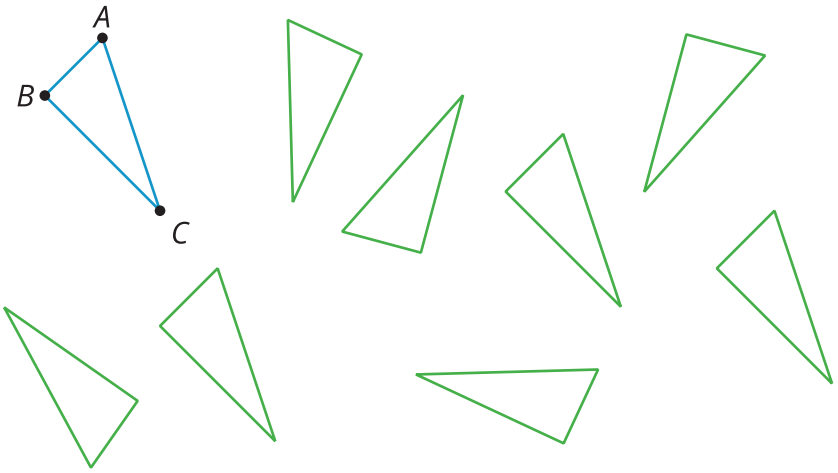
Congruence

Let's decide if two figures are congruent.

11.1 Translated Images

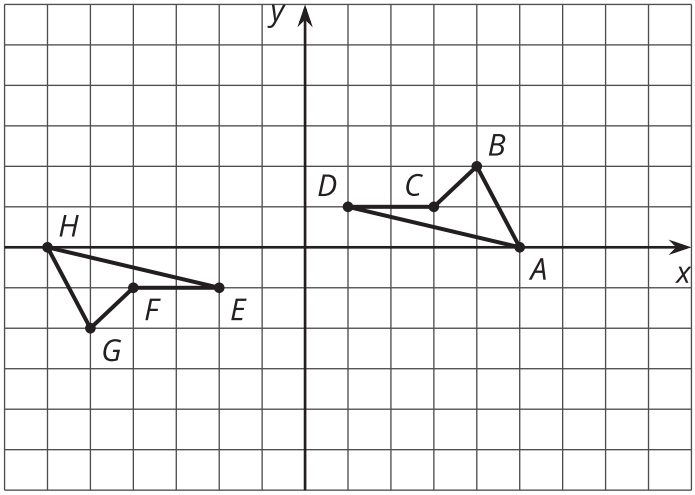
All of these triangles are congruent. Sometimes we can take one figure to another with a translation. Shade the triangles that are images of triangle ABC under a translation.



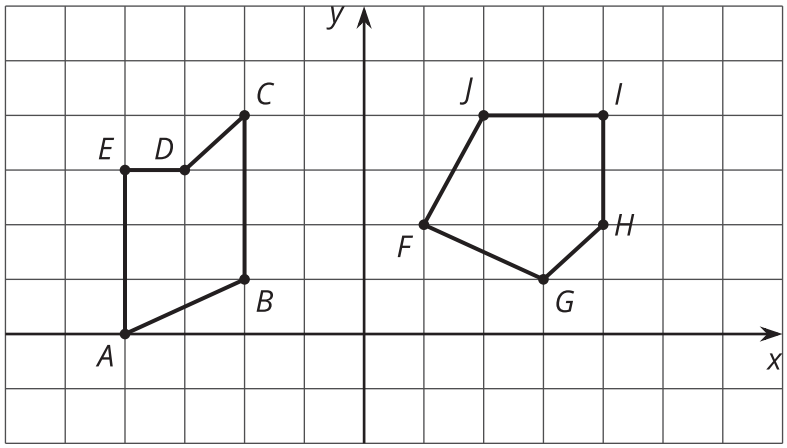
11.2 Congruent Pairs

For each of the following pairs of shapes, decide whether or not they are congruent. Explain your reasoning.

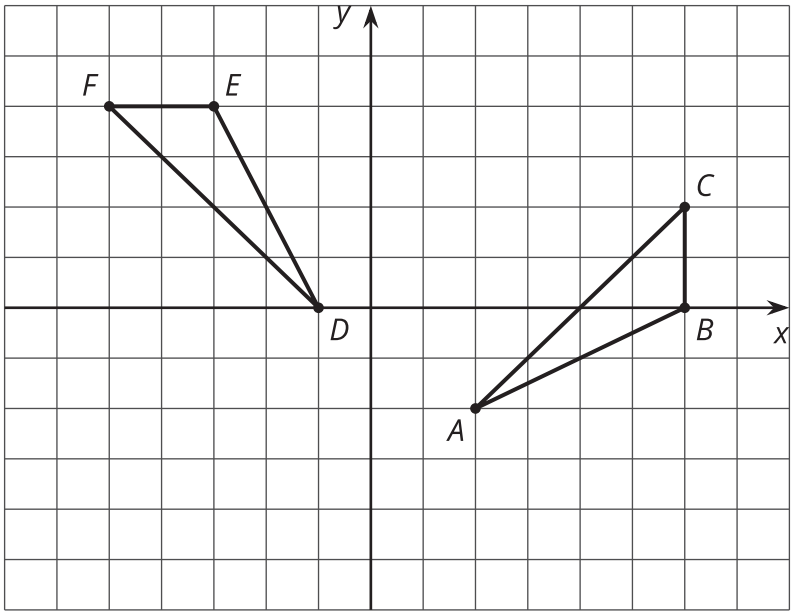
1.



