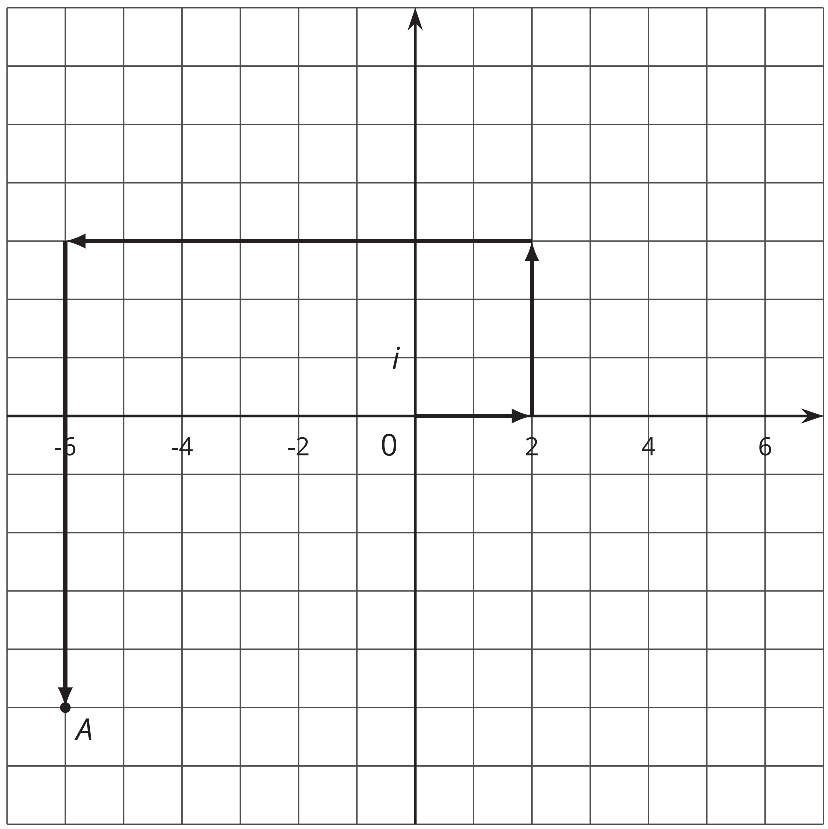
Arithmetic with Complex Numbers

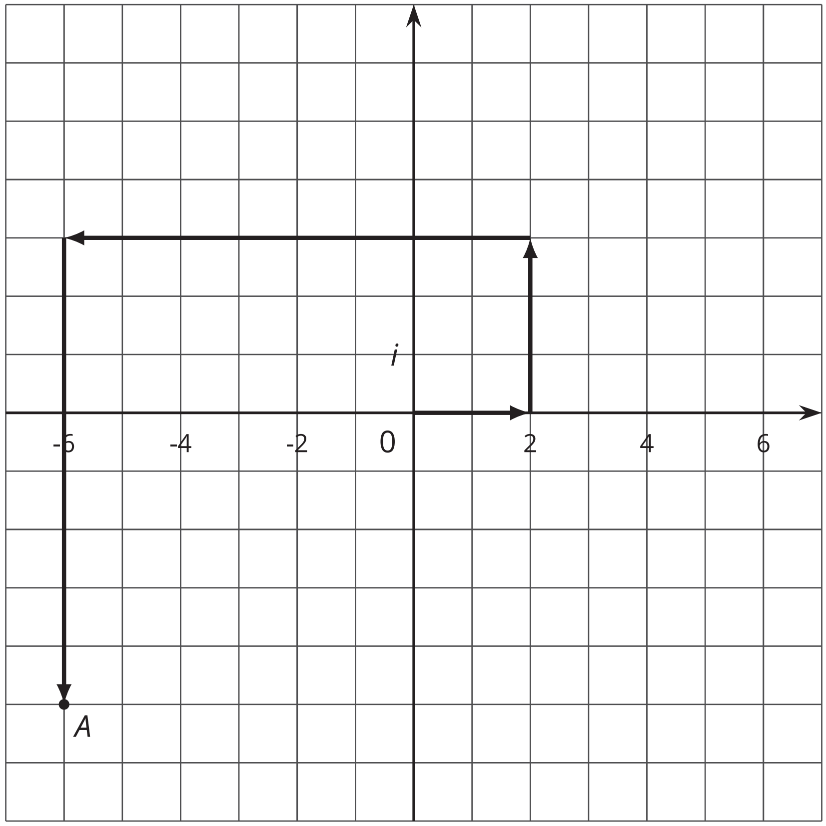
Let's work with complex numbers.

12.1 What Does This Path Mean?



12.2 Adding Complex Numbers

1. This diagram represents $(2 + 3i) + (-8 - 8i)$.



   a. How do you see $2 + 3i$ represented?

   b. How do you see $-8 - 8i$ represented?

   c. What complex number does A represent?

   d. Add "like terms" in the expression $(2 + 3i) + (-8 - 8i)$. What do you get?

