Unit 5, Lesson 9

Classifying with Slope

Goals

- Describe a figure in the coordinate plane precisely, using slopes and distances to categorize the figure, including right triangles, rectangles, squares, and parallelograms.
- Use coordinates to calculate the perimeter and area of a figure.

Learning Targets

- I can use coordinates to classify quadrilaterals and triangles and to compute their perimeter and area.

Lesson Narrative

In this lesson, students apply their work with perpendicular slopes to categorize triangles and quadrilaterals. They use slope and distance arguments to describe the type of quadrilateral that a set of coordinates determine. Using slope, students determine whether or not adjacent sides are right angles and opposite sides are parallel. They also apply the Pythagorean Theorem to determine if a figure is equilateral, and then use these calculations to more precisely describe the quadrilateral. Students use similar reasoning to categorize a set of triangles and to craft arguments to describe whether a triangle is a right triangle or not (MP3).

Standards

Addressing	**HSG-GPE.B.4, HSG-GPE.B.5, HSG-GPE.B.7**
Building Toward	**HSG-GPE.B.4**

Instructional Routines

- Card Sort
- Draw It
- MLR8: Discussion Supports
- Take Turns
- Which Three Go Together?

Required Materials

Materials to Gather

- Graph paper: Activity 2

Materials to Copy

- Triangle Types Cards (1 copy for every 2 students): Activity 3

Student Facing Learning Goals

Let's categorize some quadrilaterals and triangles.

9.1 Which Three Go Together: Coordinate Quadrilaterals

5 min

Warm-up

Activity Narrative

This *Warm-up* prompts students to compare the graphs of four quadrilaterals. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Standards

Building Toward **HSG-GPE.B.4**

Instructional Routines

- Which Three Go Together?

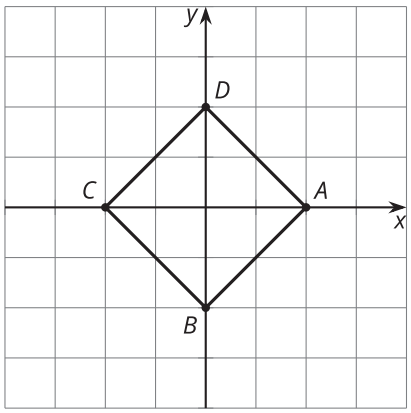
Launch

Arrange students in groups of 2–4. Display the figures. Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three figures that go together and can explain why they go together. Next, tell students to share their response with their group and then together to find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A



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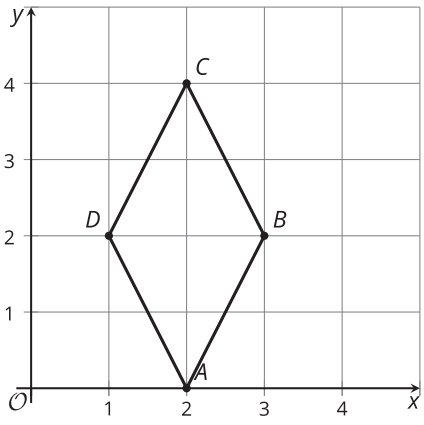

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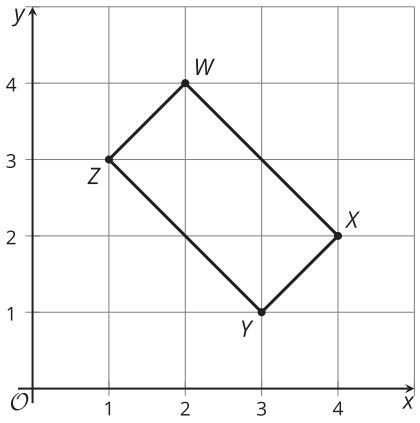

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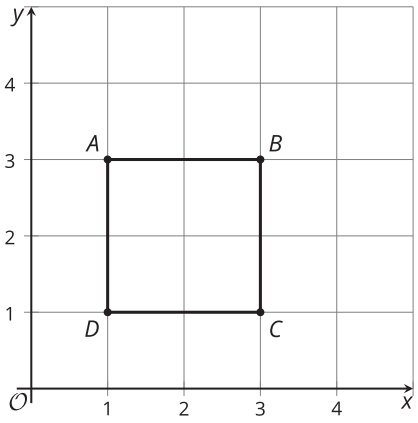


Student Response

Sample responses:

A, B, and C go together because:

- The sides are all diagonal.

