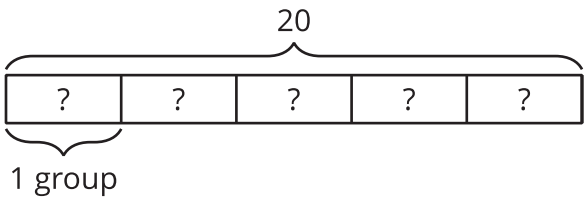
How Much in Each Group? (Part 1)

Let's look at division problems that help us find the size of one group.

5.1 Inventing a Situation

Here is a tape diagram.



1. Think of a situation with a question that the diagram can represent. Describe the situation and the question.

2. Trade descriptions with your partner. Answer your partner's question.

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5.2 How Much in One Batch?

For each question:

- Draw a diagram that represents the situation.
- Write a multiplication equation and a division equation that represent the situation.
- Answer the question.

1. To make 4 batches of pink paint, 6 teaspoons of red paint are needed. How many teaspoons of red paint are needed for 1 batch?

2. To make $\frac{1}{2}$ batch of play clay, $\frac{5}{4}$ cups of flour are needed. How many cups of flour are needed for 1 batch?

3. Two tablespoons of cornstarch make $\frac{2}{3}$ batch of glue. How many tablespoons of cornstarch are needed to make 1 batch?

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5.3 How Much in One Container?

Here are three diagrams and three descriptions that represent situations about filling containers of water.

Diagram A

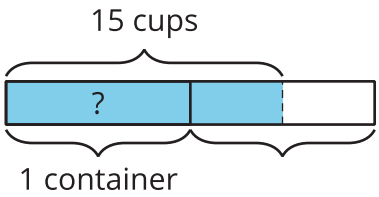


Diagram B

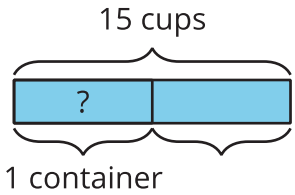
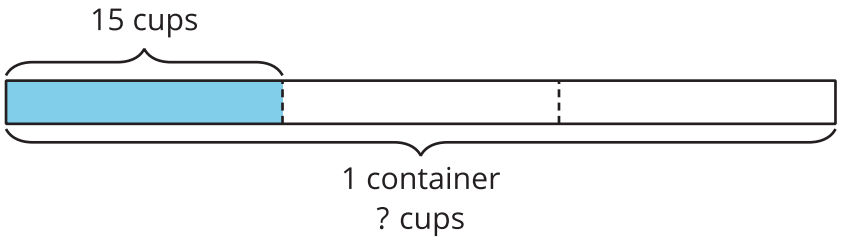


Diagram C



- Tyler filled 2 equal-size bottles with 15 cups of water. How much water was in each bottle?
- Kiran filled $1\frac{1}{2}$ pitchers with 15 cups of water. How much water was in the full pitcher?
- Priya needed 15 cups of water to fill $\frac{1}{3}$ pail. How much water is needed to fill 1 pail?

1. Match each situation to a diagram. Be prepared to explain how you know.
   a. Tyler:
   b. Kiran:
   c. Priya:
2. Choose one situation. Write a multiplication equation and a division equation to represent the situation. Then answer the question.

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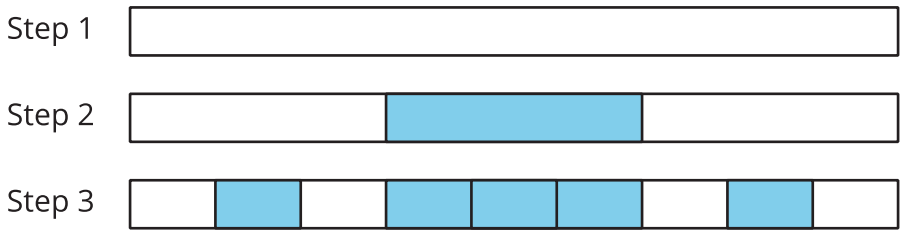
Are you ready for more?

To make a Cantor ternary set:

- Start with a tape diagram of length 1 unit. This is Step 1.
- Color in the middle third of the tape diagram. This is Step 2.
- Do the same to each remaining segment that is not colored in. This is Step 3.
- Keep repeating this process.



1. How much of the diagram is colored in after Step 2? Step 3? Step 5?

2. Elena thinks that if we could continue this process forever, eventually the entire diagram would be colored. Do you agree with her? Explain your reasoning.