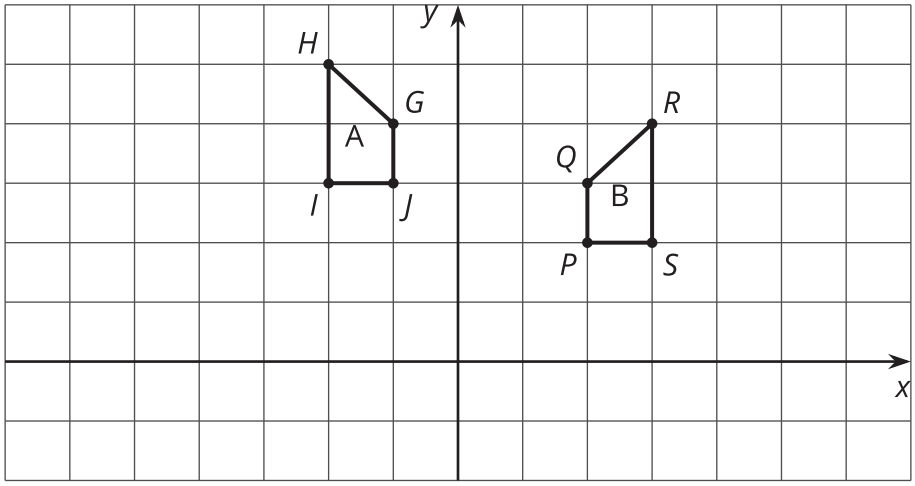
2.



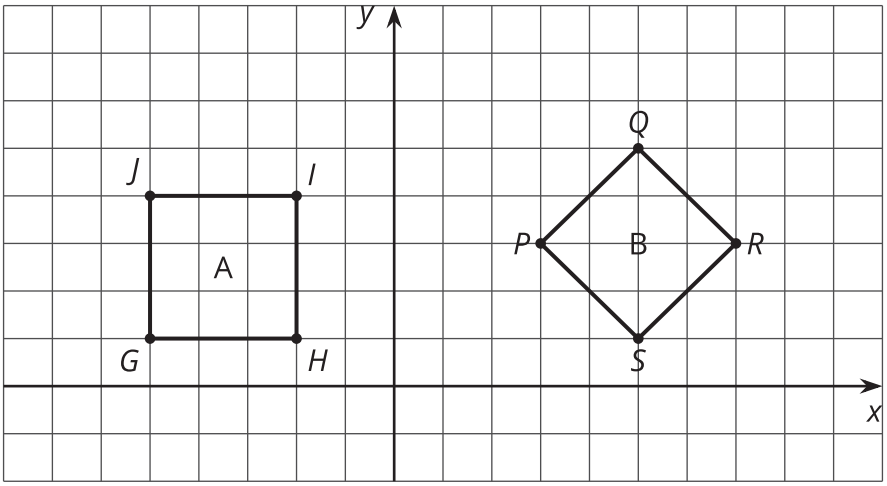
3.



4.



5.



Are you ready for more?

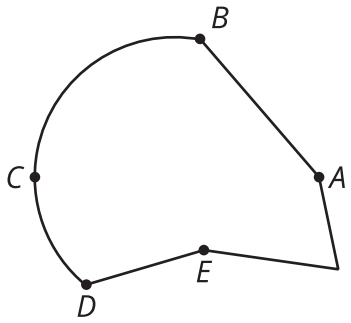
A polygon has 8 sides: five of length 1, two of length 2, and one of length 3. All sides lie on grid lines. (It may be helpful to use graph paper when working on this problem.)

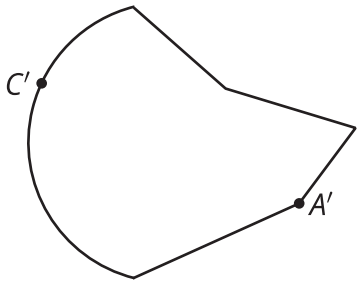
1. Find a polygon with these properties.

