Unit 9, Lesson 13

Designing Districts

Goals

- Compare and contrast different ways to distribute representatives.
- Describe (using words and other representations) how changing the way votes are grouped can affect the outcome.

Lesson Narrative

In this optional lesson, students investigate how districts can be designed to influence the outcome of an election. Students are asked to gerrymander several districts: to divide them into sections to influence the final voting result in opposite ways. The mathematics here involves geometric properties of shapes on maps: area and connectedness, as well as some proportional reasoning.

Most of the activities use students' skills from earlier units to reason abstractly and quantitatively about ratios and proportional relationships in the context of real-world problems (MP2). While some of the activities do not involve much computation, they all require serious thinking and decision making as students construct arguments and justify their plans (MP3).

Standards

Addressing **6.RP.A.3, 6.RP.A.3.c**

Instructional Routines

- MLR3: Critique, Correct, Clarify

Student Facing Learning Goals

Let's design districts.

13.1 School Mascot (Part 2)

20 min

Activity Narrative

In the previous voting activities, representatives ("advisors") were assigned to groups that couldn't be changed: schools. Sometimes the groups or districts for representatives can be changed, as in districts for the U.S. House of Representatives, and for state legislatures, wards in cities, and so on.

In this lesson, students use geometric reasoning about areas and connectedness to experiment with drawing districts in a way that predicts the outcome of elections. This is often called "gerrymandering."

As students try different configurations of districts they reason abstractly and quantitatively about how changing shapes affects outcomes (MP2).

In this activity, students critique a statement or response that is intentionally unclear, incorrect, or incomplete and improve it by clarifying meaning, correcting errors, and adding details (MP3).

Access for English Language Learners

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Standards

Addressing **6.RP.A.3, 6.RP.A.3.c**

Instructional Routines

- MLR3: Critique, Correct, Clarify

Launch

Arrange students in groups of 2–4. If needed, remind students of the school-level vote for the new mascot from an earlier activity.

Access for Students with Disabilities

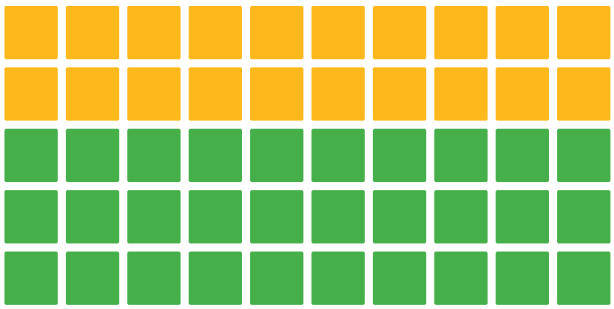
Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide snap cubes for students to build and manipulate the districts they are creating. *Supports accessibility for: Visual-Spatial Processing, Conceptual Processing*

Student Task Statement

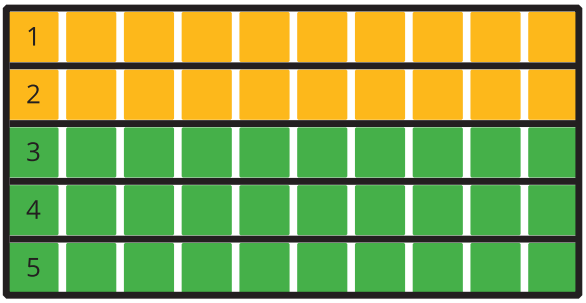
After the school mascot voting, the whole town gets interested in choosing a mascot. The mayor of the town decides to choose representatives to vote.

There are 50 blocks in the town, and the people on each block tend to have the same opinion about which mascot is best. Green blocks like sea lions, and gold blocks like banana slugs. The mayor decides to have 5 representatives, each representing a district of 10 blocks.

Here is a map of the town, with preferences shown.



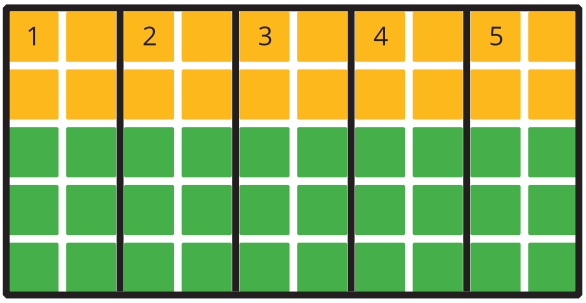
1. Suppose there were an election with each of the 50 blocks getting one vote. How many votes would be for banana slugs? For sea lions? Which mascot would win this election and what percentage of the votes would they get?
2. Suppose the blocks are in districts 1–5, as shown here. What did the people in each district prefer? What did their representative vote? Which mascot would win the election?



