Solving Any Linear Equation

Let's solve linear equations.

8.1 Math Talk: Equation Solving

Solve each equation mentally.

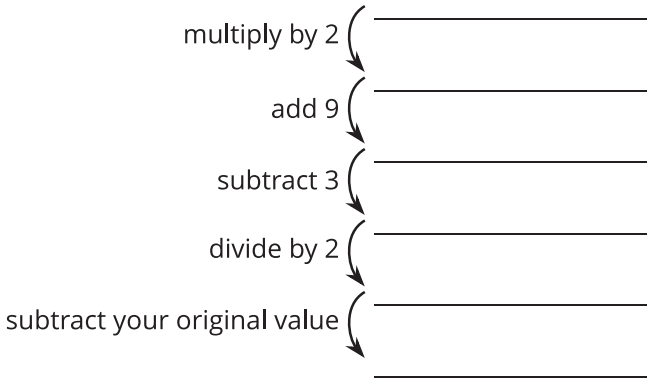
- $5 - x = 8$
- $-1 = x - 2$
- $-3x = 9$
- $-10 = -5x$

8.2 Trading Moves

Here are 4 problems. Select 2 to solve with your partner by taking turns describing a move, then writing an equivalent equation. For the other 2 problems, you and your partner should each solve 1 of the problems on your own, and then trade to check your answers.

$-6a - 7 = 4a - 2$ $\frac{1}{2}(7b - 6) = 6b - 10$ $\frac{1}{2}c + 7 = c + 13$ $2(d + 7) = -4d + 14$

8.3 A Number Puzzle



Are you ready for more?

Write another number puzzle with at least three steps.

Trade puzzles with a partner and solve theirs. Can you figure out how it works?

Compare your solutions to each puzzle. Did they solve them the same way you did?

8.4 A Puzzling Puzzle

Tyler says he invented a number puzzle. He asks Clare to pick a number, and then asks her to:

- Triple the number.
- Subtract 7.
- Double the result.
- Subtract 22.
- Divide by 6.

Clare says she now has -3. Tyler says her original number must have been a 3. How did Tyler know that?

Follow the same instructions starting with x instead of a number. Explain or show your reasoning for why the last expression means that the person started with a number 6 greater than they ended with.

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Lesson 8 Summary

When we have an equation in one variable, there are many different ways to solve it. We generally want to make moves that get us closer to an equation that clearly shows the value that makes the equation true.

For example, $x = 5$ or $t = \frac{7}{3}$ show that 5 and $\frac{7}{3}$ are solutions. Because there are many ways to do this, it helps to choose moves that leave fewer terms or factors.

If we have an equation like $3t + 5 = 7$, adding -5 to each side will leave us with fewer terms. The equation then becomes $3t = 2$.

Dividing each side of this equation by 3 results in the equivalent equation $t = \frac{2}{3}$, which is the solution.

Or, if we have an equation like $4(5 - a) = 12$, dividing each side by 4 will leave us with fewer factors on the left. The equation then becomes $5 - a = 3$.

Here is a list of valid moves that can help create equivalent equations that move toward a solution:

1. Use the distributive property so that all the expressions no longer have parentheses.
2. Collect like terms on each side of the equation.
3. Add or subtract an expression on each side so that there is a variable on just one side.
4. Add or subtract an expression on each side so that there is just a number on the side without the variable.
5. Multiply or divide by a number on each side so that the variable on one side of the equation has a coefficient of 1.

For example, suppose we want to solve $9 - 2b + 6 = -3(b + 5) + 4b$.

Use the distributive property	$9 - 2b + 6 = -3b - 15 + 4b$
Combine like terms	$15 - 2b = b - 15$
Add $2b$ to each side	$15 = 3b - 15$
Add 15 to each side	$30 = 3b$
Divide each side by 3	$10 = b$

From lots of experience, we learn when to use different valid moves that help solve an equation.