Unit 7 Family Support Materials

Exponents and Scientific Notation

Section A: Exponent Rules

This week your student will learn rules for multiplying and dividing expressions with exponents. Exponents are a way of keeping track of repeated multiplication. For example, instead of writing $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$, we can write 8^7 instead. The number repeatedly multiplied is called the base, which in this example is 8. The 7 in this example is called the exponent and tells us how many times to multiply the base.

Using our understanding of repeated multiplication, we'll figure out several "rules" for exponents. For example, suppose we want to understand the expression $10^3 \cdot 10^4$. Rewriting this to show all the factors, we get $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10)$. Since this is really seven 10s multiplied together, we can write $10^3 \cdot 10^4 = 10^7$.

Using similar reasoning, we can understand the expression $(10^3)^2$ by rewriting it to show all the factors: $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$. Because this is two groups of three factors of 10, we can write $(10^3)^2 = 10^6$.

When dividing expressions with exponents, such as $\frac{10^5}{10^2}$, we can again write out the factors to get $\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10}$. We know that $\frac{10 \cdot 10}{10 \cdot 10} = 1$, so $\frac{10^5}{10^2} = \frac{(10 \cdot 10) \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 10^3$.

Here is a task to try with your student:

1. What is the difference between the expressions $10^4 \cdot 10^5$ and $(10^4)^5$?
2. Which has a greater value, $\frac{10^7}{10^3}$ or $\frac{10^{15}}{10^{13}}$? Explain your reasoning.

Solution:

1. When we rewrite $10^4 \cdot 10^5$ to show all the factors, we get $(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$. We can see that there are a total of nine 10s being multiplied, so $10^4 \cdot 10^5 = 10^9$. When we rewrite $(10^4)^5$ as $10^4 \cdot 10^4 \cdot 10^4 \cdot 10^4 \cdot 10^4$, we can see that there are 5 groups of four 10s being multiplied together. That means $(10^4)^5 = 10^{20}$.
2. $\frac{10^7}{10^3}$ has the greater value. Sample reasoning: $\frac{10^7}{10^3} = \frac{10^3}{10^3} \cdot 10^4 = 10^4$, while $\frac{10^{15}}{10^{13}} = \frac{10^{13}}{10^{13}} \cdot 10^2 = 10^2$, and 10^4 is greater than 10^2.

Section B: More Exponent Rules

This week your student will expand their work with exponents that have bases other than 10 and will learn some new rules about exponents.

One of those rules is that any base raised to the power of 0 must equal 1. For example, $10^0 = 1$ and $2^0 = 1$.

Students will also learn about negative exponents. While 10^n represents repeated multiplication of 10, 10^{-n} represents repeated multiplication of $\frac{1}{10}$. For example $10^{-5} = (\frac{1}{10})^5 = \frac{1}{10^5}$.

Students will also see how the exponent rules work when the base of the exponential expression is a number other than 10 or, in one case, when the bases are different. For example, consider the expression $3^4 \cdot 5^4$. Rewriting this to show all the factors, we get $3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 5$. If we regroup the factors, we get $(3 \cdot 5) \cdot (3 \cdot 5) \cdot (3 \cdot 5) \cdot (3 \cdot 5)$, or $15 \cdot 15 \cdot 15 \cdot 15 = 15^4$. Note that this rule will not work for different bases if the exponents are not the same.

Here is a summary of the general rules for exponents:

$a^n \cdot a^m = a^{n+m}$	$(a^n)^m = a^{n \cdot m}$
$\frac{a^n}{a^m} = a^{n-m}$	$a^0 = 1$
$a^{-n} = \frac{1}{a^n}$	$a^n \cdot b^n = (a \cdot b)^n$

Here is a task to try with your student:

1. Write the following expressions using a single exponent:
   a. $5^5 \cdot 5^0 \cdot 5^1$
   b. $4^{-2} \cdot 4^{-4}$
   c. $\frac{(7^5)^3}{7^{10}}$
   d. $9^8 \cdot 2^8$
2. Write 3 expressions that are equivalent to 10,000.

Solution:

1.
   a. 5^6
   b. 4^{-6} or $(\frac{1}{4})^6$ or $\frac{1}{4^6}$

c. 7^5

d. $(9 \cdot 2)^8$ or 18^8

2. Sample responses: 10^4, $10 \cdot 10 \cdot 10 \cdot 10$, $\frac{10^{10}}{10^6}$, $(10^2)^2$, $2^4 \cdot 5^4$

Section C: Large and Small Numbers

This week your student will use powers of 10 to work with very large or very small numbers. For example, the United States Mint has made over 500,000,000,000 pennies. To understand this number, we can look at the number of zeros it has. The 500 followed by nine zeros tells us that the Mint made over 500 billion pennies.

