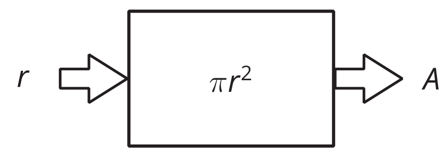
# Unit 6 Family Support Materials

# Functions and Volume

## Section A: Representing and Interpreting Functions

This week, your student will be working with **functions**. A function is a rule that produces a single output for a given input.

Here is an example of a rule that is a function: Input a number, square it, then multiply the result by $\pi$. Using $r$ for the input and $A$ for the output, we can draw a diagram to represent the function:

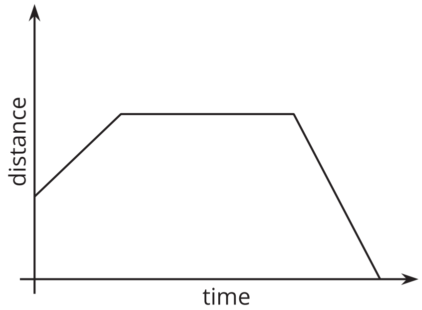



Not all rules are functions. For example, here's a rule: The input is "first letter of the month," and the output is "the month." If the input is J, what is the output? A function must give a single output, but in this case the output of this rule could be January, June, or July, so the rule is not a function.

We could also represent this function with an equation, $A = \pi r^2$. We say that the input of the function, $r$, is the **independent variable** and the output of the function, $A$, is the **dependent variable**. We can choose any value for $r$, and then the value of $A$ depends on the value of $r$.

We could also represent this function with a table or as a graph. The graph of a function is all the pairs (input, output), plotted in the coordinate plane. By convention, we always put the input first, which means the inputs are represented on the horizontal axis and the outputs are on the vertical axis.

For a graph representing a context, it is important to specify the quantities represented on each axis. For example, this graph shows Elena's distance as a function of time. If it is distance from home, then Elena starts at some distance from home (maybe at her friend's house), moves farther away from her home (maybe to a park), stays there a while, and then returns home. If it is distance from school, the story is different.




The story also changes depending on the scale on the axes: Is distance measured in miles and time in hours, or is distance measured in meters and time in seconds? Depending on the question we investigate, different representations have different advantages.

**Here is a task to try with your student:**

Jada can buy peanuts for $0.20 per ounce and raisins for $0.25 per ounce. She has $12 to spend on peanuts and raisins to make trail mix for her hiking group.

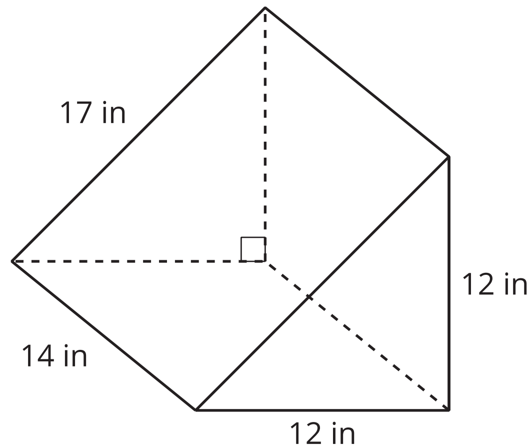
1. How much would 10 ounces of peanuts and 16 ounces of raisins cost? How much money would Jada have left?
2. Using $p$ for pounds of peanuts and $r$ for pounds of raisins, an equation relating how much of each they buy for a total of $12 is $0.2p + 0.25r = 12$. If Jada wants 20 ounces of raisins, how many ounces of peanuts can she afford?
3. Jada knows she can rewrite the equation as $r = 48 - 0.8p$. In Jada's equation, which is the independent variable? Which is the dependent variable?

Solution:

1. 10 ounces of peanuts would cost $2 since $0.2 \cdot 10 = 2$. And 16 ounces of raisins would cost $4 since $0.25 \cdot 16 = 4$. Together, they would cost Jada $6, leaving her with $6.
2. 35 ounces of peanuts. If Jada wants 20 ounces of raisins, then $0.2p + 0.25 \cdot 20 = 12$ must be true, which means $p = 35$.
3. $p$ is the independent variable and $r$ is the dependent variable for Jada's equation.