A, B, and D go together because:

- The figures are equilateral.
- They have perpendicular diagonals.
- The points are labeled A, B, C, and D.

A, C, and D go together because:

- The figures have right angles.
- They have letters in alphabetical order clockwise.

B, C, and D go together because:

- There are numbers on the grid.
- The axes show only the first quadrant.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure that the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "equilateral" or "diagonal," and to clarify their reasoning, as needed. Consider asking:

- "How do you know _____?"
- "What do you mean by _____?"
- "Can you say that in another way?"

If possible, leave the list of responses displayed until the end of class. Students will return to these images during the *Lesson Synthesis.*

9.2 Name This Quadrilateral

15 min

Activity Narrative

Students are presented with four points and are asked to fully describe the quadrilateral that has those points as its vertices. Slopes will show that all pairs of adjacent sides are perpendicular, making the shape a rectangle. Then students will use the Pythagorean Theorem to calculate both the area and the perimeter of the quadrilateral.

Making dynamic geometry software available gives students an opportunity to choose appropriate tools strategically (MP5).

Standards

Addressing **HSG-GPE.B.4, HSG-GPE.B.7**

Instructional Routines

- Draw It
- MLR8: Discussion Supports

Launch

Make graph paper available to students who would like to use it.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Provide explanations and examples of the different types of quadrilaterals, including parallelogram, rectangle, square, and rhombus, for students to use as a reference.
Supports accessibility for: Memory, Organization

Student Task Statement

A quadrilateral has vertices $(0, 0)$, $(4, 3)$, $(13, -9)$, and $(9, -12)$.

1. What type of quadrilateral is it? Explain or show your reasoning.
2. Find the perimeter of this quadrilateral.
3. Find the area of this quadrilateral.

Student Response

1. Sample response: The quadrilateral is a rectangle. The slopes are $\frac{3}{4}$, $\frac{3}{4}$, $-\frac{4}{3}$, and $-\frac{4}{3}$, so it has four right angles.
2. 40 units
3. 75 square units

Building on Student Thinking

Some students may state that the quadrilateral is a rectangle simply because it looks like one. Remind these students that we need to back up our reasoning with mathematics. Suggest that students review their reference charts for definitions and properties of rectangles.

Are You Ready for More?

1. A parallelogram has vertices $(0, 0)$, $(5, 0)$, $(-2, 10)$, and $(3, 10)$. Find the area of this parallelogram.
2. Consider a general parallelogram with vertices $(0, 0)$, (a, b), (kb, ka), and $(a - kb, b + ka)$, where a and b are positive, and a scale factor of k. Show that the parallelogram is a rectangle, then write an expression for its area in terms of a, b, and k.

Extension Student Response

1. 50 square units
2. The slopes are $\frac{b}{a}$, $\frac{b}{a}$, $-\frac{a}{b}$, and $-\frac{a}{b}$, so opposite sides are parallel and adjacent sides are perpendicular. This means that the parallelogram is a rectangle. The area is $|k|(a^2 + b^2)$, which can be found by multiplying the base of $\sqrt{a^2 + b^2}$ and the height of $\sqrt{(-kb)^2 + (ka)^2}$.

Activity Synthesis

Invite students to share their reasoning for each question. Highlight students who carried information from one question to the next, such as recognizing that in a rectangle, opposite sides have equal length, so they only need to calculate two distances (rather than all four).

