Unit 6, Lesson 4

Positive Rational Exponents

Let's use roots to write exponents that are fractions.

4.1 Math Talk: Regrouping Fractions

Find the value of each expression mentally.

- $\frac{1}{2} \cdot 5 \cdot 4$
- $\frac{5}{2} \cdot 4$
- $\frac{2}{3} \cdot 7 \cdot \frac{3}{2}$
- $7 \cdot \frac{5}{3} \cdot \frac{3}{7}$

4.2 You Can Use Any Fraction as an Exponent

1. Use exponent rules to explain why these expressions are equal to each other:

   $5^{\frac{2}{3}}$ $\qquad$ $\left(5^{\frac{1}{3}}\right)^2$ $\qquad$ $\left(5^2\right)^{\frac{1}{3}}$

2. Write $5^{\frac{2}{3}}$ using radicals.

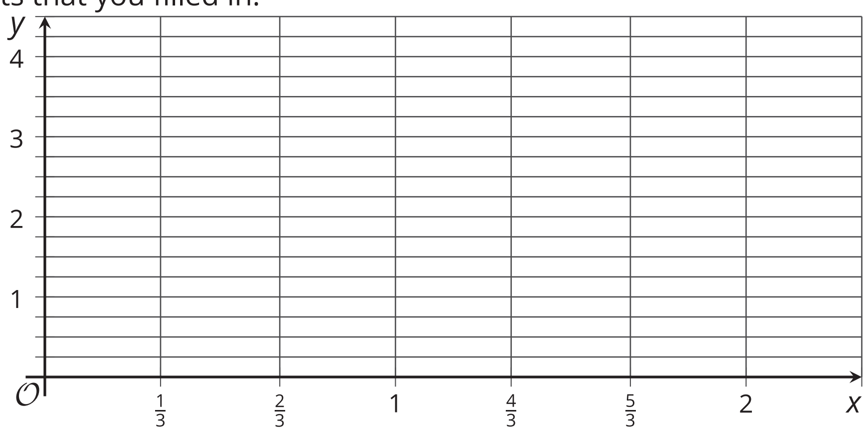
3. Write $5^{\frac{4}{3}}$ using radicals. Show your reasoning using exponent rules.

4.3 Fractional Powers Are Just Numbers

1. Complete the table as much as you can without using a calculator. (You should be able to fill in three spaces.)

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
2^x (using exponents)	2^0	$2^{\frac{1}{3}}$	$2^{\frac{2}{3}}$	2^1	$2^{\frac{4}{3}}$	$2^{\frac{5}{3}}$	2^2
2^x (decimal approximation)							

   a. Plot the points that you filled in.



   b. Connect the points as smoothly as you can.

   c. Use this graph of $y = 2^x$ to estimate the value of the other powers in the table, and write your estimates in the table.

2. Select one of the columns of the table that includes one of your estimates.

   a. Write the power from the second row of the column you chose using radical notation.

   b. What is the exact value of that number cubed?

   c. Raise your decimal estimate from the table to the third power. What should it be? How close did you get?

Are you ready for more?

Answer these questions using the fact that $(1.26)^3 = 2.000376$.

1. Explain why $\sqrt[3]{2}$ is very close to 1.26. Is it larger or smaller than 1.26?

2. Is it possible to write $\sqrt[3]{2}$ exactly with a finite decimal expansion? Explain how you know.

Lesson 4 Summary

Using exponent rules, we know $3^{\frac{1}{4}}$ is the same as $\sqrt[4]{3}$ because $\left(3^{\frac{1}{4}}\right)^4 = 3$. But what about $3^{\frac{5}{4}}$?

Using exponent rules,

$$3^{\frac{5}{4}} = \left(3^5\right)^{\frac{1}{4}} \text{ or } 3^{\frac{5}{4}} = \left(3^{\frac{1}{4}}\right)^5$$

which means that

$$3^{\frac{5}{4}} = \sqrt[4]{3^5} \text{ or } \left(\sqrt[4]{3}\right)^5$$

$3^5 = 243$, so we could also write $3^{\frac{5}{4}} = \sqrt[4]{243}$.

Here are more examples of exponents that are fractions and their equivalents:

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
5^x (using exponents)	5^0	$5^{\frac{1}{3}}$	$5^{\frac{2}{3}}$	5^1	$5^{\frac{4}{3}}$	$5^{\frac{5}{3}}$	5^2
5^x (equivalent expression)	1	$\sqrt[3]{5}$	$\sqrt[3]{5^2}$ or $\sqrt[3]{25}$	5	$\sqrt[3]{5^4}$ or $\sqrt[3]{625}$	$\sqrt[3]{5^5}$ or $\sqrt[3]{3125}$	25