Unit 5, Lesson 5

Representing Subtraction

Let's subtract signed numbers.

5.1 Equivalent Equations

Consider the equation $2 + 3 = 5$. Here are some more equations that express the same relationship in a different way:

$$3 + 2 = 5 \qquad 5 - 3 = 2 \qquad 5 - 2 = 3$$

For each equation, write two more equations that use the same numbers and express the same relationship in a different way.

1. $9 + (-1) = 8$

2. $-11 + x = 7$

5.2 Subtraction with Number Lines

1. Here is an unfinished number line diagram that represents a sum of 8.

   a. How long should the other arrow be?

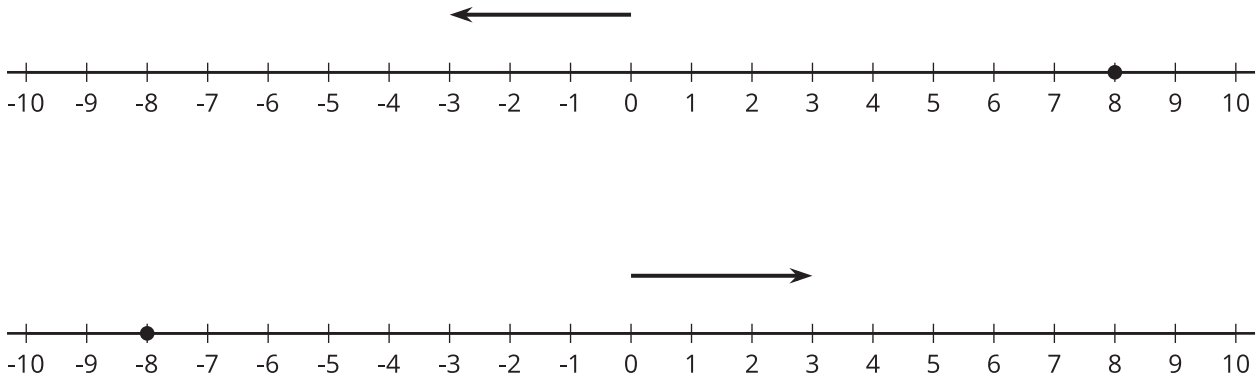
   b. For an equation that goes with this diagram:

      - Mai writes $3 + ? = 8$.
      - Tyler writes $8 - 3 = ?$.

      Do you agree with either of them?

   c. What is the unknown number? How do you know?

2. Here are two more unfinished diagrams that represent sums.



For each diagram:

a. What equation would Mai write if she used the same reasoning as before?

b. What equation would Tyler write if he used the same reasoning as before?

c. How long should the other arrow be?

d. What number would complete each equation? Be prepared to explain your reasoning.

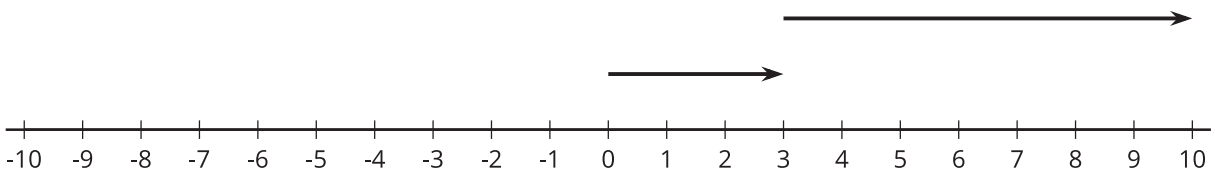
3. Draw a number line diagram for $(\text{-}8) - (\text{-}3) = ?$ What is the unknown number? How do you know?