Access for English Language Learners

MLR8 Discussion Supports. Create a visual display of the quadrilateral. As students share their strategies, annotate the display to illustrate connections. For example, next to each side, write its slope and its length. This will help students justify why the quadrilateral is a rectangle and the calculations of the area and perimeter. *Advances: Speaking, Representing*

9.3 Card Sort: Triangle Types

15 min

Activity Narrative

This activity gives students the opportunity to apply their work with distance and slope to triangles on the coordinate plane. The triangle in each figure uses the same coordinates for the longest side, allowing students to focus on the relationships between the smaller sides, including slope and side lengths.

In this partner activity, students take turns deciding whether a triangle is a right triangle or not. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and to critique the reasoning of others (MP3).

Standards

Addressing **HSG-GPE.B.4, HSG-GPE.B.5**

Instructional Routines

- Card Sort
- MLR8: Discussion Supports
- Take Turns

Launch

Tell students that the cards contain either a right triangle or a non-right triangle and that they will take turns categorizing the cards. Explain how to set up and participate in the activity. If time allows, demonstrate these steps with a student as a partner:

- Mix up the cards, and place them face up.
- Each person selects one card and silently determines whether or not it is a right triangle.
- Partners take turns being the "explainer" and the "listener."
- One person explains to the other why the triangle on the selected card is or is not a right triangle.
- The listener's job is to listen carefully and to determine whether or not the explainer is correct. If the partners don't agree, they discuss the information until they come to an agreement.
- When the partners agree on the category for the explainer's card, they switch roles.
- After both partners have explained their card, they each select a new card and repeat the process.

Consider demonstrating productive ways to agree or disagree, for example, by explaining mathematical thinking or asking clarifying questions.

Arrange students in groups of 2. Give each group a set of 8 cards cut from the blackline master.

MLR8 Discussion Supports. Students should take turns sorting their cards and explaining their reasoning to their partner. Display the following sentence frame for all to see: "I noticed _____, so I categorized the triangle as a _____" Encourage students to respectfully challenge each other when they disagree.

Student Task Statement

Your teacher will give you a set of cards. You and your partner will take turns sorting the cards, by categorizing the triangle on each card as either a right triangle or a non-right triangle.

1. For each card that you sort, explain to your partner how you know it belongs in that category.
2. For each card that your partner sorts, listen carefully to your partner's explanation. If you and your partner disagree, discuss your thinking, and work to reach an agreement.

Student Response

Cards A, B, D, E, and G are right triangles.

Building on Student Thinking

If students explain to their partner that a triangle "looks like" a right triangle, invite them to use evidence to verify their findings. Some questions that may help students think of ways to provide evidence include:

- "How could you use slope to determine if there is a right angle?"
- "How could you use the Pythagorean Theorem to determine if it is a right triangle?"
- "How could you compare it to another triangle to decide?"

Activity Synthesis

After all groups have completed the *Card Sort*, discuss:

- "Which matches were tricky? Explain why."
- "Did you need to make adjustments in your matches? What might have caused an error? What adjustments were made?"

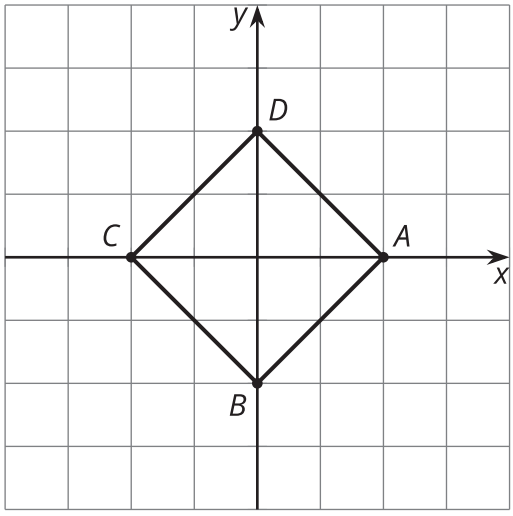
The purpose of this discussion is to make connections between slope and right triangles in the coordinate plane. Select 2–3 groups to share one of their cards and how they decided whether it was a right triangle. Discuss as many different cards as the time allows. Make sure that students explain that the slopes of the sides of the right angle in a right triangle are opposite reciprocals, and that the slopes of the relevant sides of a non-right triangle are not.

