Different Forms

Let's use the different forms of polynomials to learn about them.

6.1 Which Three Go Together: Small Differences

Which three go together? Why do they go together?

A

$$y=(x+4)(x-6)$$

B

$$y=2x^2-8x-24$$

C

$$y=x^2+5x-25$$

D

$$y=x^3+3x^2-10x-24$$

6.2 The Return of the Box

We can make a box from a piece of paper that is 8.5 inches by 14 inches by cutting squares of side length x from each corner and then folding up the sides. The volume V, in cubic inches, of the box is a function of the side length x, where $V(x)=(14-2x)(8.5-2x)(x)$.

1. Identify the degree and leading term of the polynomial. Explain or show your reasoning.

2. Without graphing, what can you say about the horizontal and vertical intercepts of the graph of V? Do these points make sense in this situation?

6.3 Using Diagrams to Multiply

1. Use the distributive property to show that each pair of expressions is equivalent.

   a. $(x+2)(x+4)$ and x^2+6x+8

   b. $(x+6)(x+\text{-}5)$ and x^2+x-30

   c. $(x^2+10x+7)(2x-1)$ and $2x^3+19x^2+4x-7$

   d. $(4x^3-8)(x^2+3)$ and $4x^5+12x^3-8x^2-24$

6.4 Spot the Differences

Let $f(x) = (x - 2)(x + 3)(x - 7)$ and $g(x) = \frac{1}{2}(x - 2)(x + 3)(x - 7)$.

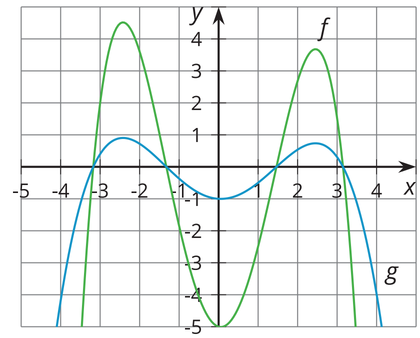
1. Use graphing technology to graph both functions in the same window of $-10 \leq x \leq 10$ and $-100 \leq y \leq 100$. Describe how the two graphs are the same and how they are different.

2. What degree do these polynomials have? Rewrite each expression in standard form to check.

3. Let $h(x) = (3x - 6)(x + 3)(x - 7)$. What do you think the graph of $y = h(x)$ will look like compared to $y = f(x)$? Use graphing technology to check your prediction.

Are you ready for more?

Here are the graphs of two polynomial functions, f and g. We know that $g(x) = k \cdot f(x)$.



1. Why do the two graphs have different vertical intercepts but the same horizontal intercepts?

2. What is the value of k?

Lesson 6 Summary

We can express polynomials in different, yet equivalent, algebraic forms that each give us different information about features of the polynomial and its graph. Earlier, we saw how expressing a polynomial function in factored form is helpful for identifying zeros. The expanded version of factored form, or standard form of a polynomial, is helpful for identifying the constant term and the degree.

For example, here are the expressions for a polynomial P written in factored form and standard form:

$$P(x) = 0.25(x-1)^2(x+2)(x-3)(x+3)$$

$$P(x) = 0.25x^5 - 3x^3 + 0.5x^2 + 6.75x - 4.5$$

The constant term, seen as -4.5 in the example, tells us the value of the function when $x = 0$. In a graph of the function, this point is known as the vertical intercept.

The degree, seen as 5 in the example, tells us about the general shape of the graph of the polynomial, which we'll learn more about in future lessons.