2. Write these sums and differences in the form $a + bi$, where a and b are real numbers.

   a. $(-3 + 2i) + (4 - 5i)$ (Check your work by drawing a diagram.)

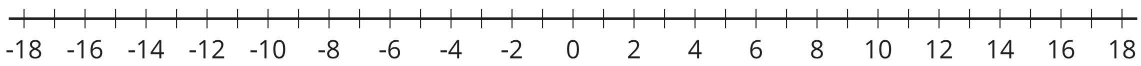
   b. $(-37 - 45i) + (11 + 81i)$

   c. $(-3 + 2i) - (4 - 5i)$

   d. $(-37 - 45i) - (11 + 81i)$

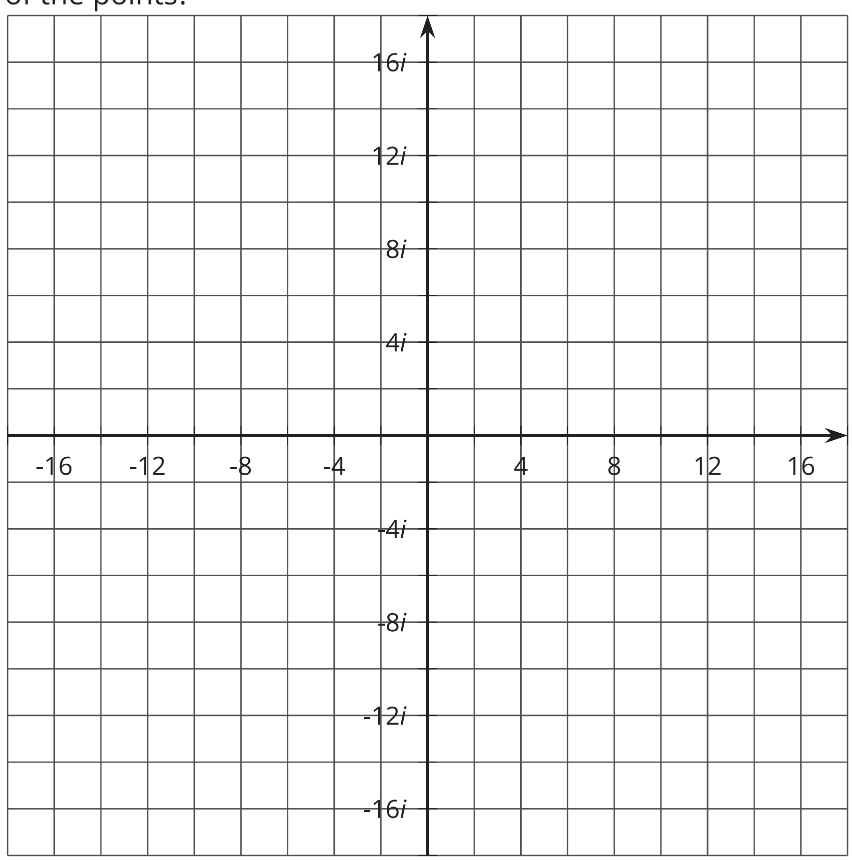
12.3 Multiplication on the Complex Plane

1. Draw points to represent 2, 2^2, 2^3, and 2^4 on the real number line. What do you notice about the arrangement of the points?



2. a. Write $2i$, $(2i)^2$, $(2i)^3$, and $(2i)^4$ in the form $a + bi$.

   b. Plot $2i$, $(2i)^2$, $(2i)^3$, and $(2i)^4$ on the complex plane. What do you notice about the arrangement of the points?



Are you ready for more?

1. If a and b are positive numbers, is it true that $\sqrt{ab} = \sqrt{a}\sqrt{b}$? Explain how you know.

2. If a and b are negative numbers, is it true that $\sqrt{ab} = \sqrt{a}\sqrt{b}$? Explain how you know.

Lesson 12 Summary

When we add a real number with an imaginary number, we get a complex number. We usually write complex numbers as:

$$a + bi$$

where a and b are real numbers. We say that a is the "real part" and b is the "imaginary part."

To add (or subtract) two complex numbers, we add (or subtract) the real parts and add (or subtract) the imaginary parts. For example:

$$(2 + 3i) + (4 + 5i) = (2 + 4) + (3i + 5i) = 6 + 8i$$

$$(2 + 3i) - (4 + 5i) = (2 - 4) + (3i - 5i) = \text{-}2 - 2i$$

In general:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

and:

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

When we raise an imaginary number to a power, we can use the fact that $i^2 = \text{-}1$ to write the result in the form $a + bi$. For example, $(4i)^3 = 4i \cdot 4i \cdot 4i$. We can group the i factors together to see how to rewrite this.

$$\begin{aligned} 4i \cdot 4i \cdot 4i &= (4 \cdot 4 \cdot 4) \cdot (i \cdot i \cdot i) \\ &= 64 \cdot (i^2 \cdot i) \\ &= 64 \cdot \text{-}1 \cdot i \\ &= \text{-}64i \end{aligned}$$