# The Number $e$

Let's learn about the number $e$.

## 12.1 Matching Situations and Equations

Match each equation to a situation it represents. Be prepared to explain how you know. Not all equations have a match.

$f(t) = 400 \cdot (0.5)^{0.1t}$

$g(t) = 400 \cdot (1.25)^{0.1t}$

$h(t) = 400 \cdot (0.75)^{0.1t}$

$j(t) = 400 \cdot (2)^{10t}$

$k(t) = 400 \cdot (2)^{0.1t}$

1. A scientist begins an experiment with 400 bacteria in a petri dish. The population doubles every 10 hours. The function gives the number of bacteria $t$ hours since the experiment began.

2. An erosion model begins with 400 kilograms of sand. The amount of sand decreases by 25% every 10 days after the experiment begins. The function gives the amount of sand left $t$ days after the experiment begins.

3. The half-life of a radioactive element is 10 years. There are 400 g of the element in a sample when it is first studied. The function gives the amount of the element remaining $t$ years later.

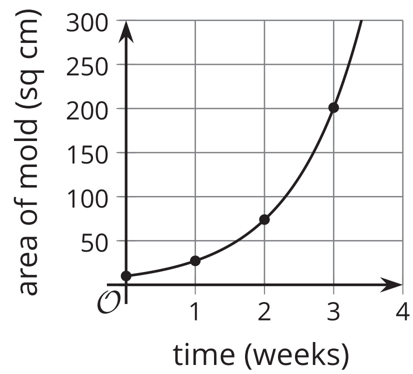
4. In a lake, the population of a species of fish is 400. The population is expected to grow by 25% in the next decade. The function gives the number of fish in the lake $t$ years after it was 400.

## 12.2 Notice and Wonder: Moldy Growth

A spot of mold is found on a basement wall. Its area is about 10 square centimeters. Here are three representations of a function that models how the mold is growing.

$$a(t) = 10 \cdot e^t$$




| time (weeks) | area of mold (sq cm) |
|---|---|
| 0 | 10 |
| 1 | 27 |
| 2 | 74 |
| 3 | 201 |
| 4 | 546 |

What do you notice? What do you wonder?

## 12.3 $(1 + \text{tiny})^{\text{huge}}$

1. Here are some functions. For each function, describe, in words, the outputs for very tiny, positive values of $x$ and for very large values of $x$.
   $a(x) = 1^x$

   $b(x) = -x$

   $d(x) = \frac{1}{x}$

   $f(x) = \left(\frac{1}{x}\right)^x$

   $g(x) = \left(1 + \frac{1}{x}\right)^x$

   $h(x) = e^x$

   $k(x) = 1 + x$

2. Remember that $e \approx 2.718$. What does the function $g$ have to do with the number $e$?

3. What do you notice about the relationship between $h$ and $k$ for very small, positive values of $x$?

## Are you ready for more?

Complete the table to show the value of each expression to the nearest hundred-thousandth. Two entries have already been completed as an example.

| $x$ | $2^x$ | $e^x$ | $3^x$ | $1+x$ |
|---|---|---|---|---|
| 0.1 | 1.07177 | 1.10517 | | |
| 0.01 | | | | |
| 0.001 | | | | |
| 0.0001 | | | | |

What do you notice about the values in the table? Which columns are the closest in value?

## Lesson 12 Summary

Scientists, economists, engineers, and others often use the number $e$ in their mathematical models. What is $e$?

$\boldsymbol{e}$ is an important constant in mathematics, just like the constant $\pi$, which is important in geometry. The value of $e$ is approximately 2.718. Like $\pi$, the number $e$ is irrational, so it can't be represented as a fraction, and its decimal representation never repeats or terminates. The number is named after the 18th-century mathematician Leonhard Euler and is sometimes called *Euler's number*.

$e$ has many useful properties and it arises in situations involving exponential growth or decay, so $e$ often appears in exponential functions.