understanding of the connection between absolute value equations and distances on a number line to describe the connection for inequalities. This will be useful in later activities when students use the connection to reason about solutions to inequalities involving absolute values.

Pay attention to the precision of student language as they describe the inequalities (MP6). In particular, phrases like "at most" and "at least" usually imply that the endpoints are included, while phrases like "less than" or "greater than" usually imply the endpoints are not included.

Standards

Addressing **HSA-CED.A.1**

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2.

Access for English Language Learners

MLR1 Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to "Describe what each of these expressions mean." Invite listeners to ask questions, and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

Student Task Statement

Describe what each of these expressions mean.

1. $|x - 8| = 4$
2. $|x - 8| < 4$
3. $|x - 8| \geq 4$

Student Response

Sample responses:

- The distance between x and 8 on the number line is 4.
- The distance between x and 8 on the number line is less than 4.
- The distance between x and 8 on the number line is at least 4.

Activity Synthesis

The purpose of this discussion is to ensure students understand inequalities involving absolute value as a range of distances. For each question, select a group to share their solution.

Ask students,

- "What is the difference between these two sentences? 'The distance between x and 8 is less than 4' and 'The distance between x and 8 is at most 4.'" (The first sentence would use the $<$ symbol and the second sentence would use the $\leq$ symbol. In the first sentence, 4 is not included as a distance that works. In the second sentence it is included.)
- "How would you write an inequality that means, 'all the values that are less than 5 away from 10?'" ($|x - 10| < 5$)

15 min

16.2 A Threaded Bolt

Activity Narrative

In this activity, students move from writing individual values that satisfy conditions for a real-world situation to writing inequalities that describe the conditions. The context of nuts and bolts is used to emphasize the importance of precision and give students a sense of how error tolerance is used in the real world.

Access for English Language Learners

This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Standards

Addressing **HSA-CED.A.1, HSA-REI.B.3**

Instructional Routines

- MLR6: Three Reads

Launch

Arrange students in groups of 2.

While an image is provided, it can be helpful to show students a bolt with a nut that fits it so that students can examine how the threads on each work together.

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the image in the problem stem, without revealing the questions.

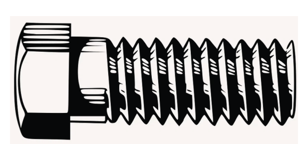
- For the first read, read the problem aloud, then ask, "What is this situation about?" (the thread distance on a bolt). Listen for and clarify any questions about the context.
- After the second read, ask students to list any quantities that can be counted or measured. (the ideal distance of threads on a bolt, the amount of error that is acceptable)
- After the third read, reveal the question: "What are 3 different distances between threads that would be acceptable for this type of bolt?" and ask, "What are some ways we might get started on this?" Invite students to name some possible starting points, referencing quantities from the second read. (think of what numbers might be within 0.1 mm of 1 mm)

Access for Students with Disabilities

Representation: Access for Perception. Use a bolt and nut to demonstrate the situation, and allow students to touch the threads.
Supports accessibility for: Conceptual Processing, Language, Memory

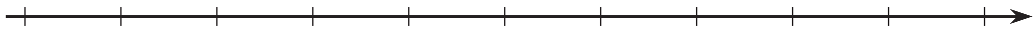
Student Task Statement

The part that winds around on a bolt is called the "thread." In order to work correctly, the threads of a certain type of bolt must be about 1 millimeter apart from one another. Due to the way they are made, it is difficult to get the threads exactly 1 millimeter apart, but they will not work correctly if they are too far off. This type of



bolt is acceptable if the threads are within 0.1 millimeter of what it is supposed to be.

1. What are 3 different distances between threads that would be acceptable for this type of bolt?
2. Use the number line to draw all of the distances between threads that are allowed.



3. Let x represent the distance between threads. Complete the two inequalities that describe what must be true for all bolts of this type that are acceptable.

   $x \leq$ ________ and $x \geq$ ________
4. Use absolute value to write these inequalities as a single inequality that expresses the distance from 1 millimeter that is acceptable.
5. Write an inequality using absolute value that represents threads for bolts that are unacceptable.