Complete the table with this election's results.

district	number of blocks for banana slugs	number of blocks for sea lions	percentage of blocks for banana slugs	representative's vote
1	10	0		banana slugs
2				
3				
4				
5				

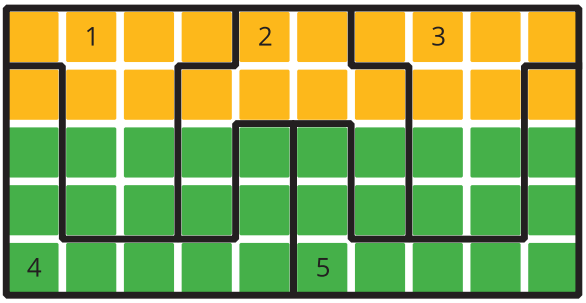
3. Suppose, instead, that the 5 districts are as shown in this new map. What did the people in each district prefer? What did their representative vote? Which mascot would win the election?



Complete the table with this election's results.

district	number of blocks for banana slugs	number of blocks for sea lions	percentage of blocks for banana slugs	representative's vote
1				
2				
3				
4				
5				

4. Suppose the 5 districts are designed in yet another way, as shown in this map. What did the people in each district prefer? What did their representative vote? Which mascot would win the election?



Complete the table with this election's results.

district	number of blocks for banana slugs	number of blocks for sea lions	percentage of blocks for banana slugs	representative's vote
1				
2				
3				
4				
5				

5. Write a headline for the local newspaper for each of the ways of splitting the town into districts.
6. Which systems of the three maps of districts do you think are more fair? Are any totally unfair?

Student Response

1. 20 votes for banana slugs, 30 votes for sea lions, so sea lions win with 60% of the vote.
2. The people in districts 1 and 2 prefer banana slugs while the people in districts 3, 4, and 5 prefer sea lions. Sea lions win with 3 of 5 representatives.

district	number of blocks choosing banana slugs	number of blocks choosing sea lions	percentage of blocks choosing banana slugs	representative's vote
1	10	0	100%	banana slugs
2	10	0	100%	banana slugs
3	0	10	0%	sea lions
4	0	10	0%	sea lions
5	0	10	0%	sea lions

3. All 5 districts have 4 blocks that prefer banana slugs and 6 blocks that prefer sea lions. Because more blocks in each district prefer sea lions, all 5 representatives vote for sea lions. Sea lions win.

district	number of blocks choosing banana slugs	number of blocks choosing sea lions	percentage of blocks choosing banana slugs	representative's vote
1	4	6	40%	sea lions
2	4	6	40%	sea lions
3	4	6	40%	sea lions
4	4	6	40%	sea lions
5	4	6	40%	sea lions

4. The people in each district are divided in their support for each mascot. 3 of the districts (1, 2, and 3) have more support for banana slugs while the other 2 districts (4 and 5) have more support for sea lions, so 3 representatives vote for banana slugs and 2 vote for sea lions. Banana slugs win with 3 of 5 representatives.

district	number of blocks choosing banana slugs	number of blocks choosing sea lions	percentage of blocks choosing banana slugs	representative's vote
1	6	4	60%	banana slugs
2	6	4	60%	banana slugs
3	6	4	60%	banana slugs
4	1	9	10%	sea lions
5	1	9	10%	sea lions

5. Sample responses:

   First map: 60% of Districts and 60% of People Vote for Sea Lions
   Second map: All Districts, but Only 60% of People, Vote for Sea Lions
   Third map: Banana Slugs Win with 60% of Districts, but Only 40% of People

6. Sample response: The first map seems the fairest since the percentages of the people and the representatives match. The second map has the same winner as the vote of the people but different percentages. The third map seems totally unfair: The percentages are reversed. More than half the people voted for sea lions, but banana slugs won.

Activity Synthesis

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response about the fairness of the district maps by correcting errors, clarifying meaning, and adding details.

- Display this first draft:
  "Both maps 1 and 2 are equally fair since 60% of people prefer sea lions. That means they show the same results."

  Ask, "What parts of this response are unclear, incorrect, or incomplete?" As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement. If not brought up by students, highlight the number of districts voting for each option in each map and how that influences the final results of the vote.
- Give students 2–4 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

13.2 Fair and Unfair Districts

30 min

Activity Narrative

Students design districts in three towns to "gerrymander" the results of elections. In two of the towns, the election results can be skewed to either color. In the third, it isn't possible to skew the results. When students design and defend their districts, they construct arguments and critique the reasoning of others (MP3).

Standards

Addressing **6.RP.A.3, 6.RP.A.3.c**

Launch

Arrange students in groups of 2–4.

Explain the history of gerrymandering: Sometimes people in charge of designing districts make them in strange shapes to produce the election results they want. One of the first was Elbridge Gerry (governor of Massachusetts in 1812), whose party designed a district that many people thought looked like a salamander. They called it a *Gerrymander*, and the name stuck. It means a very strangely shaped, spread-out district designed to produce a certain result.

Usually, districts are required to be connected: A person traveling to all parts of the district should be able to stay inside the district. There should be no "islands" that are separated by parts of other districts.

Student Task Statement

1. Smallville's map is shown, with opinions shown by block in green and gold. Decompose the map to create three connected, equal-area districts in two ways:

a. Design three districts in which green will win at least two of the three districts. Record results in Table 1.

Table 1:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

b. Design three districts in which gold will win at least two of the three districts. Record results in Table 2.

