Unit 6, Lesson 9

The Distributive Property, Part 1

Goals

- Generate equivalent numerical expressions that are related by the distributive property, and explain (orally or using other representations) the reasoning.
- Use an area diagram to make sense of equivalent numerical expressions that are related by the distributive property.

Learning Targets

- I can use a diagram of a rectangle split into two smaller rectangles to write different expressions representing its area.
- I can use the distributive property to explain how two expressions with numbers are equivalent.

Lesson Narrative

In this lesson, students interpret and generate equivalent numerical expressions using the distributive property. In previous grades, students used the distributive property informally to explain strategies as they developed multiplication fluency. They used rectangular diagrams to make sense of the distributive property and explain their reasoning, without naming the property.

The activities in this lesson are designed to elicit this understanding, from students' work with computation and with rectangular diagrams, and to support students' abstract and quantitative reasoning as they use the distributive property as a way to generate and justify equivalent numerical expressions (MP2). The term "distributive property" is used formally with students, although students may continue to explain their understanding of the property using terms that make sense to them throughout the unit.

Standards

Building On	**3.MD.C.7.c, 4.NBT.B.5, 5.NBT.B.7**
Addressing	**6.NS.B.4**
Building Toward	**6.EE.A.3, 6.EE.A.4, 6.NS.B.4**

Instructional Routines

- Math Talk
- MLR2: Collect and Display
- MLR8: Discussion Supports

Student Facing Learning Goals

Let's use the distributive property to describe expressions.

9.1 Math Talk: Ways to Multiply

Warm-up

5 min

Activity Narrative

This *Math Talk* focuses on multiplication of multi-digit numbers. It encourages students to think about decomposition of numbers by place value to rely on what they know about properties of operations to mentally solve problems. The strategies elicited here will be helpful in upcoming activities and lessons as students build on their informal understanding of the distributive property to generate and justify equivalent expressions.

Students must be precise in their word choice and use of language when describing how they may decompose numbers, how they multiply, and the sums or differences they create to find the product (MP6).

Standards

Building On **4.NBT.B.5, 5.NBT.B.7**

Building Toward **6.EE.A.3, 6.NS.B.4**

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the Activity Synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Task Statement

Find the value of each product mentally.

- $5 \cdot 102$
- $5 \cdot 98$
- $5 \cdot 999$
- $5 \cdot (0.999)$

Student Response

- 510. Sample reasoning:
  - $5 \cdot 102 = 5 \cdot 100 + 5 \cdot 2$, which is $500 + 10$.
  - $10 \cdot 102$ is 1,020 and 5 is half of 10, so $5 \cdot 102$ is half of 1,020, which is 510.
- 490. Sample reasoning:
  - 98 is 2 less than 100, so $5 \cdot 98$ is $5 \cdot 2$, or 10, less than 500, which is 490.
  - $5 \cdot 98 = 5 \cdot 90 + 5 \cdot 8 = 450 + 40$, which is 490.
    98 is 4 less than 102, so $5 \cdot 98$ is $5 \cdot 4$, or 20, less than 510, which is 490.
- 5,015. Sample reasoning:

- ◦ 999 is 1 less than 1,000, so the product is $5 \cdot 1$ less than $5 \cdot 1{,}000$ or $5{,}000 - 5$, which is 4,995.
  - ◦ $999 = 900 + 90 + 9$. Multiplying 900, 90, and 9 each by 5 gives 4,500, 450, and 45. Adding them up gives 4,995.
- 4,995. Sample reasoning:
  - The product is one-thousandth of 4,995, because 0.999 is a thousandth of 999.
  - $0.999 = 1 - 0.001$, so $5 \cdot (0.999) = 5 \cdot 1 - 5 \cdot (0.001)$, or $5 - 0.005$, which is 4.995.

Activity Synthesis

To involve more students in the conversation, consider asking:

- "Who can restate ________'s reasoning in a different way?"
- "Did anyone use the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ________'s strategy?"
- "Do you agree or disagree? Why?"
- "What connections to previous problems do you see?"