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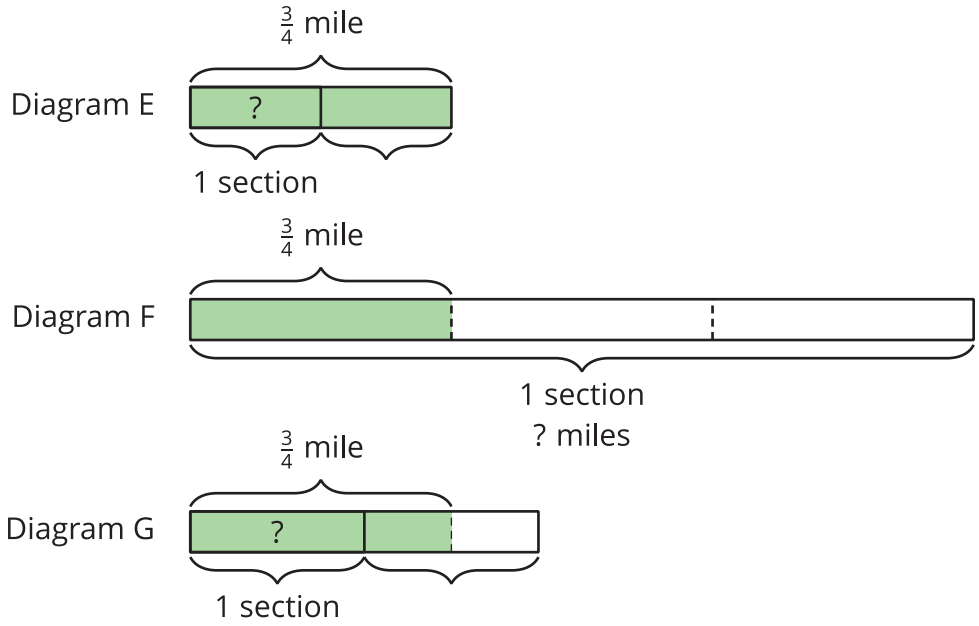
5.4 How Long Is a Section?

Here are three diagrams and three descriptions that represent situations about sections of highways.



- Priya's class has adopted two equal sections of a highway to keep clean. The combined length is $\frac{3}{4}$ of a mile. How long is each section?
- Lin's class has also adopted some sections of a highway to keep clean. If $1\frac{1}{2}$ sections are $\frac{3}{4}$ mile long, how long is each section?
- A high school adopted a section of highway to keep clean. If $\frac{1}{3}$ of the section is $\frac{3}{4}$ mile long, how long is the section?

1. Match each situation to a diagram. Be prepared to explain how you know.

   a. Priya's class:

   b. Lin's class:

   c. High school:

2. Choose one situation. Write a multiplication equation and a division equation to represent the situation. Then answer the question.

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Lesson 5 Summary

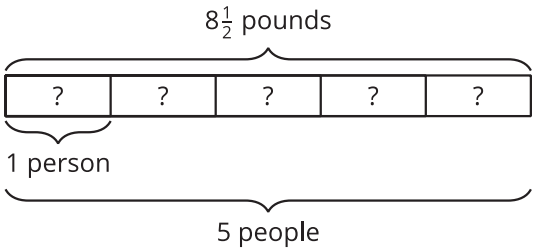
Sometimes we know the amount for *multiple* groups, but we don't know how much is in one group. We can use division to find out.

For example, if 5 people share $8\frac{1}{2}$ pounds of cherries equally, how many pounds of cherries does each person get?

We can represent this situation with a multiplication equation, a division equation, and a diagram:

$$5 \cdot ? = 8\frac{1}{2}$$

$$8\frac{1}{2} \div 5 = ?$$



$8\frac{1}{2} \div 5$ can be written as $\frac{17}{2} \div 5$. Dividing by 5 is equivalent to multiplying by $\frac{1}{5}$, and $\frac{17}{2} \cdot \frac{1}{5} = \frac{17}{10}$. Each person gets $1\frac{7}{10}$ pounds.

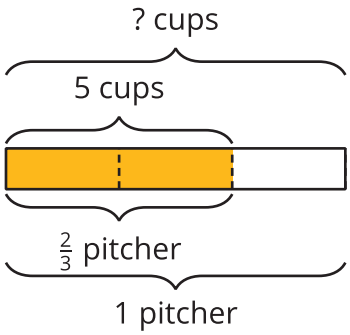
Other times, we know the amount in *a fraction* of a group, but we don't know the size of 1 group. We can also use division to find out.

For example, Jada poured 5 cups of iced tea in a pitcher and filled $\frac{2}{3}$ of the pitcher. How many cups of iced tea fill the entire pitcher?

Here are equations and a diagram that can represent this situation:

$$\frac{2}{3} \cdot ? = 5$$

$$5 \div \frac{2}{3} = ?$$



If $\frac{2}{3}$ of a pitcher is 5 cups, then $\frac{1}{3}$ of a pitcher is half of 5, which is $\frac{5}{2}$. Because there are 3 thirds in 1 whole, there would be $(3 \cdot \frac{5}{2})$ or $\frac{15}{2}$ cups in one whole pitcher. We can check our answer by multiplying: $\frac{2}{3} \cdot \frac{15}{2} = \frac{30}{6}$, and $\frac{30}{6} = 5$.

Notice that in the first example, the number of groups is greater than 1 (5 people) and in the second, the number of groups is less than 1 ($\frac{2}{3}$ of a pitcher), but the division and multiplication equations for both situations have the same structure.

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