Invite 1–2 students or groups to share how they verified their findings for each triangle type. If no students suggest it, ask how they could use slopes or the Pythagorean Theorem to verify whether each triangle is right or not.

Lesson Synthesis

Display the images from the *Warm-up* again.

A



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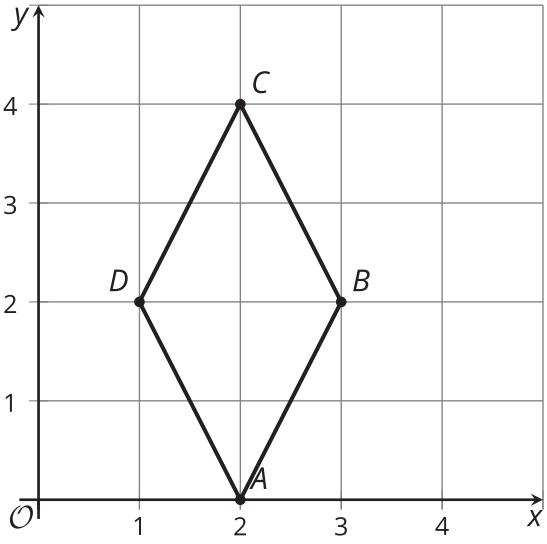

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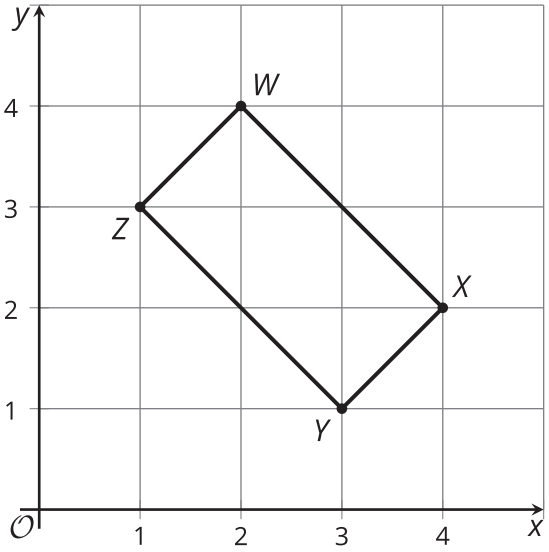

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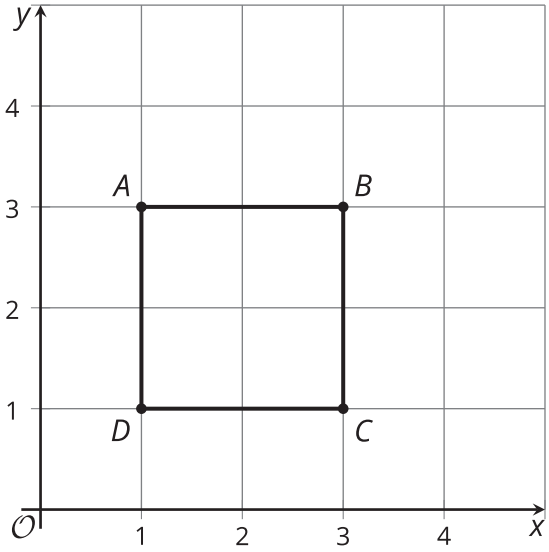


Arrange students in groups of 2–4. Invite students to each choose a different quadrilateral and to classify the quadrilateral as precisely as they can. Then ask them to calculate their quadrilateral's area and perimeter. Suggest that they refer to the list of their responses from the *Warm-up* for ideas of properties to use in their classifications.

After a few minutes of quiet work time, invite students to share, with their group, some of their classifications and reasons. Ask the other group members to listen and critique the reasoning of the person who is sharing. Repeat as time allows.

