Using the Sum

Let’s calculate some totals.

15.1 Some Interesting Sums

Recall that for any geometric sequence starting at a with a common ratio r, the sum s of the first n terms is given by $s = a\frac{1-r^n}{1-r}$. Find the approximate sum of the first 50 terms of each sequence:

1. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

2. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

15.2 That's a Lot of Houses

In 2010, about 886 thousand homes were sold in the United Kingdom. For the next 3 years, the number of homes sold increased by about 7% annually. Assuming the sales trend continues,

1. How many homes were sold in 2013? In 2014?

2. What information does the value of the expression $886\,\frac{(1-1.07^{11})}{(1-1.07)}$ tell us?

3. Predict the total number of house sales from 2010 to 2016. Explain your reasoning.

Are you ready for more?

Han and Lin each have a method to calculate $3^5+3^6+\cdots+3^n$. Han says this is $3^5\left(1+3+3^2+\cdots+3^{n-5}\right)$ and concludes that $3^5+\cdots+3^n=3^5\,\frac{3^{n-4}-1}{3-1}$. Lin says that this is a difference of terms in two geometric sequences and can be written as $\frac{3^{n+1}-1}{3-1}-\frac{3^5-1}{3-1}$. Do you agree with either Han or Lin? Explain your reasoning.

15.3 Back to Funding the Future

Let’s say you open a savings account with an interest rate of 5% compounded annually (once per year) and that you plan on contributing the same amount to it at the start of every year.

1. Predict how much you need to put into the account at the start of each year to have over \$100,000 in it when you turn 70.

2. Calculate how much the account would have after the deposit at the start of the 50th year if the amount invested each year were:
   a. \$100
   b. \$500
   c. \$1,000
   d. \$2,000

3. Say you decide to invest \$1,000 into the account at the start of each year at the same interest rate. How many years until the account reaches \$100,000? How does the amount you invest into the account compare to the amount of interest earned by the account?

Lesson 15 Summary

Let's say you plan to invest \$200 at the start of each year into an account that averages 3% interest compounded annually at the end of the year. How many years until the account has more than \$10,000? \$20,000?

We know that, at the end of year 1, the amount in the account is \$206. At the end of year 2, the amount in the account is \$418.18 since $200(1.03)^2 + 200(1.03) = 418.18$. At the start of year 30, for example, that original \$200 has been compounded a total of 29 times, while the last \$200 deposited has been compounded 0 times. Figuring out how much is in the account 30 years after the first deposit means adding up $200(1.03)^{29} + 200(1.03)^{28} + \ldots + 200(1.03) + 200$. We can use the formula for the sum of a geometric sequence, $s = a\frac{(1-r^n)}{(1-r)}$, to find the total amount in the account. The sequence starts at $a = 200$ and increases at a rate of $r = 1.03$ each year. After n years, the total s in the account is $s = 206\frac{(1-1.03^n)}{(1-1.03)}$. Now we have a simpler expression to evaluate for different n-values. It turns out that when $n = 31$, the account has about \$10,301 in it, and when $n = 47$, it has about \$20,682 in it.