5.3 We Can Add Instead

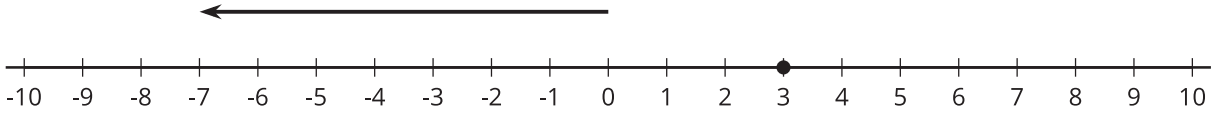
1. Match each diagram to one of these expressions:

$3 + 7$ $3 - 7$ $3 + (-7)$ $3 - (-7)$

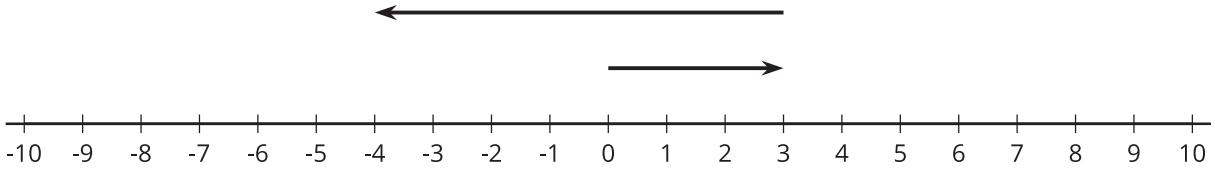
a.



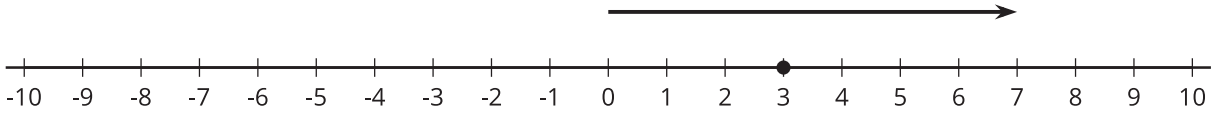
b.



c.



d.



2. Which expressions in the first question have the same value? What do you notice?

Pause here so your teacher can review your work.

3. Which expression has the same value as $8 - 12$?

   A. $-8 + 12$

   B. $8 + -12$

   C. $12 - 8$

   D. $8 - -12$

4. Which expression has the same value as $8 + -5$?

   A. $8 - 5$

   B. $-5 + -8$

   C. $8 - -5$

   D. $8 + 5$

5. Which expression has the same value as $-5 - -9$?

   A. $-9 - -5$

   B. $-9 + 5$

   C. $-5 + -9$

   D. $-5 + 9$

6. Choose one of the three previous problems. Draw and label diagrams to show that the two expressions have the same value.

Are you ready for more?

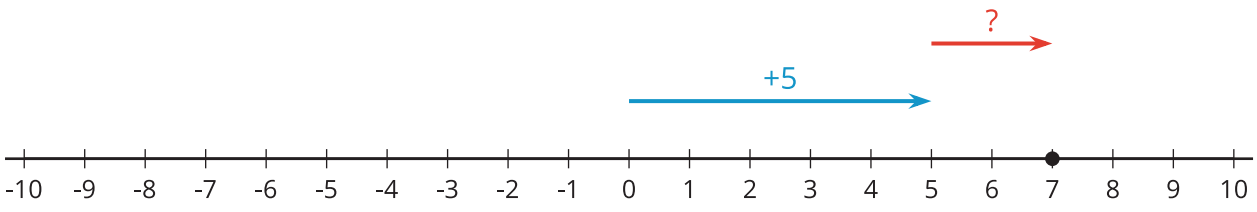
It is possible to make a new number system using *only* the numbers 0, 1, 2, and 3. We will write the symbols for adding and subtracting in this system like this: $2 \oplus 1 = 3$ and $2 \ominus 1 = 1$. The table shows some of the sums.

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3				

1. In this system, $1 \oplus 2 = 3$ and $2 \oplus 3 = 1$. How can you see that in the table?
2. What do you think $3 \oplus 1$ should be?
3. What about $3 \oplus 3$?
4. What do you think $3 \ominus 1$ should be?
5. What about $2 \ominus 3$?
6. Can you think of any uses for this number system?