Sample responses:

- Quadrilaterals A and D are squares. For Quadrilateral A, the slopes show that the sides are perpendicular, and the Pythagorean Theorem shows that the sides are all congruent. For Quadrilateral D, the sides are aligned with the coordinate grid lines, so it is easy to see the 90-degree angles and side measurements.
- Quadrilateral B is a rhombus because its sides are congruent. It is not a square because its adjacent sides aren't perpendicular.
- Quadrilateral C is a rectangle. Its adjacent sides are perpendicular.
- Quadrilaterals A, B, and D have right angles, with a pair of sides that have slopes that are opposite reciprocals.
- All of the quadrilaterals have 2 pairs of parallel sides, and we know that the sides are parallel because they have equal slopes.

	area (square units)	perimeter (units)
A	8	$4\sqrt{8}$ or about 11.3
B	4	$4\sqrt{5}$ or about 8.9
C	4	$2(\sqrt{2}+\sqrt{8})$ or about 8.5
D	4	8

9.4 Name That Quadrilateral

Cool-down

5 min

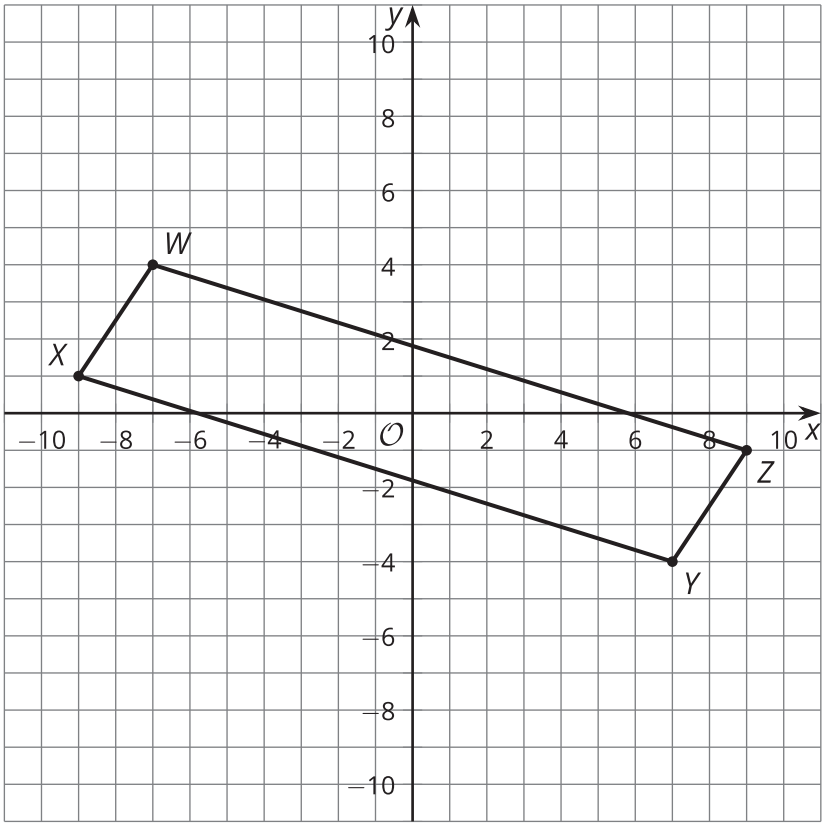
Standards

Addressing **HSG-GPE.B.4**

Student Task Statement

Diego claims that he can tell, using slopes, that this quadrilateral is a parallelogram. Noah looks at the slopes Diego calculated and says he could be even more specific and call it a rectangle.

Do you agree with either of them? Explain or show your reasoning.



Student Response

Sample response: I agree with Diego. The shape is a parallelogram. The slopes of segments WX and YZ are each $\frac{3}{2}$. The slopes of segments XY and WZ are each $-\frac{5}{16}$. Both sets of opposite sides are parallel because their slopes are the same. The adjacent sides are not perpendicular because the product of their slopes is not -1, so it is not a rectangle.

