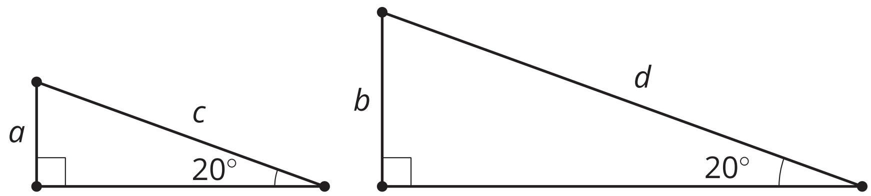
Unit 3, Lesson 4

Ratios in Right Triangles

Let's investigate ratios in the side lengths of right triangles.

4.1 Ratio Rivalry



Consider $\frac{a}{c}$ and $\frac{b}{d}$. Determine which ratio is greater or whether they are equal. Explain how you know.

4.2 Tons of Triangles

Your teacher will give you some angles.

1. Draw several right triangles using the angles you receive.
2. Precisely measure the side lengths of the triangles.
3. Complete the tables by computing 3 quotients for each of the acute angles in each triangle:
    a. the length of the leg adjacent to your angle divided by the length of the hypotenuse
    b. the length of the leg opposite from your angle divided by the length of the hypotenuse
    c. the length of the leg opposite from your angle divided by the length of the leg adjacent to your angle
4. Find the mean of each column in your table.

4.3 Tons of Ratios

1. Compare the row for 20 degrees and the row for 70 degrees in the Right Triangle Table. What is the same? What is different?

2. The row for 55 degrees is given here. Complete the row for 35 degrees.

angle	adjacent leg ÷ hypotenuse	opposite leg ÷ hypotenuse	opposite leg ÷ adjacent leg
35°			
55°	0.574	0.819	1.428

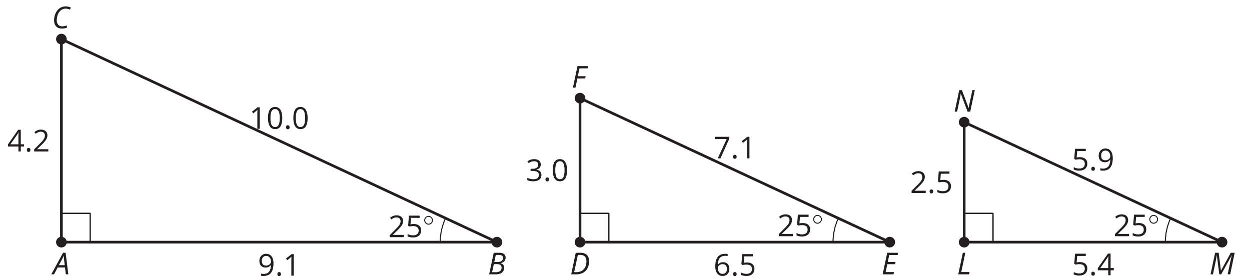
3. Tyler measured a right triangle and found a value of 0.839 for the adjacent leg divided by the hypotenuse. Estimate the angle Tyler used to build his triangle.

Are you ready for more?

1. What is the range for the possible ratios of each of the following ratios?
   a. adjacent leg ÷ hypotenuse
   b. opposite leg ÷ hypotenuse
   c. opposite leg ÷ adjacent leg
2. What would the triangle look like if the “adjacent leg ÷ hypotenuse” ratio was 1? Greater than 1?

Lesson 4 Summary

All right triangles that contain the same acute angles are similar to each other. This means that the ratios of corresponding side lengths are equal for all right triangles with the same acute angles.



These triangles are all similar by the Angle-Angle Triangle Similarity Theorem. Focusing on the 25-degree angles, we see that all 3 triangles have ratios of the adjacent leg to the hypotenuse of approximately 0.91.

Because all right triangles with the same acute angle measures have the same ratios, we can look for patterns that will help us solve problems. The values in this Right Triangle Table come from measuring triangles very precisely and then finding ratios of the sides. The results in the table are written out to the thousandths place, which is more accurate than any ratio we could calculate when measuring by hand.

angle	adjacent leg ÷ hypotenuse	opposite leg ÷ hypotenuse	opposite leg ÷ adjacent leg
25°	0.906	0.423	0.466
35°	0.819	0.574	0.700
45°	0.707	0.707	1.000
55°	0.574	0.819	1.428
65°	0.423	0.906	2.145

Some ratios in this table are repeated. Notice that the rows for 25 degrees and 65 degrees have two of the same ratios. What is special about 25 and 65? They are complementary angles, that is, two angles sum to 90 degrees. This seems to be true for other complementary angles. Notice that $35 + 55 = 90$ and those rows both have 0.819 as a ratio.

