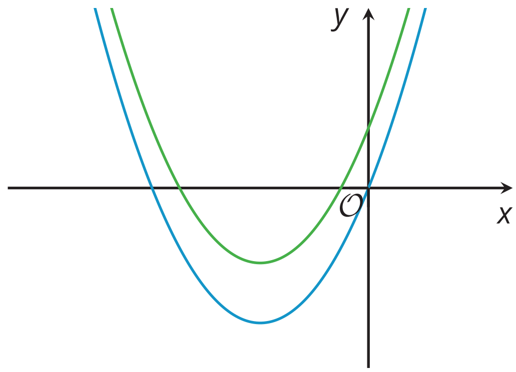
Changing the Vertex

Let’s write new quadratic equations in vertex form to produce certain graphs.

17.1 Graphs of Two Functions



Here are graphs representing two functions, f and g, given by $f(x) = x(x + 6)$ and $g(x) = x(x + 6) + 4$.

1. Which graph represents each function? Explain how you know.

2. Where does the graph of f meet the x-axis? Explain how you know.

17.2 Shifting the Graph

1. How would you change the equation $y = x^2$ so that the vertex of the graph of the new equation is located at the following coordinates and so that the graph opens as described?
   a. $(0, 11)$, opens upward

   b. $(7, 11)$, opens upward

   c. $(7, \text{-}3)$, opens downward

2. Use graphing technology to verify your predictions. Adjust your equations if necessary.
3. Kiran graphed the equation $y = x^2 + 1$ and noticed that the vertex is at $(0, 1)$. He changed the equation to $y = (x - 3)^2 + 1$ and saw that the graph shifted 3 units to the right and the vertex is now at $(3, 1)$.

   Next, he graphed the equation $y = x^2 + 2x + 1$ and observed that the vertex is at $(\text{-}1, 0)$. Kiran thought, "If I change the squared term x^2 to $(x - 5)^2$, the graph of $y = (x - 5)^2 + 2x + 1$ will be 5 units to the right and the vertex will be at $(4, 0)$."

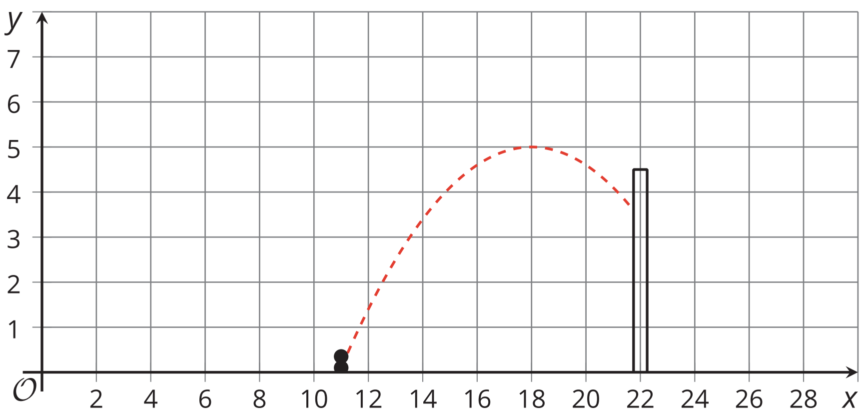
   Do you agree with Kiran? Explain or show your reasoning.

17.3 A Peanut Jumping over a Wall

Mai is learning to create computer animation by programming. In one part of her animation, she uses a quadratic function to model the path of the main character, an animated peanut, jumping over a wall.

Mai uses the equation $y = -0.1(x - h)^2 + k$ to represent the path of the jump. y represents the height of the peanut as a function of the horizontal distance, x, that it travels.

On the screen, the base of the wall is located at $(22, 0)$, with the top of the wall at $(22, 4.5)$. The dashed curve in the picture shows the graph of 1 equation that Mai tried, where the peanut fails to make it over the wall.



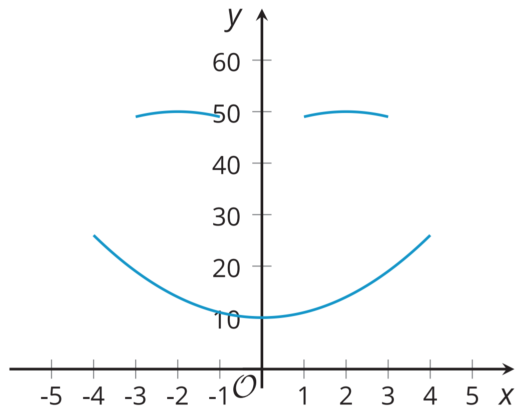
1. What are the values of h and k in this equation?

