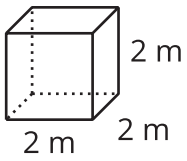
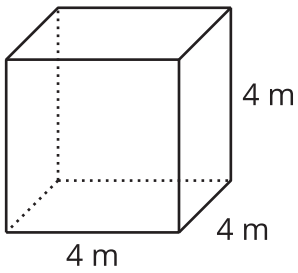
Scaling Solids

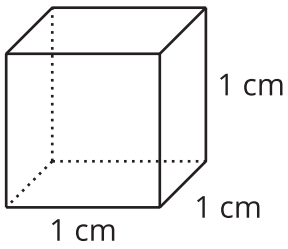
Let's see how the surface area and volume of solids change when we dilate those solids.

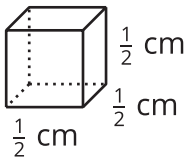
7.1 Math Talk: Cube Volumes

Find the volume of each cube mentally.

- 

- 

- 

- 

7.2 How Do Surface Area and Volume Change with Scaling?

Complete the table with the surface area and volume of each dilated cube. Then write expressions that give the surface area and volume when the scale factor is k.

Use the unit cubes to help, if you choose.

scale factor	surface area in square units	volume in cubic units
1		
2		
3		
4		
k		

7.3 Scaling All Solids

Clare says, "We know that if we dilate a cube by a factor of k, the cube's volume is multiplied by k^3. It seems like that must apply to *all* solids, but I'm not sure how to prove it."

Elena says, "Earlier in the unit, we showed that we can cover any two-dimensional shape with rectangles, so the property that area changes by k^2 when we dilate a figure by k applies to all shapes, not just rectangles. Can we do something similar here?"

1. Use Elena's line of reasoning to argue that for *any* solid, if it's dilated by a factor of k, the volume is multiplied by k^3.

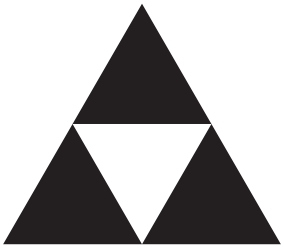
2. Suppose a triangular prism has surface area 84 square centimeters and volume 36 cubic centimeters. The prism is dilated by scale factor $k = 4$. Calculate the surface area and volume of the dilated prism.

Are you ready for more?

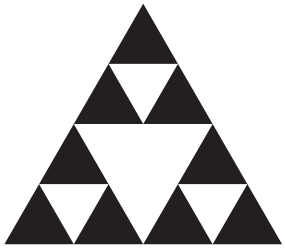
The image shows a figure called a Sierpinski (share-PEEN-skee) triangle. It's formed by starting with an equilateral triangle, then repeatedly removing equilateral triangles created by joining the midpoints of the existing triangle's sides. The first few stages are shown.

stage 1

stage 2



stage 3



If we continue this process of removal forever, we are left with some points that never get removed from the triangle. The remaining points are what we call a Sierpinski triangle. At any given stage, the triangle at the top of the figure is a scaled copy of the triangle at the previous stage.

stage 4

top triangle

For the completed Sierpinski triangle, though, the top triangle is a scaled copy not of the previous stage, but of the full Sierpinski triangle.

1. For the completed figure, what scale factor takes a Sierpinski triangle to its scaled copy at the top?

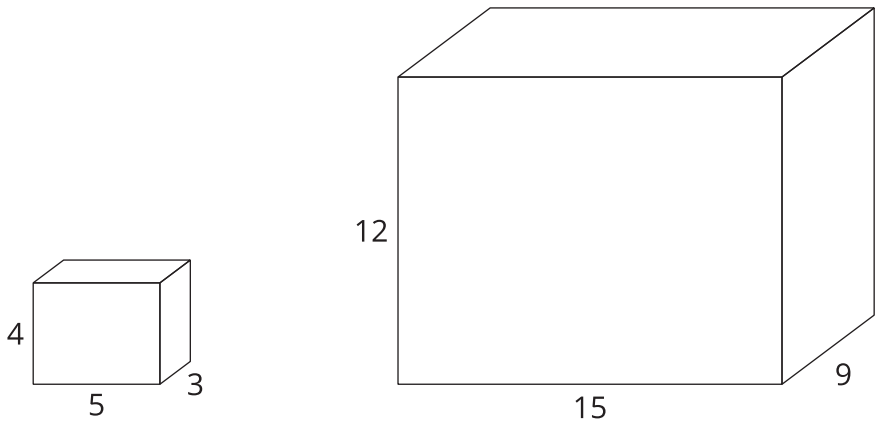
2. Based on the scale factor, what fraction of the original shaded region should be contained in the scaled copy at the top?

3. The scaled copy at the top actually contains $\frac{1}{3}$ of the shading of the original. Provide reasoning that shows that this is true.

Lesson 7 Summary

In earlier activities, we saw that if we dilate a two-dimensional shape, the area of the dilated shape is the area of the original shape multiplied by the square of the scale factor. What happens when we dilate three-dimensional solids?

Here is a rectangular prism with side lengths 3, 4, and 5 units. When we dilate the prism using a scale factor of 3, the lengths become 9, 12, and 15 units.



Because these are three-dimensional shapes, we can look at both volume and surface area. The volume of the original prism is 60 cubic units because $3 \cdot 4 \cdot 5 = 60$. The volume of the dilated prism is 1,620 cubic units because $9 \cdot 12 \cdot 15 = 1{,}620$. The volume became 27 times larger! Why? Because the side lengths tripled, when we calculated the volume we were really finding $(3 \cdot 3) \cdot (4 \cdot 3) \cdot (5 \cdot 3) = (3 \cdot 4 \cdot 5) \cdot 3^3$. The volume was multiplied by the *cube* of the scale factor, or by $3^3 = 27$.

Now let's look at surface area. How will the surface area change when the prism is dilated?

In the original prism, 2 faces have area 12 square units, 2 have area 20 square units, and 2 have area 15 square units for a total surface area of 94 square units. The corresponding faces of the dilated prism have areas 108, 180, and 135 square units. The surface area of the new prism totals 846 square units, 9 times that of the original. Just like the area of two-dimensional shapes, the surface area of the prism changed by the square of the scale factor.

In general, when we dilate any three-dimensional solid by a scale factor of k, the surface area is multiplied by k^2, and the volume is multiplied by k^3.