



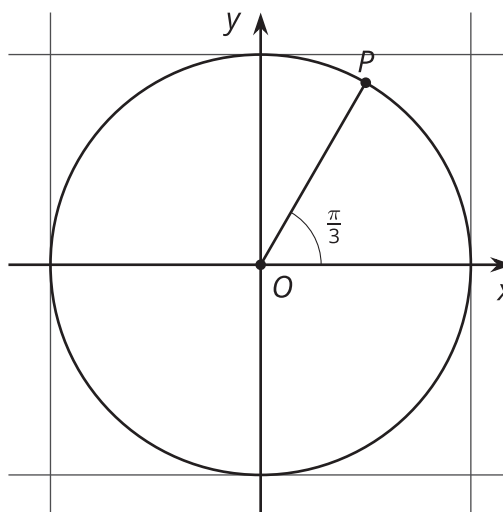
# The Pythagorean Identity (Part 1)

Let's learn more about cosine and sine.

## 5.1 Circle Equations

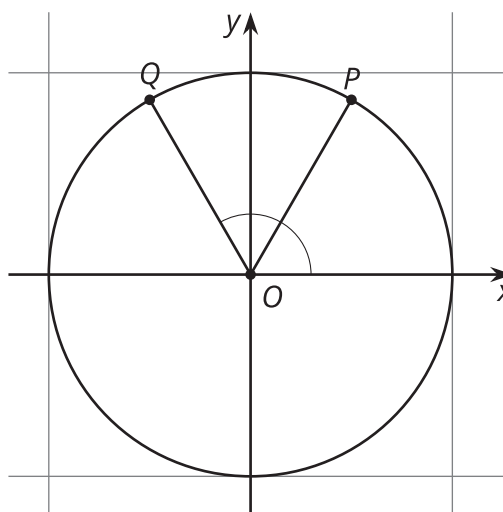
Here is a circle that is centered at  $(0, 0)$  and that has a radius of 1 unit.

What are the exact coordinates of  $P$  if  $P$  is rotated counterclockwise  $\frac{\pi}{3}$  radians from the point  $(1, 0)$ ? Explain or show your reasoning.



## 5.2 Cosine, Sine, and the Unit Circle

What are the exact coordinates of point  $Q$  if it is rotated  $\frac{2\pi}{3}$  radians counterclockwise from the point  $(1, 0)$ ? Explain or show your reasoning.



## 5.3

## A New Identity

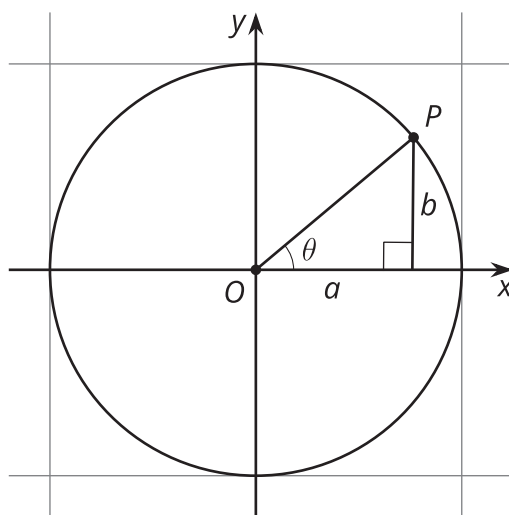
1. Is the point  $(-0.5, \sin(\frac{4\pi}{3}))$  on the unit circle? Explain or show your reasoning.
2. Is the point  $(-0.5, \sin(\frac{5\pi}{6}))$  on the unit circle? Explain or show your reasoning.
3. Suppose that  $\sin(\theta) = -0.5$  and that  $\theta$  is in Quadrant IV. What is the exact value of  $\cos(\theta)$ ? Explain or show your reasoning.

**Are you ready for more?**

Show that if  $\theta$  is an angle between 0 and  $2\pi$  and neither  $\cos(\theta) = 0$  nor  $\sin(\theta) = 0$ , then it is impossible for the sum of  $\cos(\theta)$  and  $\sin(\theta)$  to be equal to 1.

## Lesson 5 Summary

Let's say we have a point  $P$  with coordinates  $(a, b)$  on the unit circle, like the one shown here:



Using the Pythagorean Theorem, we know that  $a^2 + b^2 = 1$ . We also know this is true using the equation for a circle with radius 1 unit,  $x^2 + y^2 = 1^2$ , which is true for the point  $(a, b)$  since it is on the circle.

Another way to write the coordinates of  $P$  is to use the angle  $\theta$ , which gives the location of  $P$  on the unit circle relative to the point  $(1, 0)$ . Thinking of  $P$  this way, its coordinates are  $(\cos(\theta), \sin(\theta))$ . Since  $a = \cos(\theta)$  and  $b = \sin(\theta)$ , we can return to the Pythagorean Theorem and say that  $\cos^2(\theta) + \sin^2(\theta) = 1$  is also true.

What if  $\theta$  were a different angle and  $P$  wasn't in Quadrant I? It turns out that no matter the quadrant, the coordinates of any point on the unit circle given by an angle  $\theta$  are  $(\cos(\theta), \sin(\theta))$ . In fact, the definitions of  $\cos(\theta)$  and  $\sin(\theta)$  are the  $x$ - and  $y$ -coordinates of the point on the unit circle  $\theta$  radians counterclockwise from  $(1, 0)$ . Up until today, we've only been using the Quadrant I values for cosine and sine to find side lengths of right triangles, which are always positive.

This revised definition of cosine and sine means that  $\cos^2(\theta) + \sin^2(\theta) = 1$  is true for all values of  $\theta$  defined on the unit circle and is known as the **Pythagorean Identity**.