2. Starting with Mai's equation, choose values for h and k that will guarantee that the peanut stays on the screen but also makes it over the wall. Be prepared to explain your reasoning.

17.4 Smiley Face

Do you see 2 “eyes” and a smiling “mouth” on the graph? The 3 arcs on the graph all represent quadratic functions that were initially defined by $y = x^2$, but whose equations were later modified.

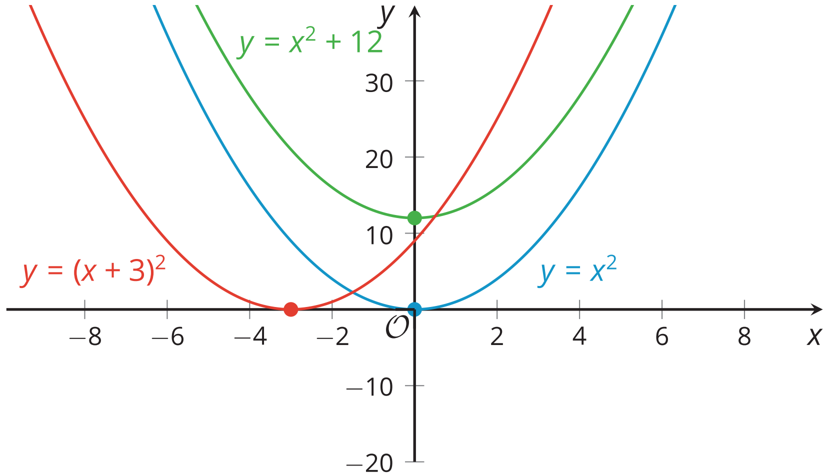
1. Write equations to represent each curve in the smiley face.



2. What domain is used for each function to create this graph?

Lesson 17 Summary

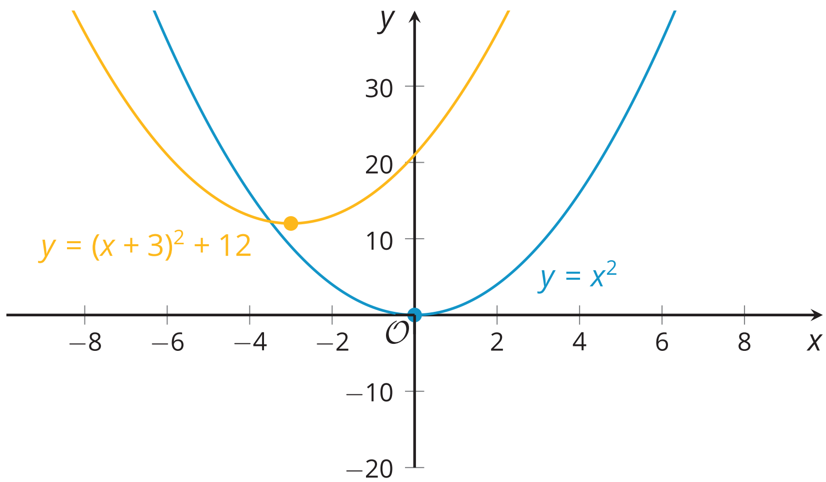
The graphs of $y = x^2$, $y = x^2 + 12$ and $y = (x + 3)^2$ all have the same shape but their locations are different. The graph that represents $y = x^2$ has its vertex at $(0, 0)$.



Notice that adding 12 to x^2 raises the graph by 12 units, so the vertex of that graph is at $(0, 12)$. Replacing x^2 with $(x + 3)^2$ shifts the graph 3 units to the left, so the vertex is now at $(-3, 0)$.

We can also shift a graph both horizontally and vertically.

The graph that represents $y = (x + 3)^2 + 12$ will have the same shape as $y = x^2$ but it will be shifted 12 units up and 3 units to the left. Its vertex is at $(-3, 12)$.



The graph representing the equation $y = -(x + 3)^2 + 12$ has the same vertex at $(-3, 12)$, but because the squared term $(x + 3)^2$ is multiplied by a negative number, the graph is flipped over horizontally, so that it opens downward.