Once students have had a chance to share a few different ways of reasoning about each product, focus on explanations using the distributive property and record the steps of reasoning for all to see. For example, when students find $5 \cdot 98$ by thinking of 98 as $100 - 2$, record:

$5 \cdot 102$
$5 \cdot (100 + 2)$
$5 \cdot 100 + 5 \cdot 2$
$500 + 10$
510

Explain to students that the strategies that involve decomposing one factor as a sum or difference of numbers and then multiplying each part by the other factor demonstrate the distributive property of multiplication. In the shown example, we are "distributing" the multiplication of 5 to the 100 and the 2. Applying the distributive property allows us to write an expression that is equivalent to a given expression but is easier to calculate. Let students know that they will spend the next few lessons deepening their understanding of this property.

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I ________ because . . . ." or "I noticed ________ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

10 min

9.2 Ways to Represent Area of a Rectangle

Activity Narrative

The purpose of this activity is to remind students of the rectangular diagrams they worked with in a previous unit to represent multiplication. Students use the structure of the diagram to make sense of equivalent expressions and revisit their understanding of the distributive property. They are also introduced to the convention that multiplication happens before addition or subtraction. For example, the expression $6 \cdot 3 + 2$ equals 20. because we first multiply $6 \cdot 3$ and then add the 2. If we want the sum to be carried out before the product, we need to use parentheses like $6 \cdot (3 + 2)$.

Standards

Building On 3.MD.C.7.c

Building Toward 6.EE.A.3, 6.EE.A.4, 6.NS.B.4

Launch

Give students 3–4 minutes of quiet think time, followed by a whole-class discussion.

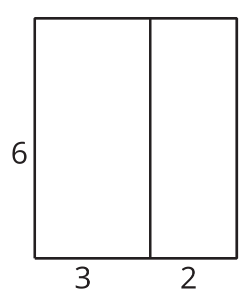
Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about finding area. Some students may benefit from a review of the rectangle diagrams they used in a previous unit to represent multiplication.
Supports accessibility for: Memory, Conceptual Processing

Student Task Statement

1. Select **all** the expressions that represent the area of the large, outer rectangle in Figure A. Explain your reasoning.

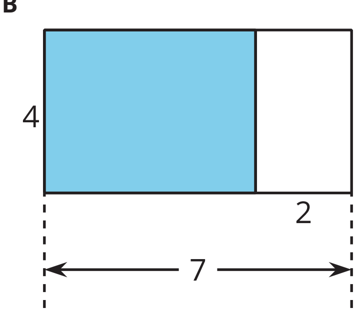
A



- $6 + 3 + 2$
- $6 \cdot 3 + 6 \cdot 2$
- $6 \cdot 3 + 2$
- $6 \cdot 5$
- $6 \cdot (3 + 2)$
- $6 \cdot 3 \cdot 2$

2. Select **all** the expressions that represent the area of the shaded rectangle on the left side of Figure B. Explain your reasoning.

B



- $4 \cdot 7 + 4 \cdot 2$
- $4 \cdot 7 \cdot 2$
- $4 \cdot 5$
- $4 \cdot 7 - 4 \cdot 2$
- $4 \cdot (7 - 2)$
- $4 \cdot (7 + 2)$
- $4 \cdot 2 - 4 \cdot 7$

Student Response

1. $6 \cdot 3 + 6 \cdot 2$, $6 \cdot 5$, and $6 \cdot (3 + 2)$. Sample reasoning:
   - These are all equal to 30.
   - $6 \cdot 3 + 6 \cdot 2$ is the sum of the areas of the two pieces, and the other two expressions are just the area of the whole rectangle.

2. $4 \cdot 5$, $4 \cdot 7 - 4 \cdot 2$, and $4 \cdot (7 - 2)$. Sample reasoning:
    - These are all equal to 20.
    - $4 \cdot 7 - 4 \cdot 2$ is the area of the whole rectangle minus the unshaded part, and the other two expressions are just the area of the shaded part.

Activity Synthesis

Invite students to share the expressions that they believe represent the area of each rectangle and ask them to explain their reasoning.