2. Is there a second polygon, not congruent to the first, with these properties?

11.3 Corresponding Points in Congruent Figures

Here are two congruent shapes with some corresponding points labeled:





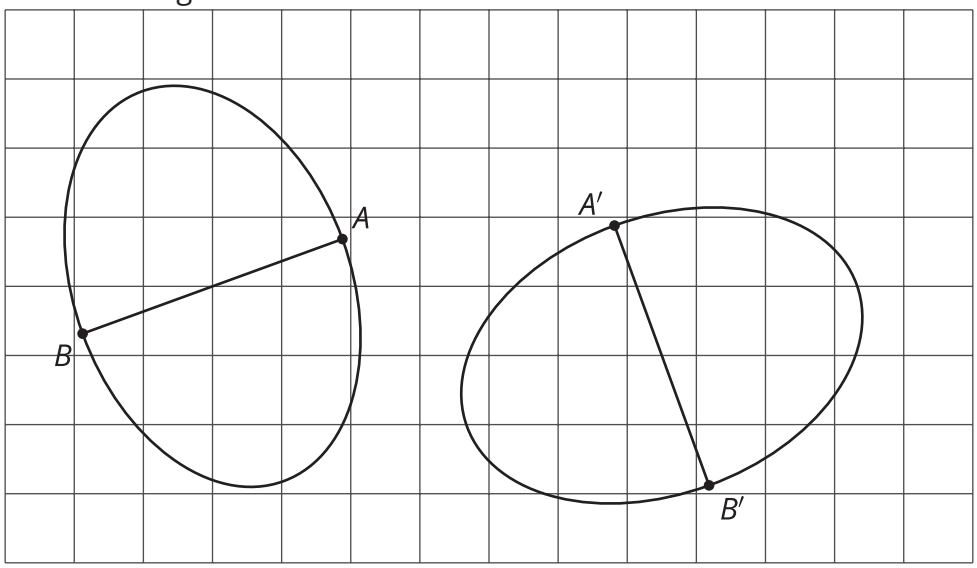
1. On the bottom figure, draw the points corresponding to B, D, and E, and label them B', D', and E'.
2. Draw line segments AD and $A'D'$ and measure them. Do the same for segments BC and $B'C'$ and for segments AE and $A'E'$. What do you notice?

3. Do you think there could be a pair of corresponding segments with different lengths? Explain.

Lesson 11 Summary

How do we know if two figures are congruent?

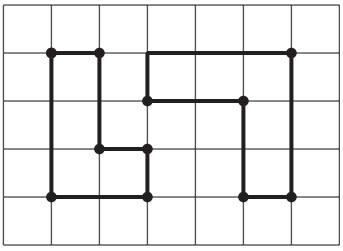
- If we copy one figure on tracing paper and move the paper so the copy covers the other figure exactly, then that suggests they are congruent.
- If we can describe a sequence of translations, rotations, and reflections that move one figure onto the other so they match up exactly, they are congruent.
- Distances between corresponding points on congruent figures are always equal, even for curved shapes. For example, corresponding segments AB and $A'B'$ on these congruent ovals have the same length:



How do we know that two figures are *not* congruent?

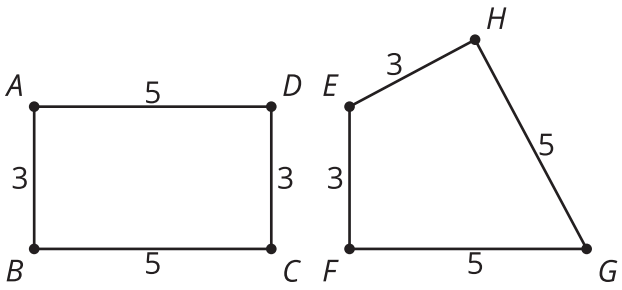
- If there is no correspondence between the figures where the parts have equal measure, that shows that the two figures are *not* congruent.
  - If two polygons have different sets of side lengths, they can't be congruent.

    For example, the figure on the left has side lengths 3, 2, 1, 1, 2, 1. The figure on the right has side lengths 3, 3, 1, 2, 2, 1. There is no way to make a correspondence between them where all corresponding sides have the same length.



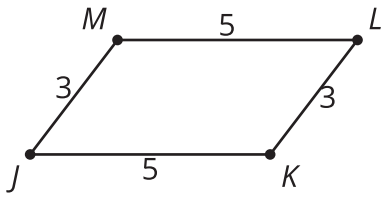
- If two polygons have the same side lengths, but not in the same order, the polygons can't be congruent.

  For example, rectangle $ABCD$ can't be congruent to quadrilateral $EFGH$. Even though they both have two sides of length 3 and two sides of length 5, they don't correspond in the same order.



- If two polygons have the same side lengths, in the same order, but different corresponding angles, the polygons can't be congruent.

  For example, parallelogram $JKLM$ can't be congruent to rectangle $ABCD$. Even though they have the same side lengths in the same order, the angles are different. All angles in $ABCD$ are right angles. In $JKLM$, angles J and L are less than 90 degrees and angles K and M are more than 90 degrees.



- If two figures have different corresponding distances, they can't be congruent.

  For example on both ovals, the longest distance is 5 units across, and the longest distance from top to bottom is 4 units. The line segment from the highest to lowest point is in the middle of the left oval, but in the right oval, it's 2 units from the right end and 3 units from the left end. This shows they are not congruent.