Student Response

1. Sample response: 0.98 mm, 0.94 mm, and 1.07 mm
2. 

3. $x \leq 1.1$ and $x \geq 0.9$
4. $|x - 1| \leq 0.1$
5. $|x - 1| > 0.1$

Activity Synthesis

The purpose of the discussion is to make a strong connection between the context, a range of distances on a number line, and inequalities involving absolute value.

Invite groups to share their inequalities that have absolute values. Then ask students,

- “What is the maximum possible thread length that is acceptable?” (1.1 millimeters)
- “What is the minimum possible thread length that is acceptable?” (0.9 millimeter)
- “A different type of bolt should have a thread length of 0.5 millimeter, but it is still acceptable if the threads are within 0.03 millimeter of that value. What is an inequality that fits this situation?” ($|x - 0.5| \leq 0.03$)
- “What would a number line that shows the solutions for the inequality you wrote in the last problem look like?” (An open circle at 0.9 millimeter with shading to the left and an open circle at 1.1 millimeters with shading to the right.)

16.3 Solving Absolute Value Inequalities

15 min

Activity Narrative

In this activity, students practice reasoning about distances on the number line to solve inequalities that include absolute value.

Standards

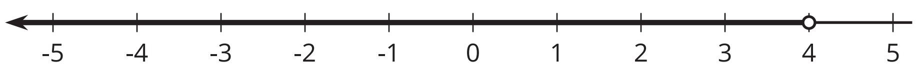
Addressing **HSA-REI.B.3**

Launch

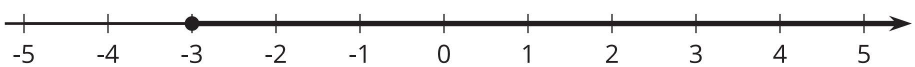
Keep students in groups of 2.

Remind students of how to graph solutions to inequalities on a number line. Pay particular attention to the closed or open circle based on the inclusivity of the inequalities. Here are some examples.

$x < 4$



$x \geq -3$



Student Task Statement

Graph the solution to each inequality on a number line.

1. $|x - 5| \leq 1$

2. $3 > |x + 3|$

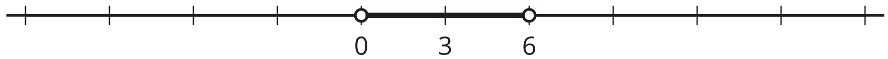
3. $|x - 0.5| \geq 2$

4. $|x - 10| > \frac{2}{3}$

Student Response

1.

2.



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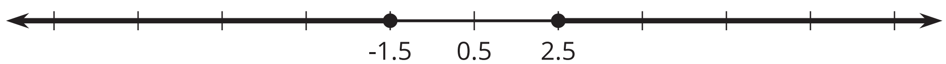

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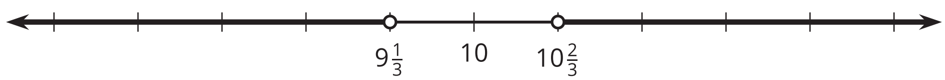


Are You Ready for More?

Use the piecewise definition of absolute value to write two new inequalities to solve each one of the inequalities here, then solve the new inequalities you have created.

1. $|2x - 3| < 5$
2. $2|x - 1| \geq 10$

Extension Student Response

Sample responses:

1. The two new inequalities are $(2x - 3) < 5$ and -$(2x - 3) < 5$. The first inequality can be solved to get $x < 4$. The second inequality can be rewritten as -$5 < (2x - 3)$ and then solved to get -$1 < x$
2. The two new inequalities are $2(x - 1) \geq 10$ and -$2(x - 1) \geq 10$. The first inequality can be solved to get $x \geq 6$. The second inequality can be rewritten as -$10 \geq 2(x - 1)$ which can be solved to get -$4 \geq x$.

Activity Synthesis

The purpose of the discussion is for students to understand how to solve inequalities that include absolute values.

