



# The Number $e$

Let's learn about the number  $e$ .

## 12.1 Matching Situations and Equations

Match each equation to a situation it represents. Be prepared to explain how you know. Not all equations have a match.

$$f(t) = 400 \cdot (0.5)^{0.1t}$$

$$j(t) = 400 \cdot (2)^{10t}$$

$$g(t) = 400 \cdot (1.25)^{0.1t}$$

$$k(t) = 400 \cdot (2)^{0.1t}$$

$$h(t) = 400 \cdot (0.75)^{0.1t}$$

1. A scientist begins an experiment with 400 bacteria in a petri dish. The population doubles every 10 hours. The function gives the number of bacteria  $t$  hours since the experiment began.
2. An erosion model begins with 400 kilograms of sand. The amount of sand decreases by 25% every 10 days after the experiment begins. The function gives the amount of sand left  $t$  days after the experiment begins.
3. The half-life of a radioactive element is 10 years. There are 400 g of the element in a sample when it is first studied. The function gives the amount of the element remaining  $t$  years later.
4. In a lake, the population of a species of fish is 400. The population is expected to grow by 25% in the next decade. The function gives the number of fish in the lake  $t$  years after it was 400.

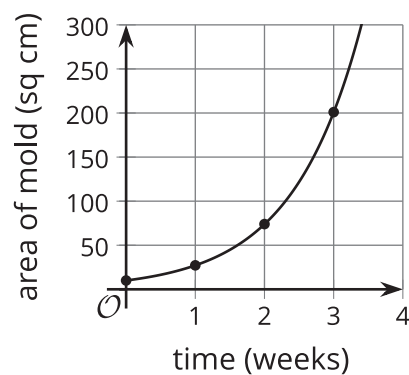


## 12.2

## Notice and Wonder: Moldy Growth

A spot of mold is found on a basement wall. Its area is about 10 square centimeters. Here are three representations of a function that models how the mold is growing.

$$a(t) = 10 \cdot e^t$$



| time (weeks) | area of mold (sq cm) |
|--------------|----------------------|
| 0            | 10                   |
| 1            | 27                   |
| 2            | 74                   |
| 3            | 201                  |
| 4            | 546                  |

What do you notice? What do you wonder?

## 12.3 $(1 + \text{tiny})^{\text{huge}}$

1. Here are some functions. For each function, describe, in words, the outputs for very tiny, positive values of  $x$  and for very large values of  $x$ .

$$a(x) = 1^x$$

$$b(x) = -x$$

$$d(x) = \frac{1}{x}$$

$$f(x) = \left(\frac{1}{x}\right)^x$$

$$g(x) = \left(1 + \frac{1}{x}\right)^x$$

$$h(x) = e^x$$

$$k(x) = 1 + x$$

2. Remember that  $e \approx 2.718$ . What does the function  $g$  have to do with the number  $e$ ?
3. What do you notice about the relationship between  $h$  and  $k$  for very small, positive values of  $x$ ?



### Are you ready for more?

Complete the table to show the value of each expression to the nearest hundred-thousandth. Two entries have already been completed as an example.

| $x$    | $2^x$   | $e^x$   | $3^x$ | $1 + x$ |
|--------|---------|---------|-------|---------|
| 0.1    | 1.07177 | 1.10517 |       |         |
| 0.01   |         |         |       |         |
| 0.001  |         |         |       |         |
| 0.0001 |         |         |       |         |

What do you notice about the values in the table? Which columns are the closest in value?

### Lesson 12 Summary

Scientists, economists, engineers, and others often use the number  $e$  in their mathematical models. What is  $e$ ?

$e$  is an important constant in mathematics, just like the constant  $\pi$ , which is important in geometry. The value of  $e$  is approximately 2.718. Like  $\pi$ , the number  $e$  is irrational, so it can't be represented as a fraction, and its decimal representation never repeats or terminates. The number is named after the 18th-century mathematician Leonhard Euler and is sometimes called *Euler's number*.

$e$  has many useful properties and it arises in situations involving exponential growth or decay, so  $e$  often appears in exponential functions.