Students may incorrectly conclude that $6 \cdot 3 + 2$ represents the area of the large outer rectangle in Figure A. This is a good opportunity to introduce a convention. Explain that when we have multiplication and addition in the same expression, it is the convention that the multiplication is done first. So, $6 \cdot 3 + 2$ equals $18 + 2$, or 20. This means that the expression $6 \cdot 3 + 2$ doesn't represent the area of the whole rectangle, which we know to be 30 square units.

Tell students that if we want the addition to be done first, we need to use parentheses in the expression. Display $6 \cdot (3 + 2)$ and show that it equals $6 \cdot 5$, or 30. Therefore, $6 \cdot (3 + 2)$ does represent the area of the large rectangle.

Applying the distributive property of multiplication, we can write $6 \cdot (3 + 2)$ as $6 \cdot 3 + 6 \cdot 2$ and know that they are equivalent. By using the rectangle, we can see that $6 \cdot (3 + 2)$ represents the same area as $6 \cdot 3 + 6 \cdot 2$, so the two expressions have the same value.

Show students that $6 \cdot (3 + 2)$ can also be written as $6(3 + 2)$. Explain that just like a number next to a variable means multiplication, a number next to parentheses means multiplication. In other words, just like $6x$ means $6 \cdot x$, the expression $6(3 + 2)$ means $6 \cdot (3 + 2)$.

Consider adding "distributive property" and equations that illustrate the property to the display started in a previous lesson.

9.3 Distributive Practice

20 min

Activity Narrative

In this activity, students practice using the distributive property to create equivalent numerical expressions. Students need to look for and make use of the structure of the expressions and the structure of the table to create equivalent sums or differences when starting with a product, and equivalent products when starting with a sum or difference (MP7).

The expressions were chosen to also elicit familiar strategies for multiplying numbers, such as by decomposing one or more factors by place value or by benchmark fractions.

Students must also reason in the other direction—write a given addition or subtraction expression as a sum or difference of two products. By the time they arrive at the expressions in the last two rows, students may have noticed that the two numbers in column 4 are results of multiplication of two pairs of numbers that share a common factor. Recognition of this structure allows students to look for a common factor for 100 and 70, and for 40 and 6.

Note that there is more than one way to rewrite the expression in the last two rows because each pair of numbers share several common factors, but the resulting expressions are equivalent. In a subsequent unit, students will explicitly study the idea of a greatest common factor.

Access for English Language Learners

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Standards

Addressing **6.NS.B.4**

Building Toward **6.EE.A.3**

Instructional Routines

- MLR2: Collect and Display

Launch

Give students 1–2 minutes to read the table and then invite them to share some observations and any questions that come to mind. Students may notice that the expressions in the first row represent a strategy for finding $5 \cdot 98$ in the W*arm-up.* Remind them that all the expressions in that row are equivalent.

If no students noticed that all expressions in each column have the same structure, draw their attention to this idea. Ask students to describe the structure that they see in each column.

Tell students that they are to complete each row in a table with expressions that are equivalent to the given expression, and to apply the distributive property (or another property) as needed.

Arrange students in groups of 2. Give students 6–8 minutes of quiet work time and then a few minutes to share their responses with their partner, followed by a whole-class discussion.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Invite students to borrow language from the display as needed, and update it throughout the lesson. Useful words and phrases to highlight for this activity include "equivalent," "sum," "difference," "product," and "factor."