Invite groups to share their solutions for the four inequalities. Ask students,

- “How do the first two inequalities compare to the last two?” (The first two involve the absolute values being less than a value, and the last two are greater than a value.)
- “How do the solutions on the number line compare for the first two inequalities and the last two inequalities?” (The first two number lines are shaded between 2 values. The last two number lines are shaded on the outside of 2 values.)
- "Does having the absolute value on the right side of the inequality change how you think about it?" (It changes how I think about it a little bit. I think about it as 3 being greater than the distance between x and -3 rather than the distance between x and -3 being less than 3.)
- “If these inequalities represent the distance you should be from an object, how would you describe the solution for

the first and last problems?" (In the first one, we need to be closer than 1 unit away from an object that is at a position of 5 units. In the last one, we need to be farther away than $\frac{2}{3}$ unit from the object that is at a position of 10 units.)

- "How does the solution to the first inequality change if it is written as $|5 - x| \leq 1$?" (It does not change the solution.)

Lesson Synthesis

The purpose of the discussion is to ensure students make the connection between absolute value inequalities and distances on a number line.

Ask students,

- "One way to refer to all of the numbers from 0 to 10 is to talk about numbers that are within 5 of the number 5. How could you write this as an inequality that uses absolute values?" ($|x - 5| \leq 5$ or $|x - 5| < 5$, depending on if we want to include 0 and 10 or not.)
- "To be safe, you should stand more than 6 feet away from a small fire when using a fire extinguisher. If the fire is at a position that is considered the 10 foot mark, how can you write this as an inequality using absolute value?" ($|x - 10| > 6$)
- "To be effective, you should stand no more than 10 feet away from a small fire when using a fire extinguisher. If the fire is still at a position that is considered the 10 foot mark, how can you write this as an inequality using absolute value?" ($|x - 10| \leq 10$)
- "When an inequality looks like $|x - a| < b$ for numbers a and b, is the solution a single shaded region or two outside regions? Explain your reasoning." (a single shaded region, because the distance from x to a number is less than b, so all of the numbers that work should be within a certain range of a)

16.4 Handball Ball

Cool-down

10 min

Standards

Addressing **HSA-CED.A.1, HSA-REI.B.3**

Student Task Statement

The international sport of handball uses a small, hollow ball that teams of players attempt to toss into a net for points. The mass of a regulation handball must be within 25 grams of the target 450 grams.

1. Write an inequality using absolute value that represents the masses of handballs that are allowed.

2. Draw the solutions on the number line.

Student Response

1. $|x - 450| \leq 25$ or $|x - 450| < 25$
2. 

Responding to Student Thinking

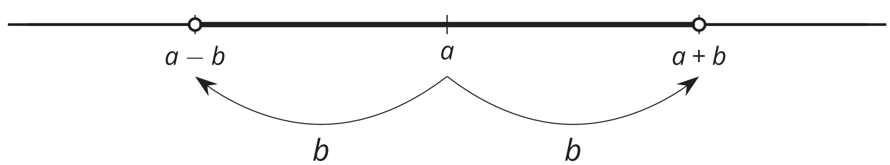
Press Pause
If most students struggle with writing and solving absolute value inequalities, and the concept is important to the standards of the course, make time to revisit absolute value inequalities. For example, review related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Integrated Math 1, Unit 8, Lesson 16, Practice Problem 1
Integrated Math 1, Unit 8, Lesson 16, Practice Problem 2

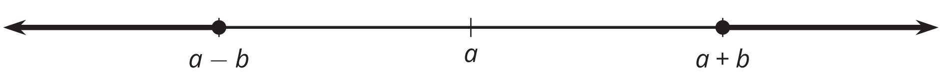
Lesson 16 Summary

In many real-world situations, it is okay if an item is not exactly the perfect length or weight. There is a range of values that will work: The length or weight of the item can be within a certain amount or at least a certain amount away. In these cases, it can be useful to write the range of values that work as inequalities that include absolute values.