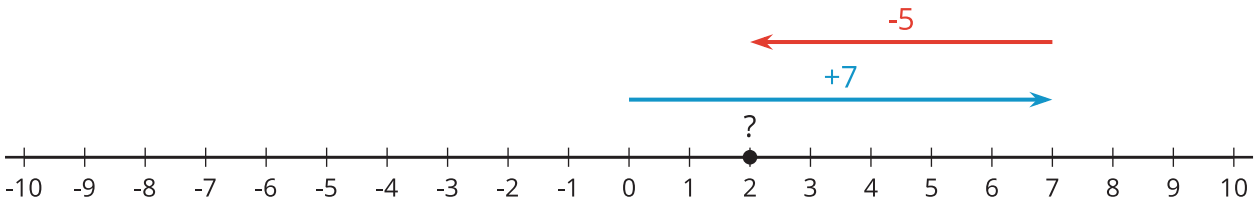
Lesson 5 Summary

We can use the relationship between addition and subtraction to reason about subtracting signed numbers. For example, the equation $7 - 5 = ?$ is equivalent to $5 + ? = 7$. Here is a diagram that represents the addition equation.



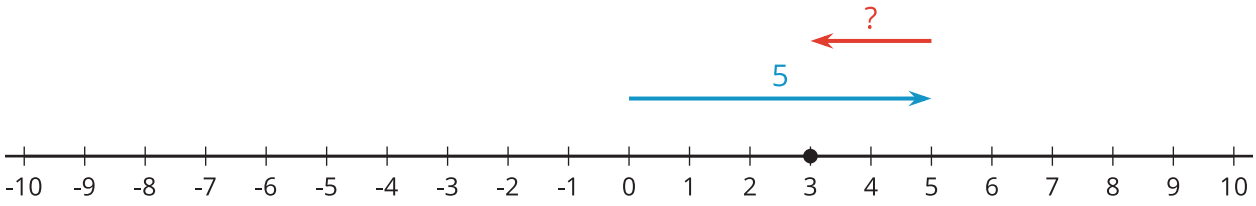
To get to the sum of 7, the second arrow must be 2 units long, pointing to the right. This tells us that positive 2 is the number that completes each equation: $5 + 2 = 7$ and $7 - 5 = 2$.

Notice that the addition expression $7 + (\text{-}5)$ also equals 2.



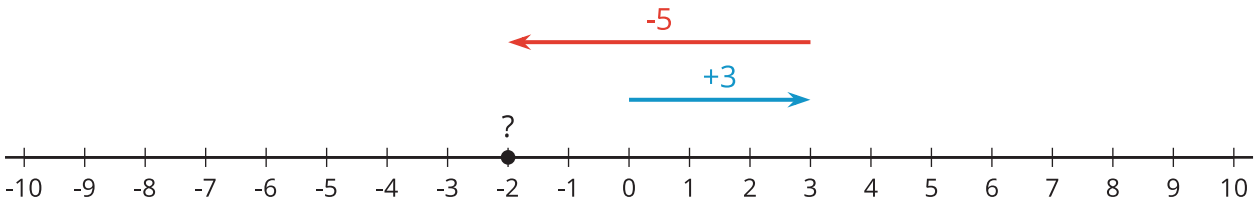
So we can see that $7 - 5 = 7 + (\text{-}5)$.

Here's another example. The equation $3 - 5 = ?$ is equivalent to $5 + ? = 3$.



To get the to the sum of 3, the second arrow must be 2 units long, pointing to the left. This tells us that -2 is the number that completes each equation: $5 + \text{-}2 = 3$ and $3 - 5 = \text{-}2$.

Notice that the addition expression $3 + (\text{-}5)$ also equals -2.



So we can see that $3 - 5 = 3 + (\text{-}5)$.

This pattern always works. In general:

$$a - b = a + (\text{-}b)$$