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, after students have completed the first two rows of the table, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.
Supports accessibility for: Organization, Attention

Student Task Statement

Complete the table. If you get stuck, consider skipping an entry and coming back to it, or drawing a diagram of two rectangles that share a side.

column 1	column 2	column 3	column 4	value
$5 \cdot 98$	$5(100-2)$	$5 \cdot 100 - 5 \cdot 2$	$500-10$	490
$33 \cdot 12$	$33(10+2)$			
		$3 \cdot 10 - 3 \cdot 4$	$30-12$	
	$100(0.4+0.06)$			
		$8 \cdot \frac{1}{2} + 8 \cdot \frac{1}{4}$		
			$100+70$	
			$40-16$	

Student Response

column 1	column 2	column 3	column 4	value
$5 \cdot 98$	$5(100-2)$	$5 \cdot 100 - 5 \cdot 2$	$500-10$	490
$33 \cdot 12$	$33(10+2)$	$33 \cdot 10 + 33 \cdot 2$	$330+66$	396
$3 \cdot 6$	$3(10-4)$	$3 \cdot 10 - 3 \cdot 4$	$30-12$	18
$100 \cdot (0.46)$	$100(0.4+0.06)$	$100 \cdot (0.4) + 100 \cdot (0.06)$	$40+6$	46
$8 \cdot \frac{3}{4}$	$8\left(\frac{1}{2}+\frac{1}{4}\right)$	$8 \cdot \frac{1}{2} + 8 \cdot \frac{1}{4}$	$4+2$	6
$10 \cdot 17$	$10(10+7)$	$10 \cdot 10 + 10 \cdot 7$	$100+70$	170
$8 \cdot 3$	$8(5-2)$	$8 \cdot 5 - 8 \cdot 2$	$40-16$	24

Note that there is more than one correct response for the last two rows. For example, $40 - 16$ could also be rewritten as $4(10-4)$ or $2(20-8)$.

Building on Student Thinking

Students might understand how to write a given product of a number and a sum, such as $33(10+2)$ in column 2, as a sum of two products, such as $33 \cdot 10 + 33 \cdot 2$ in column 3, but they might be unsure how to reason in reverse and write an expression such as $3 \cdot 10 - 3 \cdot 4$ in column 3 as a product of a number and difference. Likewise, they might move from column 3 to column 4 with ease but be unsure how to reason the other way around.

Ask students to observe how the expressions in column 2 and column 3 (or column 3 and column 4) are alike and how they are different. As needed, invite them to refer to the rectangular diagrams they have seen, draw a diagram of a partitioned rectangle to represent the expressions in column 3 and column 4 for one of their completed rows, and make connections between the expressions and the diagram. Then urge them to draw another rectangle to represent the expressions in those columns for the row they are working on.

Are You Ready for More?

1. Use the distributive property to write two expressions that equal 360. (There are many correct ways to do this.)
2. Is it possible to write an expression like $a(b + c)$ that equals 360 where a is a fraction? Either write such an expression, or explain why it is impossible.
3. Is it possible to write an expression like $a(b - c)$ that equals 360? Either write such an expression, or explain why it is impossible.
4. How many ways do you think there are to represent 360 using the distributive property?

Extension Student Response

1. Sample responses: $36(7 + 3)$, $10(20 + 16)$
2. Yes. Sample reasoning: $\frac{1}{2}(700 + 20) = 360$
3. Yes. Sample reasoning: $12(50 - 20) = 360$
4. Sample response: There are an infinite number of ways, if fractions or decimals are allowed, and a very large number even if they are not allowed.

Activity Synthesis

Invite students to share the strategies and reasoning they used to complete the table. Include students who took advantage of the structure of the expressions in each column and those who used diagrams of partitioned rectangles. To help make students' reasoning explicit, discuss questions such as:

- "How did you use the expression in column 3 to write an expression for column 4 (or column 2)?"
- "How did you use the expression in column 4 to write an expression in column 3? How did you figure out what numbers to use?"
- "Did you notice any pattern in your reasoning as you move from column to column, or as you write equivalent expressions?

Lesson Synthesis

The purpose of this discussion is to help students use their informal understanding of the distributive property to make sense of more formal definition and notation for the distributive property.

Remind students that when multiplying two factors, we often think of one of the factors as being composed of two or more numbers, either by addition or subtraction. For example, in the *Warm-up*, we thought of 102 as $100 + 2$ and 98 as

$100 - 2$ to make it easier to find $5 \cdot 102$ and $5 \cdot 98$.

Explain that the distributive property is a statement or rule that describes something about multiplication that students have observed and used for some time.

