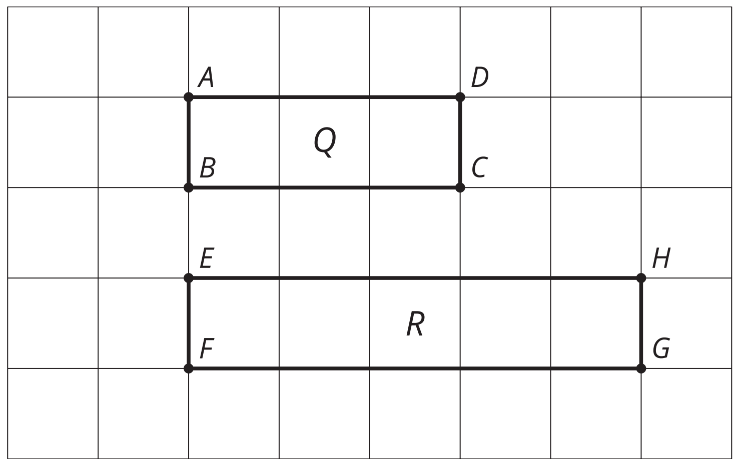
Are They All Similar?

Let's prove that figures are similar.

8.1 Stretched or Distorted? Rectangles

Are these rectangles similar? Explain your reasoning.



8.2 Faulty Logic

Tyler wrote a proof that all rectangles are similar. Make the image Tyler describes in each step in his proof. Which step makes a false assumption? Why is it false?

Step 1. Draw 2 rectangles. Label one $ABCD$ and the other $PQRS$.

Step 2. Translate rectangle $ABCD$ by the directed line segment from A to P. A' and P now coincide. The points coincide because that's how we defined our translation.

Step 3. Rotate rectangle $A'B'C'D'$, using A' as the center, so that D'' is along ray PS.

Step 4. Dilate rectangle $A''B''C''D''$, using center A'' and a scale factor of $\frac{PS}{AD}$. Segments $A'''D'''$ and PS now coincide. The segments coincide because A'' was the center of the rotation, so A'' and P don't move, and because D'' and S are on the same ray from A'', when we dilate D'' by the right scale factor, it will stay on ray PS but be the same distance from A'' as S is, so S and D''' will coincide.

Step 5. Because all angles of a rectangle are right angles, segment $A'''B'''$ now lies on ray PQ. This is because the rays are on the same side of PS and make the same angle with it. (If $A'''B'''$ and PQ don't coincide, reflect across PS so that the rays are on the same side of PS.)

Step 6. Dilate rectangle $A'''B'''C'''D'''$, using center A''' and a scale factor of $\frac{PQ}{AB}$. Segments $A''''B''''$ and PQ now coincide by the same reasoning as in step 4.

Step 7. Due to the symmetry of a rectangle, if 2 rectangles coincide on 2 sides, they must coincide on all sides.

8.3 Always? Prove It!

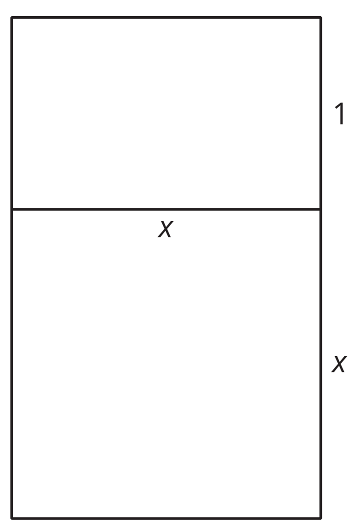
1. Decide if the statement is true. If it is true, write a proof. If it is not, provide a counterexample.
   "All circles are similar."

2. Select one of the conjectures from this list, and either prove that it is true or provide a counterexample.
   - All equilateral triangles are similar.
   - All isosceles triangles are similar.
   - All right triangles are similar.

Are you ready for more?

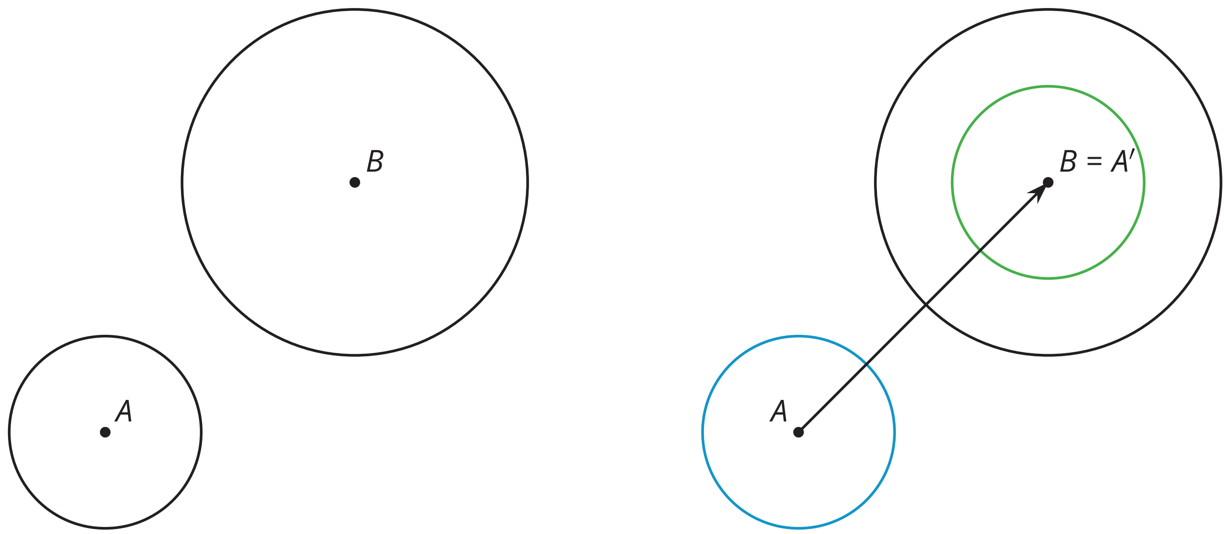
A golden rectangle is a rectangle that can be divided into a square and a rectangle similar to the original rectangle. In this image there are two golden rectangles: the entire image and the rectangle on top.

Solve for x so that the golden rectangles are similar. Explain or show your reasoning.



Lesson 8 Summary

One figure is similar to another if there is a sequence of rigid motions and dilations that takes the first figure so that it fits exactly over the second. Consider any two circles, one centered at A, with a radius of length r, and the other centered at B, with a radius of length R. Translate the circle centered at A along directed line segment AB.



Now dilate the image using center B and a scale factor of $\frac{R}{r}$. The circles should now coincide, proving that these circles are similar. Because any two circles could be described by the initial statement, this proves that any two circles are similar.

We can also show that all equilateral triangles are similar. Because we are talking about triangles, we can use the theorem that having all pairs of corresponding angles congruent and all pairs of corresponding side lengths in the same proportion is enough to prove that the triangles are similar. All the pairs of corresponding angles are congruent because all the angles in any equilateral triangle measure 60°. All the pairs of corresponding side lengths must be in the same proportion, because within each triangle, all the sides are congruent. Therefore, whatever scale factor works for one pair of sides will work for all 3 pairs of corresponding sides. If all pairs of corresponding sides are in the same proportion and all pairs of corresponding angles are congruent, then all equilateral triangles are similar.