Responding to Student Thinking

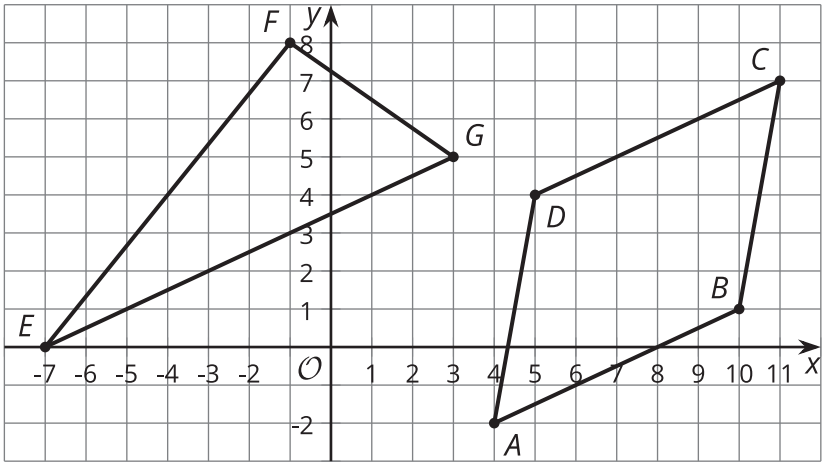
Press Pause

By this point in the unit, there should be some student mastery of perpendicular and parallel lines. If most students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help

students re-engage with earlier work.

Integrated Math 1, Unit 5, Section B Theorems about Lines

Lesson 9 Summary



What can we tell about each of these shapes? We can use slopes to check whether or not quadrilateral $ABCD$ has two pairs of parallel line segments. Sides AB and CD each have a slope of $\frac{1}{2}$. Sides BC and DA both have a slope of 6. We can also tell that it does not have any right angles because $\frac{1}{2}$ and 6 are not opposite reciprocals. So, we can tell that it is a parallelogram but not a rectangle.

Next, we can use the Pythagorean Theorem to see the lengths of each side. The lengths of segments AB and CD are $\sqrt{45}$ units, and the lengths of segments BC and DA are $\sqrt{37}$ units. All side lengths are between 6 and 7 units long, but they are not exactly the same. This means that quadrilateral $ABCD$ is a parallelogram, but not a rhombus or a square.

Can we find the area of triangle EFG? That seems tricky, because we don't know the height of the triangle using EG as the base. However, angle EFG seems like it could be a right angle. In that case, we could use sides EF and FG as the base and height.

To see if EFG is a right angle, we can calculate slopes. The slope of EF is $\frac{8}{6}$ or $\frac{4}{3}$, and the slope of FG is $-\frac{3}{4}$. Since the slopes are opposite reciprocals, the segments are perpendicular, and angle EFG is indeed a right angle. This means that we can think of EF as the base and FG as the height. The length of EF is 10 units, and the length of FG is 5 units. So the area of triangle EFG is 25 square units because $\frac{1}{2} \cdot 10 \cdot 5 = 25$.

Lesson 9 Practice Problems

1 Student Task Statement

A quadrilateral has vertices $A(0, 0)$, $B(1, 3)$, $C(0, 4)$, and $D(-1, 1)$. Prove that $ABCD$ is a parallelogram.

Solution

Sample response: Side AB is parallel to side CD because both have slopes of 3. Side AD is parallel to side BC because both have slopes of -1. A quadrilateral with 2 pairs of parallel sides is a parallelogram.

2 Student Task Statement

A rhombus has vertices at $(0, 0)$, $(5, 0)$, $(3, 4)$, and $(8, 4)$.

a. Find the slopes of the 2 diagonals of the rhombus.

b. What do the slopes tell you about the diagonals in this rhombus?

Solution

a. -2, $\frac{1}{2}$

b. Sample response: Since the product of the slopes is -1, the diagonals are perpendicular.

3 Student Task Statement

a. Show that the triangle formed with vertices at $(0, 0)$, $(4, 3)$, and $(-2, 11)$ is a right triangle.

b. Find the area of the triangle.