To be within a certain distance of a value, we can write an inequality of the form $|x - a| < b$. The solutions to this inequality can be drawn on a number line like this:



To be at least a certain distance away from a value, we can write an inequality of the form $|x - a| \geq b$. The solutions to this inequality can be drawn on a number line like this:



For example, to fence in a circular area with a radius of 100 meters, we should use the equation $C = 2\pi r$ and buy 200π meters of fencing. This is difficult to do because π is an irrational number. Maybe it's okay if we don't make a perfect circle or if it's a little off, so if we buy x amount of fence, where x is a solution to $|x - 200\pi| < 1$, we will have the right amount of fencing within 1 meter of the exact value, and that should be close enough.

Lesson 16 Practice Problems

1 Student Task Statement

Solve the inequalities. Graph the solutions on the number lines.

a. $|x - 3| \leq 5$

b. $|x - 3| < 5$

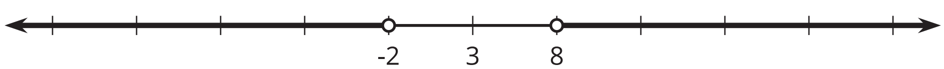
c. $|x - 3| > 5$

d. $|x - 3| \geq 5$

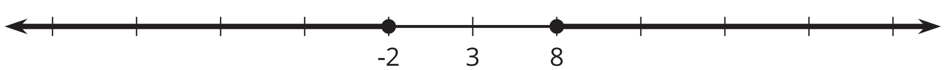
Solution

a.

b.



c.



d.

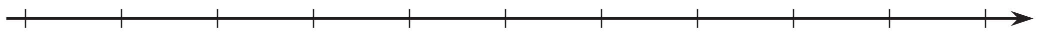
2 Student Task Statement

Solve the inequalities. Graph the solutions on the number lines.

a. $|x - 1.2| \leq 3.1$

b. $|x - \frac{3}{4}| > 5$

c. $|x + 2| < \frac{1}{3}$

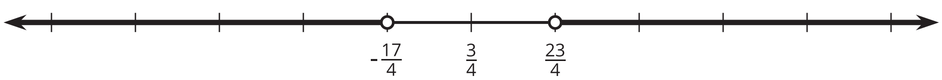


d. $|x - 7| \geq 2.3$

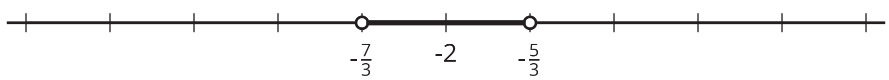
Solution

a.

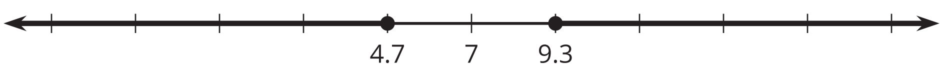
b.



c.



d.



3 Student Task Statement

Use distance on a number line to explain why the solutions to $|3 - x| \geq 2$ include all numbers less than or equal to 1 as well as all numbers greater than or equal to 5.

Solution

Sample response: The inequality says that the distance between 3 and x is at least 2. This means that 1 and any numbers farther away from 3 than 1 are solutions as well as 5 and all numbers farther away from 3 than 5.

4 Student Task Statement

a. Explain what the solutions to the inequality $|x - 4.26| \geq 0$ mean based on distance on a number line. What are the solutions?

b. Explain what the solutions to the inequality $|x - 2| < 0$ mean based on distance on a number line. What are the solutions?

Solution

Sample responses:

a. The inequality solutions are all the values that are at least 0 away from 4.26 on the number line. This means that any number is a solution.

b. The inequality solutions are all the values that are less than 0 away from 2 on the number line. This means that there are no solutions.

5 Student Task Statement

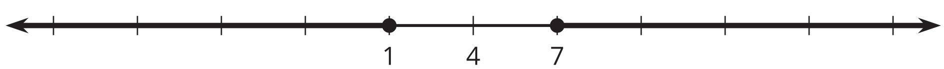
Solve the inequalities. Graph the solutions on the number lines.

a. $|x - 4| + 2 \geq 5$

b. $2|x + 3| < 10$

Solution

a.



b.