Using powers of 10, we can write this as: $500 \cdot 10^9$ (five hundred times a billion), or even as: $5 \cdot 10^{11}$. In this example, notice how 500 got 100 times smaller and 10^9 got 10^2 (or 100) times bigger, keeping the value of the expression the same.

The advantage to using powers of 10 to write a large number is that they help us see right away how large the number is by looking at the exponent.

The same is true for small quantities. For example, a single atom of carbon weighs about 0.0000000000000000000000199 grams.

We can write this using powers of 10 as $199 \cdot 10^{-25}$, or equivalently, $(1.99) \cdot 10^{-23}$.

Here is a task to try with your student:

1. Write the number 47,000,000 as a multiple of a power of 10.
2. Write the number 47,000,000 as a different multiple of a power of 10.
3. Write the number 0.00038 as a multiple of a power of 10.
4. Write the number 0.00038 as a different multiple of a power of 10.

Solution:

1. Sample responses: $47 \cdot 10^6$, $4.7 \cdot 10^7$, $0.47 \cdot 10^8$, $470 \cdot 10^5$
2. See above.
3. Sample responses: $38 \cdot 10^{-5}$, $3.8 \cdot 10^{-4}$, $0.338 \cdot 10^{-3}$, $380 \cdot 10^{-6}$
4. See above.

Section D: Scientific Notation

This week your student will be introduced to a specific way of writing numbers called scientific notation. Scientific notation is a way to write very large or very small numbers. We write these numbers by multiplying a number between 1 and 10 by a power of 10.

For example, the number 425,000,000 in scientific notation is 4.25×10^8. The number 0.0000000000783 in scientific notation is 7.83×10^{-11}. Notice how for both examples, the first factor is greater than or equal to 1 but less than 10.

Scientific notation is useful for writing large and small numbers because the power of 10 can quickly show us how big or small a number is without having to count all the zeros. Scientific notation also makes it easier to compare large and small numbers — we can begin a comparison by simply looking at the exponent to see which number is larger. If two numbers are multiplied by the same power of 10, we can easily compare the other factors because we know they have the same place values.

For example, given these three values, 2.1×10^5, 3×10^6, and 1.4×10^5, we can easily see that 3×10^6 has the greatest value since it has the largest power of 10. The other two numbers are both multiplied by 10^5, but by looking at the other factor, we can see that 2.1×10^5 will be greater than 1.4×10^5.

Here is a task to try with your student:

This table shows the top speeds of different vehicles.

vehicle	speed (kilometers per hour)
sports car	$(4.15) \cdot 10^2$
Apollo command and service module	$(3.99) \cdot 10^4$
jet boat	$(5.1) \cdot 10^2$
autonomous drone	$(2.1) \cdot 10^4$

1. Order the vehicles from fastest to slowest.
2. The top speed of a rocket sled is 10,326 kilometers per hour. Is this faster or slower than the autonomous drone?
3. Estimate how many times as fast the Apollo command and service module (CSM) is than the sports car.

Solution:

1. The order from fastest to slowest is: Apollo CSM, autonomous drone, jet boat, sports car. Because all of these values are in scientific notation, we can begin by looking at the power of 10 to compare them. After comparing the powers of 10, we compare the other factors. The speeds of the Apollo CSM and autonomous drone both have the highest power of 10, (10^4), so they are fastest. The Apollo CSM is faster than the drone because 3.99 is greater than 2.1. Similarly, the jet boat is faster than the sports car because their speeds both have the same power of 10, (10^2), but 5.1 is greater than 4.15.
2. The autonomous drone is faster than the rocket sled. In scientific notation, the rocket sled's speed is $1.0326 \cdot 10^4$, and the drone's speed is $2.1 \cdot 10^4$, and 2.1 is greater than 1.0326.
3. To find how many times as fast the Apollo CSM is than the sports car, we are trying to find out what number times $4.15 \cdot 10^2$ equals $3.99 \cdot 10^4$. So we are trying to compute $\frac{3.99 \cdot 10^4}{4.15 \cdot 10^2}$. Because we are estimating, we can simplify the calculation to $\frac{4 \cdot 10^4}{4 \cdot 10^2}$. Using exponent rules and our understanding of fractions, we have $\frac{4 \cdot 10^4}{4 \cdot 10^2} = 1 \cdot 10^{4-2} = 10^2$, so the Apollo CSM is about 100 times as fast as the sports car!