Table 2:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

2. Squaretown's map is shown, with opinions by block shown in green and gold. Decompose the map to create five connected, equal-area districts in two ways:

a. Design five districts in which green will win at least three of the five districts. Record the results in Table 3.

Table 3:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				
4				
5				

b. Design five districts in which gold will win at least three of the five districts. Record the results in Table 4.

Table 4:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				
4				
5				

3. Mountain Valley's map is shown, with opinions by block shown in green and gold. (This is a town in a narrow valley in the mountains.) Decompose the map to create 3 connected, equal-area districts in 2 ways.

   a. Design three districts in which green will win at least 2 of the 3 districts. Record the results in Table 5.

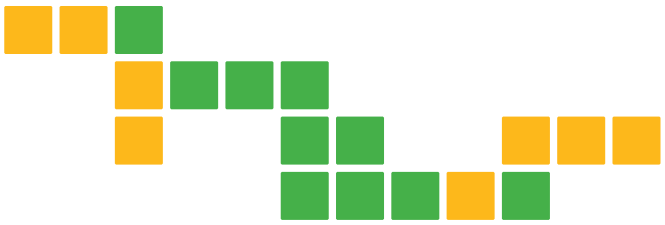


Table 5:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

   b. Design three districts in which gold will win at least 2 of the 3 districts. Record the results in Table 6.

Table 6:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

Student Response

1. a. Sample response:

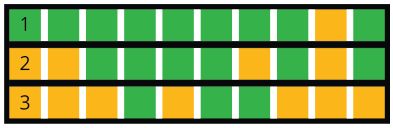


Table 1 shows that green wins.

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1	9	1	90%	green
2	6	4	60%	green
3	3	7	30%	gold

b.

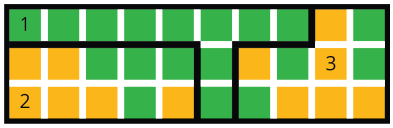


Table 2 shows that gold wins.

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1	10	0	100%	green
2	4	6	40%	gold
3	4	6	40%	gold

2. a. Sample response:

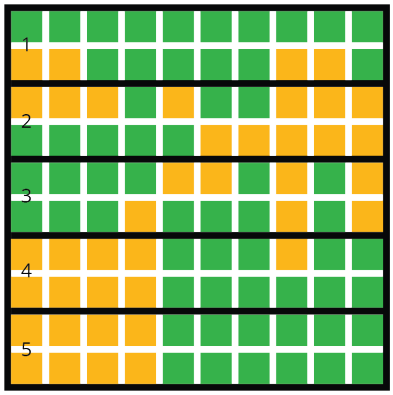


Table 3 shows that green wins.

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1	16	4	80%	green
2	8	12	40%	gold
3	13	7	65%	green
4	11	9	55%	green
5	12	8	60%	green

b.

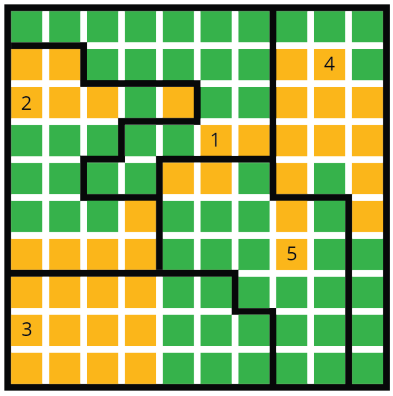


Table 4 shows that gold wins.

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1	18	2	90%	green
2	9	11	45%	gold
3	8	12	40%	gold
4	9	11	45%	gold
5	16	4	80%	green

3. a. It is not possible for three districts to be drawn where green wins at least 2 of the 3 districts. It is impossible to complete table 5.

b.

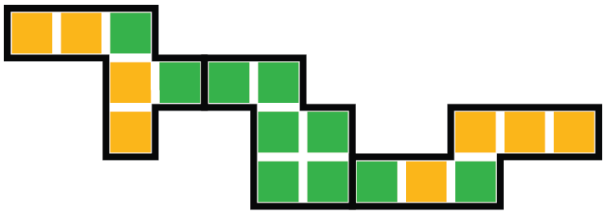


Table 6:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1	2	4	33%	gold
2	6	6	100%	green
3	2	4	33%	gold

Activity Synthesis

The goal of this activity is to understand that it is possible to design fair districts (where the result of the vote is similar to the vote if all individual votes were counted) and unfair districts. Here are some questions for discussion:

- "In the Smallville maps, which map is more fair, the one where green wins or the one where gold wins?" (Green because there are 18 green blocks and 12 gold blocks, so a green result better represents what the people want.)
- "In the Squaretown maps, which map is more fair, the one where green wins or the one where gold wins?" (Green because there are 60 green blocks and 40 gold blocks, so a green result better represents what the people want.)

- "All of the maps we made followed the rules: Districts must be the same size and connected. Are there reasons maps would be made that are unfair?" (If the people designing the maps are biased, they could make maps that favor their choice.)
- "What other rules about drawing maps could we add to make sure the maps are as fair as possible?"