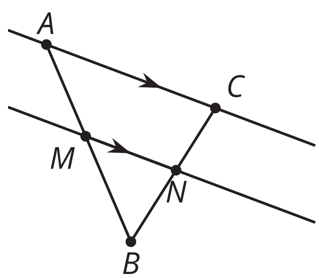
Splitting Triangle Sides with Dilation (Part 2)

Let's investigate parallel segments in triangles.

11.1 Notice and Wonder: Parallel Segments

What do you notice? What do you wonder?

$\overleftrightarrow{AC} \parallel \overleftrightarrow{MN}$



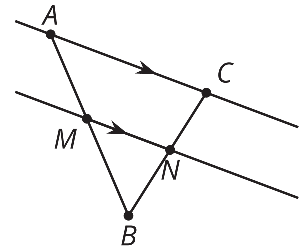
11.2 Prove It: Parallel Segments

Does a line parallel to one side of a triangle always create similar triangles?

1. Create several examples. Decide if the conjecture is true or false. If it's false, make a more specific true conjecture.

2. Find any additional information that you can be sure is true. Label it on the diagram.

$\overleftrightarrow{AC} \parallel \overleftrightarrow{MN}$

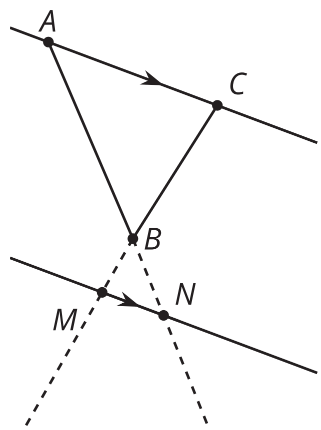


3. Write an argument that would convince a skeptic that your conjecture is true.

Are you ready for more?

If the line parallel to one side of the triangle does not intersect the other sides of the triangle, does it still create a similar triangle if the sides of the original triangle are extended? Modify your reasoning from this activity to show that triangle ABC is similar to triangle NBM.

$\overleftrightarrow{AC} \parallel \overleftrightarrow{NM}$

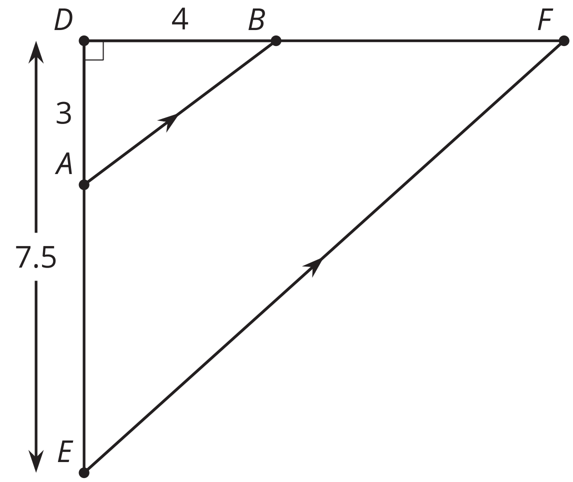


11.3 Preponderance of Proportional Relationships

Find the length of each unlabelled side.

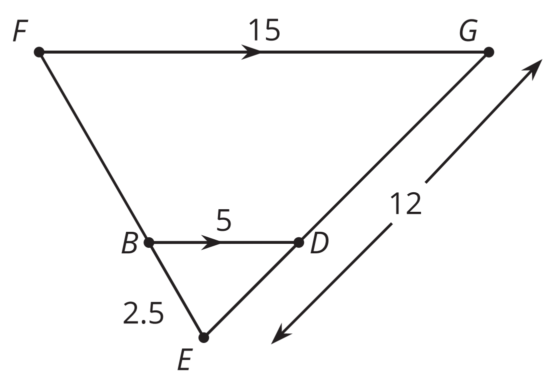
1. Segments AB and EF are parallel.
   - $AB =$
   - $DF =$
   - $EF =$

$\overline{AB} \parallel \overline{EF}, \overline{AD} \perp \overline{DB}$



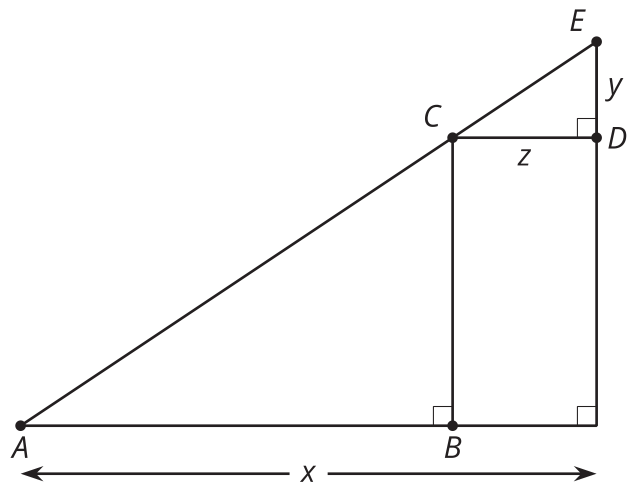
2. Segments BD and FG are parallel. Segment EG is 12 units long. Segment EB is 2.5 units long.
   - $EF =$
   - $ED =$

$\overline{BD} \parallel \overline{FG}$



Are you ready for more?

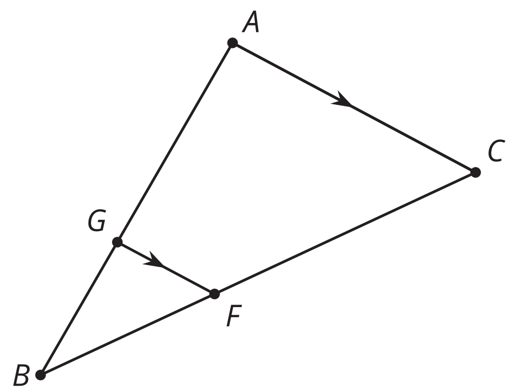
Find the lengths of sides CE, CB, and CA in terms of x, y, and z. Explain or show your reasoning.



Lesson 11 Summary

In triangle ABC, segment GF is parallel to segment AC. We can show that corresponding angles in triangle ACB and triangle GFB are congruent, so the triangles are similar by the Angle-Angle Triangle Similarity Theorem. There must be a dilation that sends triangle GFB to triangle ACB, and so pairs of corresponding side lengths are in the same proportion. Then we can show that segment GF divides segments AB and CB proportionally. In other words, $\frac{BG}{GA}=\frac{BF}{FC}$.

$\overline{FG} \parallel \overline{AC}$



For example, suppose G is $\frac{2}{3}$ of the way from A to B and F is $\frac{2}{3}$ of the way from C to B. Then if $BA = 9$ and $BC = 12$, we know that $GA = 6$ and $FC = 8$. What will BG and BF equal? Since $BG = 3$ and $BF = 4$, we know that $\frac{3}{6} = \frac{4}{8}$ and can show that $\frac{BG}{GA}=\frac{BF}{FC}$.

This argument holds in general. A segment in a triangle that is parallel to one side of the triangle divides the other two sides of the triangle proportionally.