It says that we can multiply a factor and an addition (or a subtraction) expression by “distributing” the multiplication to each number in the expression, and then adding (or subtracting) the products. Doing this will not change the result of the original multiplication.

Display $33 \cdot 12$ for all to see and discuss this example:

- To compute $33 \cdot 12$, we can think of the 12 as the sum of 10 and 2 and write $33 \cdot (10 + 2)$.
- Then we can “distribute” the multiplication of 33 to 10 and 2 by writing $33 \cdot 10 + 33 \cdot 2$.
- These two expressions, $33 \cdot (10 + 2)$ and $33 \cdot 10 + 33 \cdot 2$, are equivalent.

By the distributive property, we can also rewrite an expression like $4 \cdot 7 - 4 \cdot 2$ as $4 \cdot (7 - 2)$, because we know that the subtraction expression $4 \cdot 7 - 4 \cdot 2$ is a result of multiplying 4 and each number in $7 - 2$, so the two expressions are equivalent.

Tell students that they will continue to explore the distributive property of multiplication in upcoming lessons.

9.4 Complete the Equation

Cool-down

5 min

Standards

Building Toward **6.EE.A.3**

Student Task Statement

Write a number or expression in each empty box to create true equations.

1. $7 \cdot (3 + 5) = \boxed{} + \boxed{}$
2. $5 \cdot 3 - 5 \cdot 2 = \boxed{} \cdot (3 - 2)$

Student Response

1. $7 \cdot (3 + 5) = \boxed{21} + \boxed{35}$ or $7 \cdot (3 + 5) = \boxed{7 \cdot 3} + \boxed{7 \cdot 5}$ (or equivalent)
2. $5 \cdot 3 - 5 \cdot 2 = \boxed{5} \cdot (3 - 2)$

Responding to Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 9 Summary

When we need to do mental calculations, we often come up with ways to make the calculation easier to do mentally.

Suppose we are grocery shopping and need to know how much it will cost to buy 5 cans of beans at 79 cents a can. We may calculate mentally in this way:

$5 \cdot 79$
$5 \cdot (70 + 9)$
$5 \cdot 70 + 5 \cdot 9$
$350 + 45$
395

When we think, "79 is the same as $70 + 9$. I can just multiply $5 \cdot 70$ and $5 \cdot 9$ and add the products together" we are using the distributive property.

In general, when we multiply two factors, we can break up one of the factors into parts, multiply each part by the other factor, and then add the products. The result will be the same as the product of the two original factors. When we break up one of the factors and multiply the parts we are using the distributive property of multiplication.

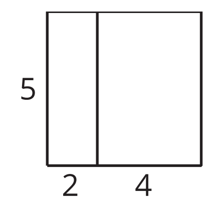
The distributive property also works with subtraction. Here is another way to find $5 \cdot 79$:

$5 \cdot 79$
$5 \cdot (80 - 1)$
$5 \cdot 80 - 5 \cdot 1$
$400 - 5$
395

Lesson 9 Practice Problems

1 Student Task Statement

Select **all** the expressions that represent the area of the large, outer rectangle.



A. $5(2+4)$

B. $5 \cdot 2 + 4$

C. $5 \cdot 2 + 5 \cdot 4$

D. $5 \cdot 2 \cdot 4$

E. $5 + 2 + 4$

F. $5 \cdot 6$

Solution

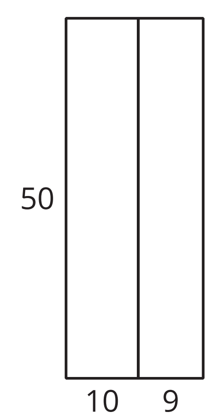
A, C, F

2 Student Task Statement

Explain how each expression is equivalent to $50 \cdot 19$.
Use the diagram if needed.

a. $50 \cdot (10 + 9)$

b. $50 \cdot 10 + 50 \cdot 9$



Solution

Sample responses:

a. 50 matches the length of the rectangle and $10 + 9$ matches the total width.

b. $50 \cdot 10$ matches the area of the larger rectangle on the left and $50 \cdot 9$ matches the area of the other rectangle on the right. The diagram shows you can decompose 19 into 10 and 9.

