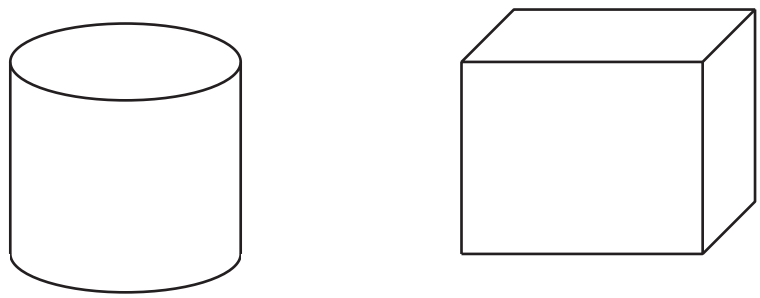
Cylinder Volumes

Let’s analyze cylinder volumes.

9.1 The Same but Different

Here are two solids.

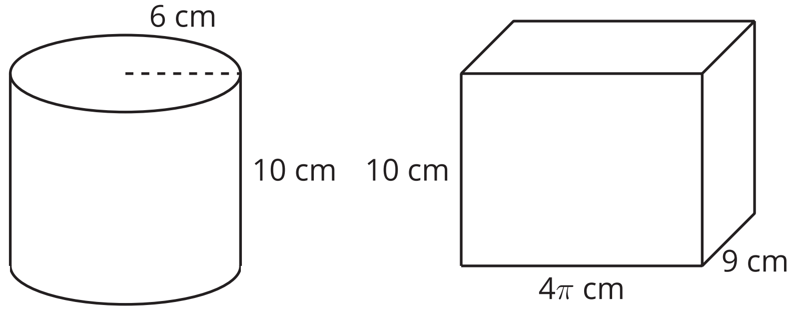


1. What information would you need to calculate the volume of each solid?

2. What is the same and different about how you would find the volume of each solid?

9.2 Water Transfer

Here are two containers. All measurements are in centimeters.



1. Suppose the prism contains water that reaches a height of 1 cm.
   a. Draw a representation of this situation.

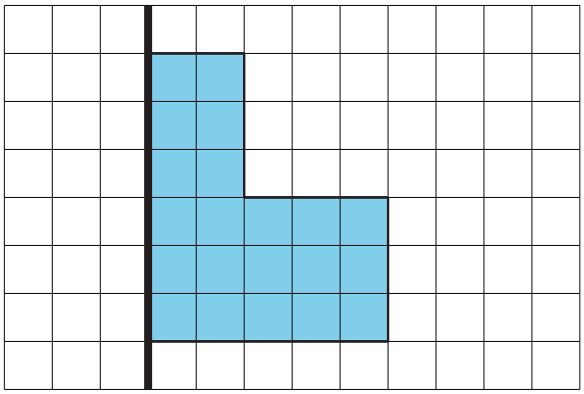
   b. The water is poured from the prism into the cylinder. What is the height of the water in the cylinder? Explain your reasoning.

2. Suppose the prism contained water that reached a height of 3 cm instead of 1 cm. If the water were poured into the empty cylinder, what would the height of the water in the cylinder be?

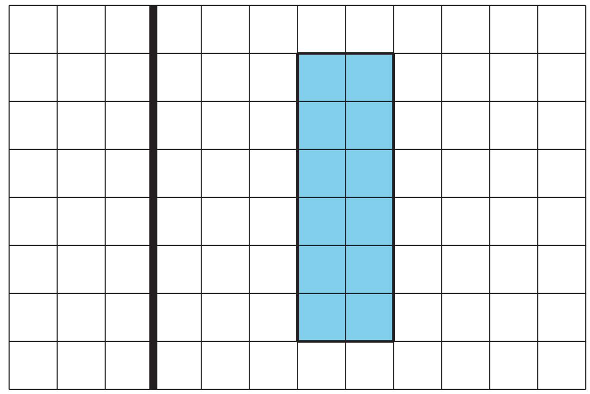
9.3 Revisiting Rotation

Suppose each two-dimensional figure is rotated around the vertical axis shown. Each small square in the grid represents 1 square centimeter.

A



B



For each solid:

1. Either sketch or describe in words the three-dimensional solid that would form.

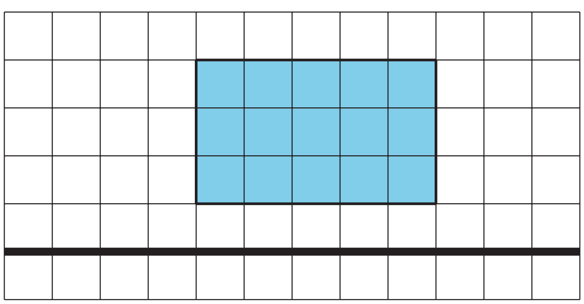
2. Find the solid's volume.

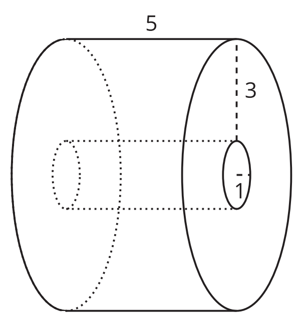
Are you ready for more?

1. Given a cylinder with a radius of r and a height of h, write an expression for the volume if each of these changes were made.
   a. The height is tripled to $3h$ and the radius remains r.
   b. The radius is tripled to $3r$ and the height remains h.
2. Given a cube of side length s, write an expression for the volume if the side length were tripled to $3s$.
3. Which change affected the shape's volume the most? The least? Explain or show your reasoning.

Lesson 9 Summary

Cylinder and prism volumes can be found by multiplying the area of the figure's base by its height. The formula $V = Bh$, where V represents volume, B is the area of the base, and h is height, captures this concept. Consider the solid formed by rotating this rectangle around the horizontal axis shown. The result is a hollow cylinder of height 5 units with inner radius 1 unit and outer radius 4 units.





To calculate the volume of the outer cylinder, start by finding the area of the circular base. The circle's radius measures 4 units, so its area is 16π square units because $\pi(4)^2 = 16\pi$. Multiply that by the cylinder's height of 5 units to get 80π cubic units.

For the inner cylinder, the area of the base is π square units, because $\pi(1)^2 = \pi$. The volume is therefore 5π cubic units. Now subtract the volume of the inner, hollow part from the volume of the outer cylinder to get the volume of the solid: 75π cubic units because $80\pi - 5\pi = 75\pi$.