# Section B: Prisms and Cylinders

This week your student will be thinking about the surface area and **volume** of three-dimensional figures. Here is a triangular **prism**. Its **base** is a right triangle with sides that measure 12, 12, and 17 inches.



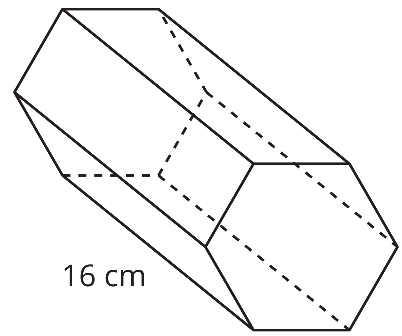

In general, we can find the volume of any prism by multiplying the area of its base times its height. For this prism, the area of the triangular base is 72 $in^2$, so the volume is $72 \cdot 14$, or 1,008, $in^3$.

To find the surface area of a prism, we can find the area of each of the faces and add them up. The example prism has two faces that are triangles and three faces that are rectangles. When we add all of these areas together, we see that the prism has a total surface area of $72 + 72 + 168 + 168 + 238$, or 718, $in^2$.

**Here is a task to try with your student:**

The base of this prism is a hexagon where all the sides measure 5 cm. The area of the base is about 65 $\text{cm}^2$.

1. What is the volume of the prism?
2. What is the surface area of the prism?




Solution:

1. The volume of the prism is about 1,040 $\text{cm}^3$, because $65 \cdot 16 = 1{,}040$.
2. The surface area of the prism is 610 $\text{cm}^2$, because $16 \cdot 5 = 80$ and $65 + 65 + 80 + 80 + 80 + 80 + 80 + 80 = 610$.

# Section C: Cones and Spheres

This week, your student will compare the volumes of different objects. Many common objects, from water bottles to buildings to balloons, are similar in shape to rectangular prisms, cylinders, cones, and spheres—or even combinations of these shapes. We can use the volume formulas for these shapes to compare the volume of different types of objects.

For example, let's say we want to know which has more volume: a cube-shaped box with an edge length of 3 centimeters or a sphere with a radius of 2 centimeters.

The volume of the cube is 27 cubic centimeters since $\text{edge}^3 = 3^3 = 27$. The volume of the sphere is about 33.51 cubic centimeters since $\frac{4}{3}\pi \cdot \text{radius}^3 = \frac{4}{3}\pi \cdot 2^3 \approx 33.51$. Therefore, we can tell that the cube-shaped box holds less than the sphere.

**Here is a task to try with your student:**

A globe fits tightly inside a cubic box. The box has an edge length of 8 cm.

1. What is the volume of the box?
2. Estimate the volume of the globe: Is it more or less than the volume of the box? How can you tell?
3. What is the diameter of the globe? The radius?
4. The formula for the volume of a sphere (like a globe) is $V = \frac{4}{3}\pi r^3$. What is the actual volume of the sphere? How close was your estimate in the previous problem?

Solution:

1. 512 cm$^3$. The box is a cube, so its volume is $8^3$ cubic centimeters.
2. Answers vary. The number should be less than 512 cm$^3$ since the volume of the globe must be less than the volume of the box. Possible explanation: It fits entirely inside the box, so it takes up less space. Since we can fit the globe inside the box and there is still space left over, the box has more volume.
3. Since the globe fits tightly inside the cubic box, the diameter of the globe must be the same as the edge length of the box, 8 cm. This means the radius is 4 cm.
4. $\frac{256}{3}\pi$, or about 268, cm$^3$. Since the side length of the cube is 8 cm, the radius of the globe is half of that, or 4 cm. The volume of the globe is, therefore, $\frac{4}{3}\pi \cdot 4^3 = \frac{256}{3}\pi$.