3 Student Task Statement

Complete each calculation using the distributive property.

$98 \cdot 24$	$21 \cdot 15$	$0.51 \cdot 40$
$(100 - 2) \cdot 24$	$(20 + 1) \cdot 15$	$(0.5 + 0.01) \cdot 40$

Solution

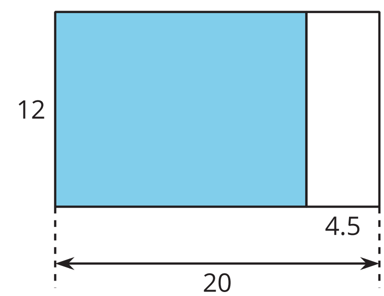
a. $(100 - 2) \cdot 24 = 2{,}400 - 48 = 2{,}352$

b. $(20 + 1) \cdot 15 = 300 + 15 = 315$

c. $(0.5 + 0.01) \cdot 40 = 20 + 0.4 = 20.4$

4 Student Task Statement

Select **all** the expressions that represent the area of the shaded rectangle on the left side of the figure.



A. $12(20 + 4.5)$

B. $20 \cdot 12 \cdot 15.5$

C. $15.5 \cdot 12$

D. $15.5 \cdot 12 + 4.5 \cdot 12$

E. $20 \cdot 12 - 4.5 \cdot 12$

F. $12(20 - 4.5)$

Solution

C, E, F

5 from Unit 6, Lesson 8

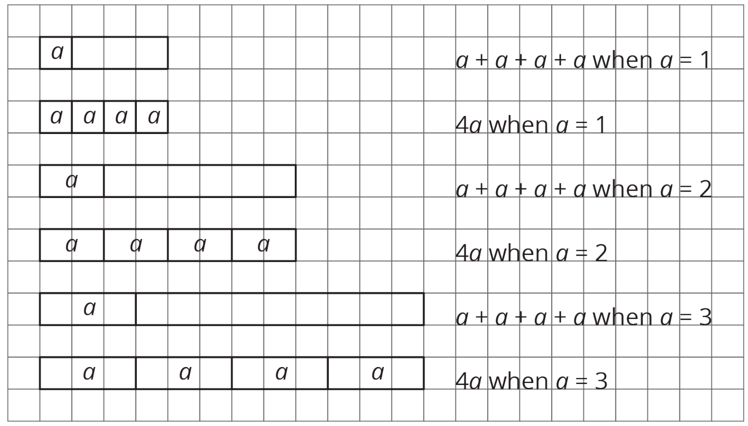
Student Task Statement

a. On graph paper, draw diagrams that represent $a + a + a + a$ and $4a$ when a is 1, 2, and 3. What do you notice?

b. Do $a + a + a + a$ and $4a$ have the same value for any value of a? Explain how you know.

Solution

a. Sample response: I notice that for each value of a, the diagrams for $a + a + a + a$ and $4a$ are the same length.



b. Yes. Sample reasoning: $4a$ can be rewritten as $a + a + a + a$, and this is true for any value of a. This can also be shown with a tape diagram.

6 from Unit 6, Lesson 6

Student Task Statement

120% of x is equal to 78.

a. Write an equation that shows the relationship of 120%, x, and 78.

b. Use your equation to find x. Show your reasoning.

Solution

a. $1.2x = 78$ (or equivalent)

b. $x = 65$. Sample reasoning: $78 \div 1.2 = 65$

7 from Unit 6, Lesson 7

Student Task Statement

Kiran's aunt is 17 years older than Kiran.

a. How old will Kiran's aunt be when Kiran is:
15 years old? 30 years old? x years old?

b. How old will Kiran be when his aunt is 60 years old?

Solution

a. 32 years old ($15 + 17 = 32$), 47 years old ($30 + 17 = 47$), $x + 17$ years old

b. 43 years old, since $x + 17 = 6$ is true when x is 43.