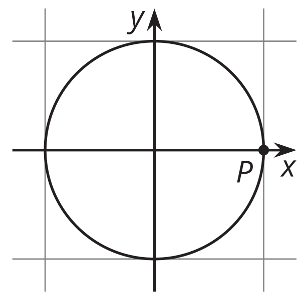
Unit 6, Lesson 9

Introduction to Trigonometric Functions

Let’s graph cosine and sine.

9.1 An Angle and a Circle

Suppose there is a point P at $(1, 0)$ on the unit circle.

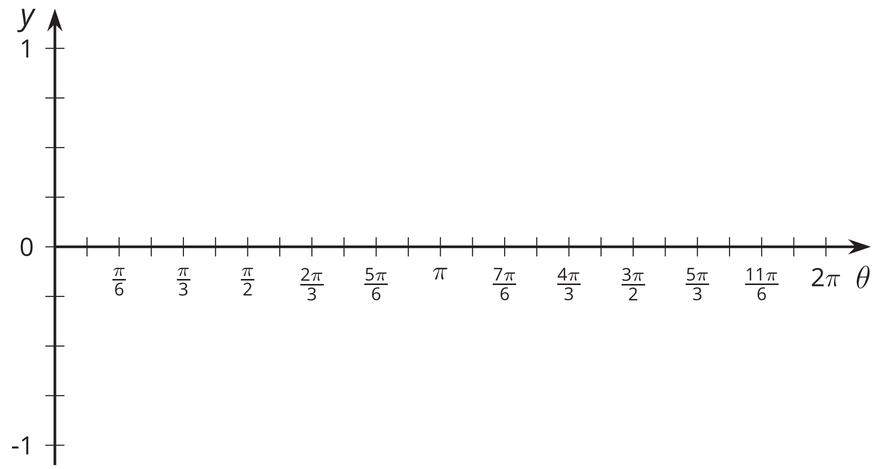


1. Describe how the x-coordinate of P changes as it rotates once counterclockwise around the circle.

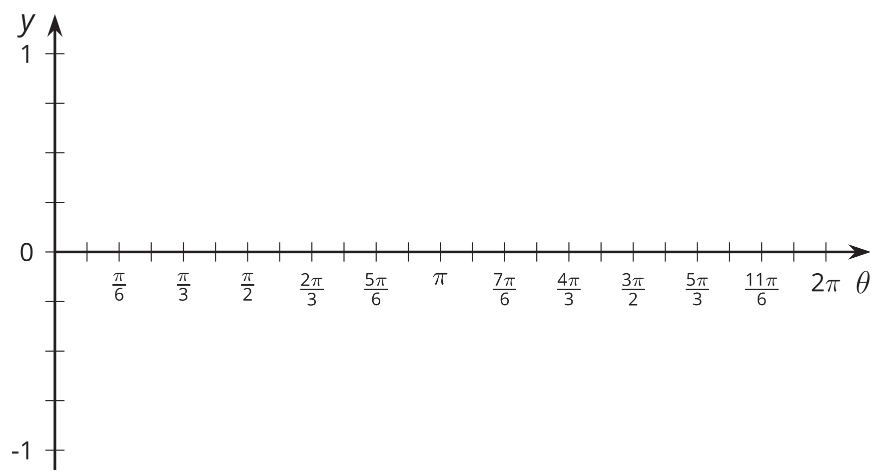
2. Describe how the y-coordinate of P changes as it rotates once counterclockwise around the circle.

9.2 Do the Wave

1. For each tick mark on the horizontal axis, plot the value of $y = \cos(\theta)$, where θ is the measure of an angle in radians. Use the class display of the unit circle, the unit circle from an earlier lesson, or technology to estimate the value of $\cos(\theta)$.



2. For each tick mark on the horizontal axis, plot the value of $y = \sin(\theta)$. Use the class display of the unit circle, the unit circle from an earlier lesson, or technology to estimate the value of $\sin(\theta)$.



3. What do you notice about the two graphs?

4. Explain why any angle measure between 0 and 2π gives a point on each graph.

5. Could these graphs represent functions? Explain your reasoning.

9.3 Graphs of Cosine and Sine

1. Looking at the graphs of $y = \cos(\theta)$ and $y = \sin(\theta)$, at what values of θ do $\cos(\theta) = \sin(\theta)$? To where on the unit circle do these points correspond?

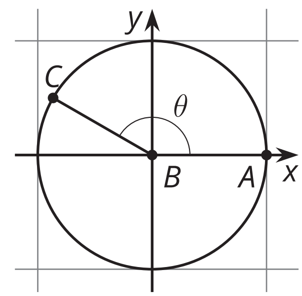
2. For each of these equations, first predict what the graph looks like, and then check your prediction using technology.
   a. $y = \cos(\theta) + \sin(\theta)$
   b. $y = \cos^2(\theta)$
   c. $y = \sin^2(\theta)$
   d. $y = \cos^2(\theta) + \sin^2(\theta)$

Are you ready for more?

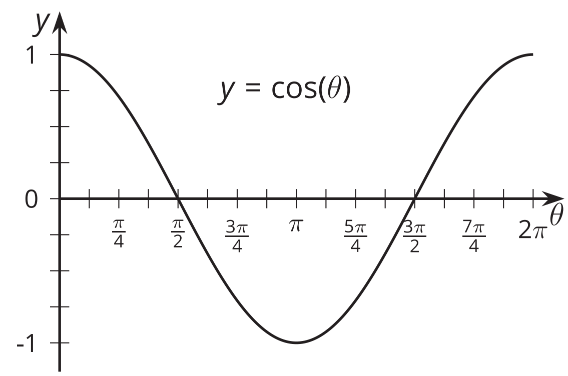
For the equation given, predict what the graph looks like, and then check your prediction using technology: $y = \theta + \cos(\theta)$.

Lesson 9 Summary

Using the unit circle, we can make sense of $\cos(\theta)$ and $\sin(\theta)$ for any angle measure θ between 0 and 2π radians. For an angle θ, starting at the positive x-axis, there is a point, C, where the terminal ray of the angle intersects the unit circle. The coordinates of that point are $(\cos(\theta), \sin(\theta))$.

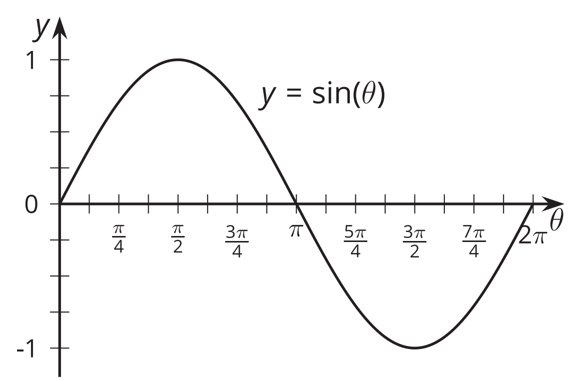


But what if we wanted to think about just the horizontal position of point C as θ goes from 0 to 2π? The horizontal location is defined by the x-coordinate, which is $\cos(\theta)$. If we graph $y = \cos(\theta)$, we get:



This graph is 1 when θ is 0 because the x-coordinate of the point at 0 radians on the unit circle is $(1, 0)$. The graph then decreases to -1 (the smallest x-value on the unit circle) before increasing back to 1.

We can do the same for the y-coordinate of a point on the unit circle by graphing $y = \sin(\theta)$:



This graph is 0 when θ is 0, increases to 1 (the greatest y-value on the unit circle), then decreases to -1 before returning to 0.