Unit 4, Lesson 4

Equations and Their Solutions

Let's recall what we know about solutions to equations.

4.1 What Is a Solution?

Cement is made by mixing water and concrete mix. The amount of water, w, added to the concrete mix can affect the strength of the material. One liter of water weighs 2.2 pounds.

The equation $2.2w + 80 = 86.05$ represents the relationship between these quantities.

1. What could the 80 represent in this situation?

2. Priya said that neither 2 nor 3 could be the solution to the equation. Explain why she is correct.

3. Find the solution to the equation.

4.2 Weekend Earnings

Jada has time on the weekends to earn some money. A local bookstore is looking for someone to help sort books and will pay $12.20 an hour. To get to and from the bookstore on a work day, however, Jada would have to spend $7.15 on bus fare.

1. Write an equation that represents Jada's take-home earnings in dollars, E, if she works at the bookstore for h hours.

2. One day, Jada takes home $90.45 after working h hours and after paying the bus fare. Write an equation to represent this situation.

3. Is 4 a solution to the last equation you wrote? What about 7?
   - If so, be prepared to explain how you know one or both of them are solutions.
   - If not, be prepared to explain why they are not solutions. Then, find the solution.

4. In this situation, what does the solution to the equation tell us?

Are you ready for more?

Jada has a second option to earn money—she could help some neighbors with errands and computer work for $11 an hour. After reconsidering her schedule, Jada realizes that she has about 9 hours available to work one day of the weekend.

Which option should she choose—sorting books at the bookstore or helping her neighbors? Explain your reasoning.

4.3 Carbon Dioxide Production

One gallon of gasoline produces about 20 pounds of carbon dioxide. One gallon of pure ethanol produces about 13 pounds of carbon. A car engine that can run on gasoline or ethanol produces 100 pounds of carbon dioxide from g gallons of gasoline and c gallons of ethanol.
The equation $20g + 13c = 100$ represents the relationship between these quantities.

1. Determine if each pair of values could be the number of gallons of gasoline and ethanol burned in the car engine. Be prepared to explain your reasoning.
    a. 5 gallons of gasoline and 7.7 gallons of ethanol
    b. 3.05 gallons of gasoline and 3 gallons of ethanol
    c. 2 gallons of gasoline and 4.5 gallons of ethanol
2. If the car engine burned 4 gallons of gasoline, how many gallons of ethanol were burned? Show your reasoning.
3. In this situation, what does a solution to the equation $20g + 13c = 100$ tell us? Give an example of a solution.

Lesson 4 Summary

An equation that contains only one unknown quantity or one quantity that can vary is called an *equation in one variable*.

For example, the equation $2\ell + 2w = 72$ represents the relationship between the length, ℓ, and the width, w, of a rectangle that has a perimeter of 72 units. If we know that the length is 15 units, we can rewrite the equation as:

$$2(15) + 2w = 72.$$

This is an equation in one variable, because w is the only quantity that we don't know. To solve this equation means to find a value of w that makes the equation true.

In this case, 21 is the solution because substituting 21 for w in the equation results in a true statement.

$$\begin{aligned} 2(15) + 2w &= 72 \\ 2(15) + 2(21) &= 72 \\ 30 + 42 &= 72 \\ 72 &= 72 \end{aligned}$$

An equation that contains two unknown quantities or two quantities that vary is called an *equation in two variables*. A solution to such an equation is *a pair* of numbers that makes the equation true.

Suppose Tyler spends \$45 on T-shirts and socks. A T-shirt costs \$10 and a pair of socks costs \$2.50. If t represents the number of T-shirts and p represents the number of pairs of socks that Tyler buys, we can can represent this situation with the equation:

$$10t + 2.50p = 45$$

This is an equation in two variables. More than one pair of values for t and p make the equation true.

$t = 3$ and $p = 6$

$$\begin{aligned} 10(3) + 2.50(6) &= 45 \\ 30 + 15 &= 45 \\ 45 &= 45 \end{aligned}$$

$t = 4$ and $p = 2$

$$\begin{aligned} 10(4) + 2.50(2) &= 45 \\ 40 + 5 &= 45 \\ 45 &= 45 \end{aligned}$$

$t = 2$ and $p = 10$

$$\begin{aligned} 10(2) + 2.50(10) &= 45 \\ 20 + 25 &= 45 \\ 45 &= 45 \end{aligned}$$

In this situation, one constraint is that the combined cost of shirts and socks must equal \$45. Solutions to the equation are pairs of t and p values that satisfy this constraint.

Combinations such as $t = 1$ and $p = 10$ or $t = 2$ and $p = 7$ are *not* solutions because they don’t meet the constraint. When these pairs of values are substituted into the equation, they result in statements that are false.