

Polynomial Division (Part 2)

Let's learn a different way to divide polynomials.

13.1 Notice and Wonder: Different Divisions

What do you notice? What do you wonder?

$$\begin{array}{r} 2 \\ 11 \overline{) 2772} \\ \underline{22} \\ 5 \end{array}$$

$$\begin{array}{r} 25 \\ 11 \overline{) 2772} \\ \underline{22} \\ 57 \\ \underline{55} \\ 2 \end{array}$$

$$\begin{array}{r} 252 \\ 11 \overline{) 2772} \\ \underline{22} \\ 57 \\ \underline{55} \\ 22 \\ \underline{22} \\ 0 \end{array}$$

$$\begin{array}{r} 2x^2 \\ x+1 \overline{) 2x^3 + 7x^2 + 7x + 2} \\ \underline{-2x^3 - 2x^2} \\ 5x^2 + 7x \end{array}$$

13.2 Polynomial Long Division

1. Diego used the long division shown here to figure out that $6x^2 - 7x - 5 = (2x + 1)(3x - 5)$. Show what it would look like if he had used a diagram.

$$\begin{array}{r} 3x - 5 \\ 2x + 1 \overline{) 6x^2 - 7x - 5} \\ \underline{-6x^2 - 3x} \\ -10x - 5 \\ \underline{10x + 5} \\ 0 \end{array}$$

| | | |
|----|-----------------|--|
| | | |
| 2x | 6x ² | |
| 1 | | |

Pause here for a whole-class discussion.

2. $(x - 2)$ is a factor of $2x^3 - 7x^2 + x + 10$, which means there is some other factor A where $2x^3 - 7x^2 + x + 10 = (x - 2)(A)$. Finish the division started here to find the value of A .

$$\begin{array}{r} 2x^2 \\ x - 2 \overline{) 2x^3 - 7x^2 + x + 10} \\ \underline{-2x^3 + 4x^2} \end{array}$$

3. Jada used the diagram shown here to figure out that $2x^3 + 13x^2 + 16x + 5 = (2x + 1)(x^2 + 6x + 5)$. Show what it would look like if she had used long division.

| | | | |
|------|--------|---------|-------|
| | x^2 | $6x$ | 5 |
| $2x$ | $2x^3$ | $12x^2$ | $10x$ |
| 1 | x^2 | $6x$ | 5 |

$$2x + 1 \overline{) 2x^3 + 13x^2 + 16x + 5}$$

Are you ready for more?

1. What is $(x^4 - 1) \div (x - 1)$?

2. Use your response to predict what $(x^7 - 1) \div (x - 1)$ is, and then use division to check your prediction.

13.3 More Long Division

Here are some polynomial functions with known factors. Rewrite each polynomial as a product of linear factors using long division.

1. $A(x) = x^3 - 7x^2 - 16x + 112, (x - 7)$

$$\begin{array}{r} x^2 \\ x - 7 \overline{) x^3 - 7x^2 - 16x + 112} \\ \underline{-x^3 + 7x^2} \end{array}$$

2. $C(x) = x^3 - 3x^2 - 13x + 15, (x + 3)$



13.4 Missing Numbers

Here are pairs of equivalent expressions, one in standard form and the other in factored form. Find the missing numbers.

1. $x^2 + 9x + 14$ and $(x + 2)(x + \boxed{})$

2. $x^2 - 9x + 20$ and $(x - \boxed{})(x - \boxed{})$

3. $2x^2 + 2x - 24$ and $2(x + \boxed{})(x - 3)$

4. $\boxed{}x^3 + 11x^2 - 17x + 6$ and $(-x + 3)(2x - 1)(x - 2)$

5. $6x^3 + 2x^2 - 16x + 8$ and $(x - 1)(2x + 4)(\boxed{}x - 2)$

6. $2x^3 + 7x^2 - 7x - 12$ and $(2x - 3)(x + \boxed{})(x + \boxed{})$

7. $x^3 + 6x^2 + \boxed{}x - 10$ and $(x + 2)(x - 1)(x + \boxed{})$



Lesson 13 Summary

In earlier grades, we learned that one way to divide numbers, like 1,573 divided by 11, is by using long division.

$$\begin{array}{r} 1 \\ 11 \overline{) 1573} \\ \underline{11} \\ 4 \end{array}$$

$$\begin{array}{r} 14 \\ 11 \overline{) 1573} \\ \underline{11} \\ 47 \\ \underline{44} \\ 3 \end{array}$$

$$\begin{array}{r} 143 \\ 11 \overline{) 1573} \\ \underline{11} \\ 47 \\ \underline{44} \\ 33 \\ \underline{33} \\ 0 \end{array}$$

Here the division has been completed in stages, focusing on the highest power of 10 (1,000) in the dividend 1,573, and working down. This long division shows that $1,573 = (11)(143)$.

Similar to integers, we can also use long division on polynomials. Instead of focusing on powers of 10, in polynomial long division we focus on powers of x . Just as we started with the highest power or 10, we start with the highest power of x , the leading term, and work down to the constant term. For example, here is $x^3 + 5x^2 + 7x + 3$ divided by $x + 1$ completed in three stages. Notice how terms of the same degree are in the same columns.

$$\begin{array}{r} x^2 \\ x+1 \overline{) x^3 + 5x^2 + 7x + 3} \\ \underline{-x^3 - x^2} \\ 4x^2 + 7x \end{array}$$

$$\begin{array}{r} x^2 + 4x \\ x+1 \overline{) x^3 + 5x^2 + 7x + 3} \\ \underline{-x^3 - x^2} \\ 4x^2 + 7x \\ \underline{-4x^2 - 4x} \\ 3x + 3 \end{array}$$

$$\begin{array}{r} x^2 + 4x + 3 \\ x+1 \overline{) x^3 + 5x^2 + 7x + 3} \\ \underline{-x^3 - x^2} \\ 4x^2 + 7x \\ \underline{-4x^2 - 4x} \\ 3x + 3 \end{array}$$

At each stage, the focus is only on the term with the largest exponent that's left. At the conclusion, we can see that $x^3 + 5x^2 + 7x + 3 = (x + 1)(x^2 + 4x + 3)$.