Solution

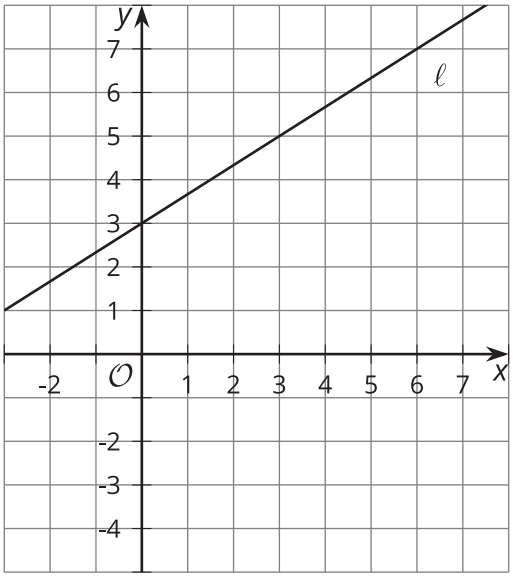
a. Sample response: The slope of one side is $\frac{3}{4}$ and another side has slope $-\frac{4}{3}$, so they form a right angle.

b. The area is 25 square units.

4 from Unit 5, Lesson 7

Student Task Statement

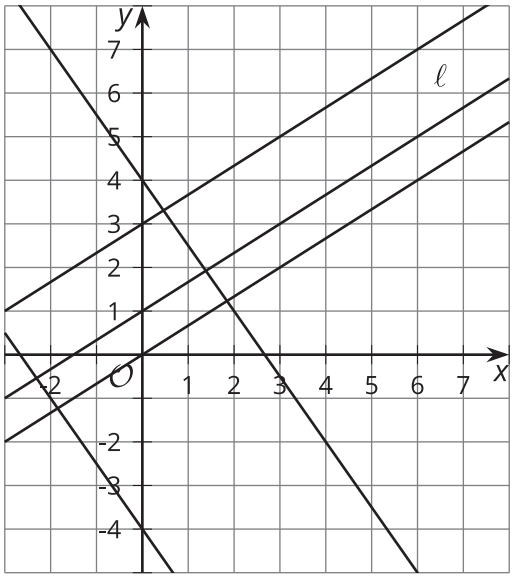
Here is a line ℓ. Write equations for and graph 2 different lines perpendicular to ℓ and 2 different lines parallel to ℓ.



Solution

Sample response:

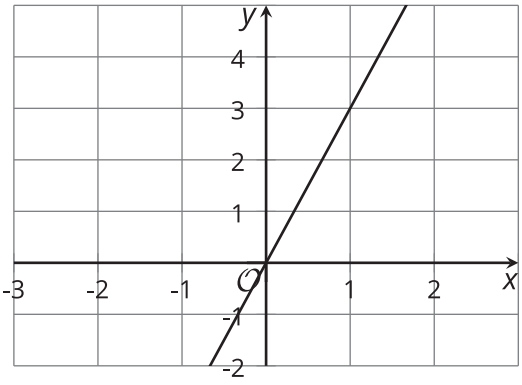
- perpendicular lines: $2y + 3x = 8$, $y = -1.5x - 4$
- parallel lines: $y = \frac{2}{3}x + 1$, $y = \frac{2}{3}x$



5 from Unit 5, Lesson 6

Student Task Statement

The line shown is rotated 90 degrees clockwise around the origin. What is the slope of its image?



Solution

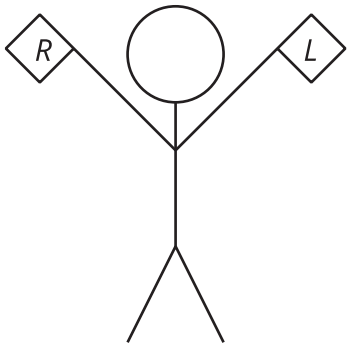
$-\frac{1}{3}$

6 from Unit 1, Lesson 13

Student Task Statement

The semaphore alphabet is a way to use flags to signal messages. Here's how to signal the letter U. Describe a transformation that would take the left hand flag to the right hand flag.

the letter U in semaphore



Solution

Sample response: Reflect Figure L across the vertical line.

Sample response: Rotate Figure L 90 degrees counterclockwise around the point of intersection of the arms. Reflect across the right arm.